Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/15-1.1.1.4-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [159]. This is test number [15].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.37 (158)	0.63 (1)
Mathematica	97.48 (155)	2.52 (4)
Maple	80.50 (128)	19.50 (31)
Fricas	41.51 (66)	58.49 (93)
Mupad	30.82 (49)	69.18 (110)
Giac	26.42 (42)	73.58 (117)
Maxima	24.53 (39)	75.47 (120)
Sympy	20.75 (33)	79.25 (126)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

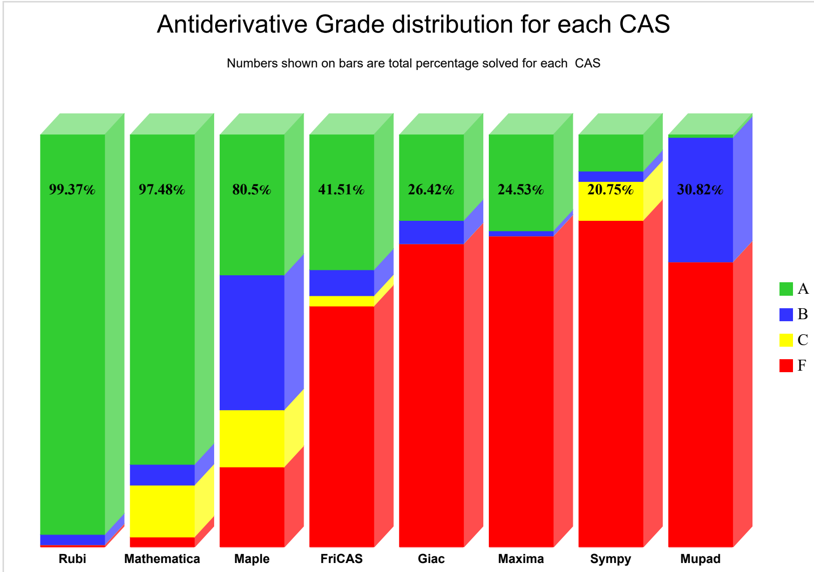
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

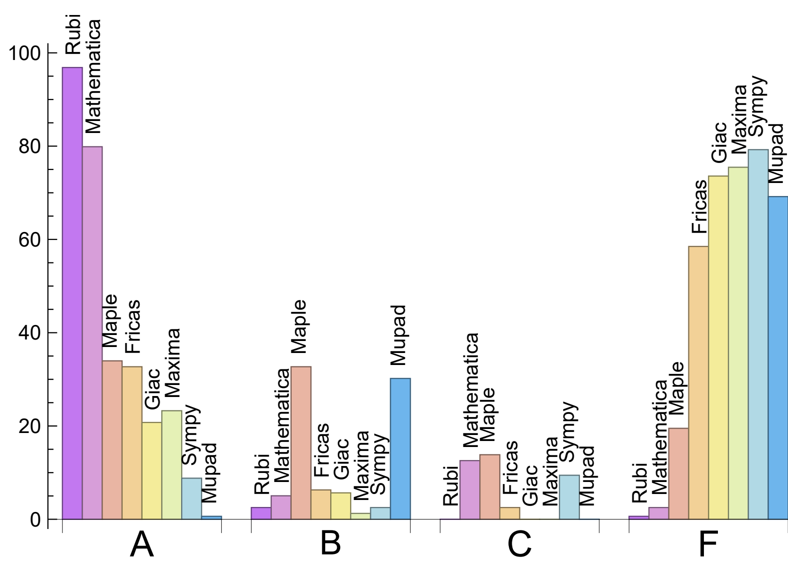
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.86	2.52	0.00	0.63
Mathematica	79.87	5.03	12.58	2.52
Maple	33.96	32.70	13.84	19.50
Fricas	32.70	6.29	2.52	58.49
Maxima	23.27	1.26	0.00	75.47
Giac	20.75	5.66	0.00	73.58
Sympy	8.81	2.52	9.43	79.25
Mupad	N/A	30.19	0.00	69.18

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	31	100.00 %	0.00 %	0.00 %
Fricas	93	84.95 %	15.05 %	0.00 %
Giac	117	94.87 %	1.71 %	3.42 %
Maxima	120	95.00 %	0.00 %	5.00 %
Sympy	126	53.97 %	30.16 %	15.87 %
Mupad	110	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

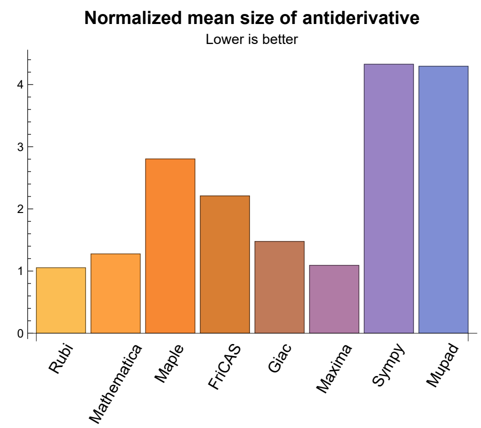
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

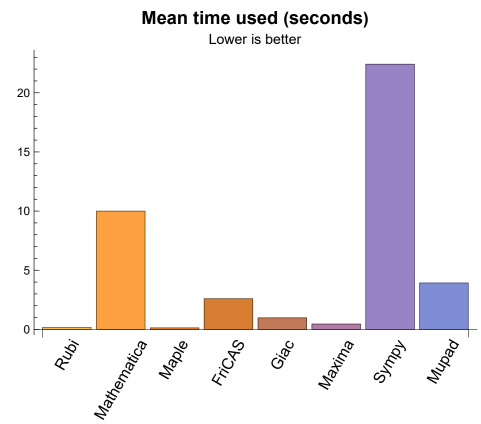
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	224.08	1.05	191.50	1.00
Mathematica	10.00	387.90	1.28	165.00	0.95
Maple	0.13	1061.84	2.80	182.50	1.65
Maxima	0.45	101.92	1.09	84.00	1.08
Fricas	2.59	374.89	2.21	73.00	1.17
Sympy	22.41	491.91	4.33	167.00	2.59
Giac	0.98	165.14	1.48	108.50	1.35
Mupad	3.92	572.84	4.30	244.00	2.38

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{143}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{107\}$

Mathematica $\{77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 93, 94, 95, 96, 99, 101, 102, 107, 111, 132, 133, 146, 154, 155, 156, 157, 158, 159\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

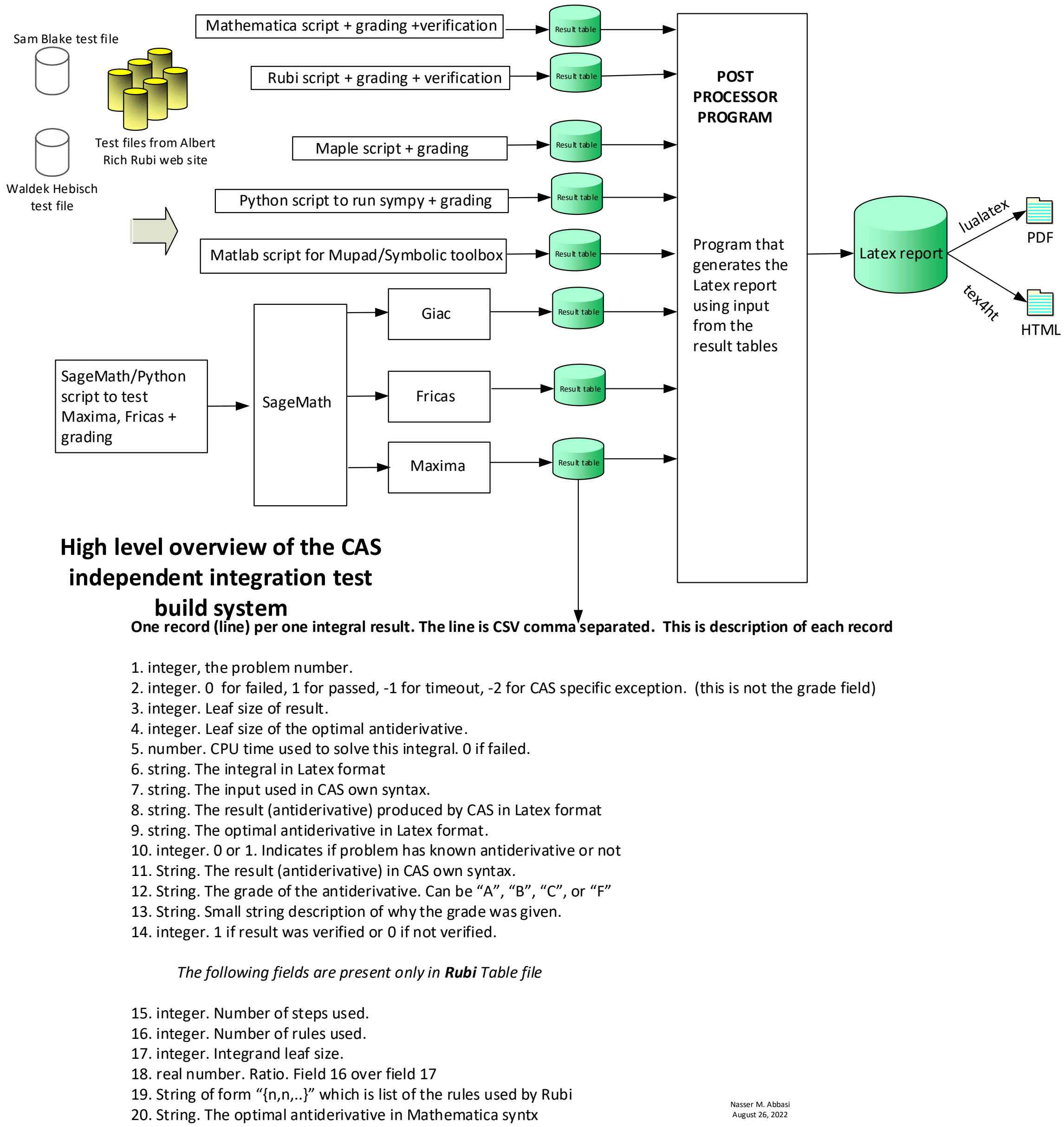
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 158, 159 }

B grade: { 97, 155, 156, 157 }

C grade: { }

F grade: { 111 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 100, 101, 102, 103, 104, 105, 106, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

B grade: { 25, 26, 54, 97, 99, 107, 108, 111 }

C grade: { 33, 34, 43, 58, 59, 68, 69, 70, 71, 72, 73, 74, 75, 76, 120, 124, 125, 126, 132, 133 }

F grade: { 139, 140, 141, 144 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 31, 32, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 60, 61, 62, 63, 64, 65, 68, 70, 77, 88, 95, 96, 102, 104, 109, 143 }

B grade: { 33, 34, 40, 41, 42, 43, 49, 50, 56, 57, 58, 59, 66, 67, 69, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 105, 106, 107, 108, 110, 111, 119, 130, 131, 134, 135 }

C grade: { 22, 23, 24, 25, 26, 27, 28, 29, 30, 97, 103, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

F grade: { 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 143, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159 }

B grade: { 119, 155 }

C grade: { }

F grade: { 12, 13, 14, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 44, 45, 46, 47, 51, 52, 53, 60, 61, 62, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

B grade: { 13, 14, 20, 21, 26, 119, 130, 131, 134, 135 }

C grade: { 33, 34, 68, 69 }

F grade: { 5, 39, 40, 41, 42, 43, 48, 49, 50, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.6 Sympy

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19 }

B grade: { 3, 13, 20, 119 }

C grade: { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 151, 152, 156, 157 }

F grade: { 4, 5, 14, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 154, 155, 158, 159 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 143, 149, 150, 154, 155, 156, 157 }

B grade: { 5, 26, 27, 28, 29, 30, 119, 158, 159 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 151, 152, 153 }

2.1.8 Mupad

A grade: { 143 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 119, 130, 131, 134, 135, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	109	114	118	148	150	115
N.S.	1	1.00	1.00	0.97	1.02	1.05	1.32	1.34	1.03
time (sec)	N/A	0.103	0.035	0.007	0.297	0.908	0.016	0.584	2.383

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	194	168	183	146	208	174
N.S.	1	1.00	0.98	1.54	1.33	1.45	1.16	1.65	1.38
time (sec)	N/A	0.138	0.064	0.099	0.307	0.842	0.277	0.643	2.537

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	85	102	109	125	507	112	105
N.S.	1	1.00	1.01	1.21	1.30	1.49	6.04	1.33	1.25
time (sec)	N/A	0.061	0.046	0.114	0.297	1.117	196.579	0.554	2.970

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	102	108	140	163	0	162	127
N.S.	1	1.00	0.94	1.00	1.30	1.51	0.00	1.50	1.18
time (sec)	N/A	0.079	0.054	0.138	0.283	109.750	0.000	0.775	4.168

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	164	321	0	0	363	317
N.S.	1	1.00	1.01	1.01	1.97	0.00	0.00	2.23	1.94
time (sec)	N/A	0.146	0.145	0.210	0.295	0.000	0.000	0.604	6.622

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.008	0.005	0.106	0.321	1.266	0.043	0.581	0.076

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	34	34	53	32	31	29
N.S.	1	1.00	0.77	0.79	0.79	1.23	0.74	0.72	0.67
time (sec)	N/A	0.024	0.020	0.118	0.306	1.693	0.057	0.760	0.123

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	197	301	246	643	274	338	413
N.S.	1	1.00	0.87	1.33	1.08	2.83	1.21	1.49	1.82
time (sec)	N/A	0.176	0.319	0.104	0.530	1.868	15.113	0.629	0.161

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	176	156	403	167	201	263
N.S.	1	1.00	0.90	1.21	1.07	2.76	1.14	1.38	1.80
time (sec)	N/A	0.065	0.207	0.104	0.504	1.067	11.051	0.711	2.621

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	81	89	94	217	92	105	136
N.S.	1	1.00	1.05	1.16	1.22	2.82	1.19	1.36	1.77
time (sec)	N/A	0.018	0.114	0.099	0.476	1.437	10.997	1.220	0.091

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	62	115	54	57	45
N.S.	1	1.00	0.98	0.85	1.15	2.13	1.00	1.06	0.83
time (sec)	N/A	0.012	0.051	0.119	0.562	1.698	2.522	1.082	0.072

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	103	0	458	97	112	2368
N.S.	1	1.00	1.00	1.02	0.00	4.53	0.96	1.11	23.45
time (sec)	N/A	0.082	0.221	0.115	0.000	1.423	9.975	1.178	2.871

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	123	134	0	1008	1204	142	1827
N.S.	1	1.00	0.97	1.06	0.00	7.94	9.48	1.12	14.39
time (sec)	N/A	0.077	0.494	0.119	0.000	1.874	35.375	0.956	0.599

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	195	219	0	2199	0	300	2500
N.S.	1	1.00	0.94	1.05	0.00	10.57	0.00	1.44	12.02
time (sec)	N/A	0.181	1.004	0.110	0.000	2.409	0.000	0.859	4.543

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	236	301	245	639	274	338	413
N.S.	1	1.00	1.04	1.33	1.08	2.83	1.21	1.50	1.83
time (sec)	N/A	0.183	0.310	0.110	0.501	1.509	15.122	1.212	2.528

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	157	176	156	401	167	201	263
N.S.	1	1.00	1.08	1.21	1.08	2.77	1.15	1.39	1.81
time (sec)	N/A	0.065	0.207	0.102	0.553	2.160	11.016	1.200	0.089

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	91	89	93	215	92	105	136
N.S.	1	1.00	1.18	1.16	1.21	2.79	1.19	1.36	1.77
time (sec)	N/A	0.018	0.150	0.099	0.503	1.620	10.758	1.049	2.487

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	62	115	54	57	45
N.S.	1	1.00	0.98	0.85	1.15	2.13	1.00	1.06	0.83
time (sec)	N/A	0.012	0.057	0.102	0.501	1.216	2.457	1.064	0.069

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	103	0	457	100	112	2355
N.S.	1	1.00	1.00	1.02	0.00	4.52	0.99	1.11	23.32
time (sec)	N/A	0.082	0.209	0.111	0.000	0.897	8.744	1.549	2.819

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	122	137	0	998	1149	142	1814
N.S.	1	1.00	0.95	1.07	0.00	7.80	8.98	1.11	14.17
time (sec)	N/A	0.077	0.492	0.106	0.000	0.971	35.601	1.259	2.952

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	194	221	0	2195	0	301	2500
N.S.	1	1.00	0.95	1.08	0.00	10.71	0.00	1.47	12.20
time (sec)	N/A	0.192	1.101	0.106	0.000	1.447	0.000	0.963	4.632

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	107	132	105	65	484	46	345
N.S.	1	1.00	0.96	1.19	0.95	0.59	4.36	0.41	3.11
time (sec)	N/A	0.032	0.142	0.125	0.548	0.926	53.185	1.156	7.778

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	99	111	83	57	393	40	269
N.S.	1	1.00	1.14	1.28	0.95	0.66	4.52	0.46	3.09
time (sec)	N/A	0.023	0.113	0.105	0.541	1.004	17.457	0.903	5.922

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	91	90	61	49	269	34	191
N.S.	1	1.00	1.44	1.43	0.97	0.78	4.27	0.54	3.03
time (sec)	N/A	0.016	0.103	0.104	0.553	0.787	8.955	1.024	4.527

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	79	70	41	43	133	28	118
N.S.	1	1.00	2.14	1.89	1.11	1.16	3.59	0.76	3.19
time (sec)	N/A	0.009	0.076	0.105	0.537	1.058	4.882	1.290	3.447

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	69	69	41	47	71	51	47
N.S.	1	1.00	2.38	2.38	1.41	1.62	2.45	1.76	1.62
time (sec)	N/A	0.010	0.085	0.104	0.517	1.312	9.880	1.460	2.985

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	29	29	42	27	107	88	24
N.S.	1	1.00	0.64	0.64	0.93	0.60	2.38	1.96	0.53
time (sec)	N/A	0.007	0.093	0.109	0.568	1.002	5.976	1.252	2.747

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	37	62	35	189	130	32
N.S.	1	1.00	0.51	0.51	0.85	0.48	2.59	1.78	0.44
time (sec)	N/A	0.012	0.110	0.129	0.549	0.868	9.038	1.089	2.731

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	45	45	84	43	274	175	40
N.S.	1	1.00	0.46	0.46	0.87	0.44	2.82	1.80	0.41
time (sec)	N/A	0.020	0.123	0.107	0.490	0.895	12.016	1.086	2.766

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	53	53	106	51	359	217	48
N.S.	1	1.00	0.44	0.44	0.88	0.42	2.97	1.79	0.40
time (sec)	N/A	0.026	0.141	0.111	0.492	0.925	17.243	0.846	2.829

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	44	21	40	117	43	65
N.S.	1	1.00	0.95	1.13	0.54	1.03	3.00	1.10	1.67
time (sec)	N/A	0.005	0.067	0.115	0.519	0.801	26.123	0.782	4.080

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	44	21	40	117	43	444
N.S.	1	1.00	0.95	1.13	0.54	1.03	3.00	1.10	11.38
time (sec)	N/A	0.008	0.004	0.113	0.504	0.680	41.074	0.717	5.273

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	309	604	0	1230	0	0	-1
N.S.	1	1.00	2.13	4.17	0.00	8.48	0.00	0.00	-0.01
time (sec)	N/A	0.073	16.145	0.128	0.000	0.256	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	312	940	0	1124	0	0	-1
N.S.	1	1.00	1.41	4.25	0.00	5.09	0.00	0.00	-0.00
time (sec)	N/A	0.137	18.822	0.118	0.000	0.269	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	135	154	0	43	0	0	-1
N.S.	1	1.00	0.48	0.55	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.257	10.837	0.152	0.000	0.203	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	130	149	0	38	0	0	-1
N.S.	1	1.00	0.53	0.61	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.193	8.853	0.130	0.000	0.196	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	125	144	0	33	0	0	-1
N.S.	1	1.00	0.65	0.75	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.052	1.544	0.135	0.000	0.243	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	120	139	0	28	0	0	-1
N.S.	1	1.00	0.74	0.86	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.041	1.827	0.129	0.000	0.190	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	139	174	0	0	0	0	-1
N.S.	1	1.00	0.76	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	5.853	0.141	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	302	0	0	0	0	-1
N.S.	1	1.00	0.69	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	6.016	0.141	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	134	434	0	0	0	0	-1
N.S.	1	1.00	0.59	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	6.214	0.145	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	139	566	0	0	0	0	-1
N.S.	1	1.00	0.53	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.254	6.480	0.145	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	1254	3670	0	0	0	0	-1
N.S.	1	1.00	2.20	6.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.874	30.381	0.161	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	130	149	0	38	0	0	-1
N.S.	1	1.00	0.53	0.61	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.185	11.354	0.393	0.000	0.346	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	144	0	33	0	0	-1
N.S.	1	1.00	0.61	0.70	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.135	8.854	0.144	0.000	0.284	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	120	139	0	28	0	0	-1
N.S.	1	1.00	0.74	0.86	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.043	2.175	0.170	0.000	0.326	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	115	134	0	23	0	0	-1
N.S.	1	1.00	0.88	1.02	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.033	1.253	0.132	0.000	0.306	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	95	67	0	0	0	0	-1
N.S.	1	1.00	0.63	0.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	2.504	0.129	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	302	0	0	0	0	-1
N.S.	1	1.00	0.69	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	5.967	0.138	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	135	434	0	0	0	0	-1
N.S.	1	1.00	0.60	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	5.834	0.142	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	144	0	33	0	0	-1
N.S.	1	1.00	0.61	0.70	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.138	18.714	0.132	0.000	0.252	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	120	139	0	28	0	0	-1
N.S.	1	1.00	0.72	0.83	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.094	16.194	0.133	0.000	0.297	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	115	134	0	23	0	0	-1
N.S.	1	1.00	0.88	1.02	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.032	2.164	0.132	0.000	0.219	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	111	33	0	0	0	0	-1
N.S.	1	1.00	2.36	0.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	1.523	0.129	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	52	0	0	0	0	-1
N.S.	1	1.00	0.68	0.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	2.497	0.130	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	302	0	0	0	0	-1
N.S.	1	1.00	0.69	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	6.169	0.161	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	142	434	0	0	0	0	-1
N.S.	1	1.00	0.63	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	5.936	0.181	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	202	668	0	0	0	0	-1
N.S.	1	1.00	0.69	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	19.183	0.119	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1198	1551	0	0	0	0	-1
N.S.	1	1.00	2.67	3.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.469	20.170	0.123	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	125	144	0	33	0	0	-1
N.S.	1	1.00	0.62	0.71	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.133	31.369	0.153	0.000	0.248	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	120	139	0	28	0	0	-1
N.S.	1	1.00	0.73	0.84	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.094	28.483	0.137	0.000	0.243	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	115	134	0	23	0	0	-1
N.S.	1	1.00	0.89	1.04	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.040	26.241	0.141	0.000	0.208	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	187	51	0	0	0	0	-1
N.S.	1	1.00	1.91	0.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	8.263	0.159	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	79	33	0	0	0	0	-1
N.S.	1	1.00	1.65	0.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	1.164	0.122	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	99	34	0	0	0	0	-1
N.S.	1	1.00	1.94	0.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	2.475	0.127	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	302	0	0	0	0	-1
N.S.	1	1.00	0.69	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	5.015	0.138	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	142	434	0	0	0	0	-1
N.S.	1	1.00	0.63	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	6.839	0.140	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	180	210	0	718	0	0	-1
N.S.	1	1.00	1.31	1.53	0.00	5.24	0.00	0.00	-0.01
time (sec)	N/A	0.044	11.417	0.112	0.000	0.334	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	319	566	0	725	0	0	-1
N.S.	1	1.00	1.12	1.99	0.00	2.55	0.00	0.00	-0.00
time (sec)	N/A	0.116	20.063	0.107	0.000	0.360	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	226	222	0	0	0	0	-1
N.S.	1	1.00	1.37	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	14.543	0.117	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	322	1978	0	0	0	0	-1
N.S.	1	1.00	0.82	5.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	22.835	0.116	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	875	875	4180	16647	0	0	0	0	-1
N.S.	1	1.00	4.78	19.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	35.322	0.198	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	203	184	0	0	0	0	-1
N.S.	1	1.00	2.74	2.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	20.691	0.128	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	203	181	0	0	0	0	-1
N.S.	1	1.00	2.74	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	10.112	0.128	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	218	212	0	0	0	0	-1
N.S.	1	1.00	2.53	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	20.666	0.131	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	218	205	0	0	0	0	-1
N.S.	1	1.00	2.53	2.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	10.123	0.123	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	350	841	0	0	0	0	-1
N.S.	1	1.00	0.74	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	45.864	0.177	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	345	836	0	0	0	0	-1
N.S.	1	1.00	0.80	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	44.122	0.147	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	831	0	0	0	0	-1
N.S.	1	1.00	0.87	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	39.876	0.143	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	826	0	0	0	0	-1
N.S.	1	1.00	0.99	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	28.770	0.160	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	330	821	0	0	0	0	-1
N.S.	1	1.00	0.95	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	37.472	0.132	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	366	1077	0	0	0	0	-1
N.S.	1	1.00	0.94	2.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	34.422	0.135	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	251	913	0	0	0	0	-1
N.S.	1	1.00	0.76	2.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	39.736	0.134	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	259	1088	0	0	0	0	-1
N.S.	1	1.00	0.70	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	34.951	0.135	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	345	836	0	0	0	0	-1
N.S.	1	1.00	0.80	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	40.170	0.137	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	831	0	0	0	0	-1
N.S.	1	1.00	0.87	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	38.226	0.135	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	826	0	0	0	0	-1
N.S.	1	1.00	0.99	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	30.216	0.135	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	318	821	0	0	0	0	-1
N.S.	1	1.00	0.87	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	4.810	0.128	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	326	816	0	0	0	0	-1
N.S.	1	1.00	1.17	2.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	17.618	0.132	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	246	738	0	0	0	0	-1
N.S.	1	1.00	0.85	2.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	27.219	0.131	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	251	913	0	0	0	0	-1
N.S.	1	1.00	0.76	2.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	32.108	0.133	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	258	1088	0	0	0	0	-1
N.S.	1	1.00	0.70	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	28.129	0.134	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	831	0	0	0	0	-1
N.S.	1	1.00	0.87	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	31.257	0.138	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	349	826	0	0	0	0	-1
N.S.	1	1.00	0.99	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	24.051	0.135	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	347	821	0	0	0	0	-1
N.S.	1	1.00	0.95	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	6.206	0.129	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	170	134	0	0	0	0	-1
N.S.	1	1.00	1.68	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	4.176	0.129	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	195	237	563	0	0	0	0	-1
N.S.	1	3.25	3.95	9.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	29.666	0.147	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	248	738	0	0	0	0	-1
N.S.	1	1.00	0.86	2.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	31.906	0.132	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	721	721	6667	15274	0	0	0	0	-1
N.S.	1	1.00	9.25	21.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	48.808	0.126	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	208	206	4553	0	0	0	0	-1
N.S.	1	1.29	1.28	28.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	23.740	0.122	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	826	0	0	0	0	-1
N.S.	1	1.00	0.99	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	23.515	0.135	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	347	821	0	0	0	0	-1
N.S.	1	1.00	0.74	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	9.120	0.134	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	162	0	0	0	0	-1
N.S.	1	1.00	0.95	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	3.825	0.131	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	90	133	0	0	0	0	-1
N.S.	1	1.00	1.27	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	3.200	0.131	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	270	237	563	0	0	0	0	-1
N.S.	1	1.38	1.22	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	16.300	0.130	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	246	738	0	0	0	0	-1
N.S.	1	1.00	0.85	2.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	32.316	0.136	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	968	968	6638	17031	0	0	0	0	-1
N.S.	1	1.00	6.86	17.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	29.868	0.118	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	584	1250	0	0	0	0	-1
N.S.	1	1.00	2.56	5.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.110	28.315	0.118	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	198	227	270	0	0	0	0	-1
N.S.	1	1.23	1.41	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	24.274	0.118	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	462	9326	0	0	0	0	-1
N.S.	1	1.00	1.08	21.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	35.395	0.124	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	B	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	786	0	7075	22970	0	0	0	0	-1
N.S.	1	0.00	9.00	29.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.006	35.295	0.189	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	285	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	1.019	0.041	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	0.538	0.048	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	153	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.184	0.030	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.131	0.030	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.107	0.028	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	177	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.401	0.018	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	228	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.603	0.032	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	241	726	484	892	8221	1665	819
N.S.	1	1.00	1.44	4.35	2.90	5.34	49.23	9.97	4.90
time (sec)	N/A	0.092	0.389	0.105	0.351	1.081	1.918	0.803	2.945

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	136	165	0	0	0	0	0	-1
N.S.	1	1.01	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.343	0.020	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.205	0.029	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	229	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.576	0.049	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	104	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.247	0.020	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	197	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.113	0.455	0.014	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	264	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	0.545	0.026	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	195	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	0.379	0.015	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	221	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	0.271	0.023	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	205	198	0	0	0	0	0	-1
N.S.	1	1.01	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.279	0.021	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	237	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.106	0.342	0.021	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	220	894	0	1654	0	0	1895
N.S.	1	1.00	0.61	2.47	0.00	4.57	0.00	0.00	5.23
time (sec)	N/A	0.236	0.457	0.115	0.000	1.349	0.000	0.000	4.487

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	279	2343	0	3440	0	0	2500
N.S.	1	1.00	0.55	4.62	0.00	6.79	0.00	0.00	4.93
time (sec)	N/A	0.360	0.619	0.117	0.000	1.603	0.000	0.000	6.752

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	815	803	3579	0	0	0	0	0	-1
N.S.	1	0.99	4.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.904	36.873	0.025	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	572	566	3413	0	0	0	0	0	-1
N.S.	1	0.99	5.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	7.023	0.023	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	360	227	906	0	1586	0	0	1890
N.S.	1	0.99	0.63	2.50	0.00	4.37	0.00	0.00	5.21
time (sec)	N/A	0.245	0.524	0.115	0.000	1.551	0.000	0.000	4.277

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	186	181	509	0	898	0	0	869
N.S.	1	0.99	0.96	2.71	0.00	4.78	0.00	0.00	4.62
time (sec)	N/A	0.062	0.159	0.112	0.000	1.417	0.000	0.000	3.407

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	190	199	0	0	0	0	0	-1
N.S.	1	1.07	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.379	0.017	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	174	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.205	0.018	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	184	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.347	0.019	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	2.815	0.040	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	1.008	0.037	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.131	0.636	0.031	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.004	0.268	0.041	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.565	0.033	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	208	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.347	0.058	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	215	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.101	0.423	0.057	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	261	223	0	0	0	0	0	-1
N.S.	1	0.99	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.317	0.033	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	508	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	1.481	0.033	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	89	139	87	78	0	76	244
N.S.	1	1.00	1.13	1.76	1.10	0.99	0.00	0.96	3.09
time (sec)	N/A	0.092	0.279	0.121	0.503	0.889	0.000	1.154	7.441

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	82	117	57	67	0	60	232
N.S.	1	1.00	1.30	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.038	0.252	0.110	0.514	1.053	0.000	0.971	6.988

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	93	96	57	81	245	0	122
N.S.	1	1.00	1.94	2.00	1.19	1.69	5.10	0.00	2.54
time (sec)	N/A	0.113	0.264	0.126	0.525	0.818	43.371	0.000	3.924

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	93	97	57	84	221	0	114
N.S.	1	1.00	1.94	2.02	1.19	1.75	4.60	0.00	2.38
time (sec)	N/A	0.112	0.319	0.117	0.519	1.090	41.364	0.000	3.741

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	98	65	0	0	312
N.S.	1	1.00	0.97	1.52	1.38	0.92	0.00	0.00	4.39
time (sec)	N/A	0.120	0.314	0.111	0.489	0.953	0.000	0.000	5.854

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	151	74	137	100	73	0	105	318
N.S.	1	1.74	0.85	1.57	1.15	0.84	0.00	1.21	3.66
time (sec)	N/A	0.092	0.194	0.112	0.304	1.406	0.000	1.220	12.354

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	135	63	120	90	61	0	80	312
N.S.	1	2.60	1.21	2.31	1.73	1.17	0.00	1.54	6.00
time (sec)	N/A	0.047	0.180	0.111	0.333	1.213	0.000	0.690	12.400

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	135	69	95	56	73	240	71	118
N.S.	1	2.45	1.25	1.73	1.02	1.33	4.36	1.29	2.15
time (sec)	N/A	0.119	0.162	0.109	0.506	1.033	41.285	0.695	3.975

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	135	69	96	56	82	216	83	118
N.S.	1	2.45	1.25	1.75	1.02	1.49	3.93	1.51	2.15
time (sec)	N/A	0.116	0.149	0.113	0.528	1.033	40.181	1.130	3.859

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	129	60	103	61	69	0	145	316
N.S.	1	1.55	0.72	1.24	0.73	0.83	0.00	1.75	3.81
time (sec)	N/A	0.117	0.144	0.108	0.497	1.056	0.000	1.350	9.891

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	171	71	123	86	90	0	197	304
N.S.	1	1.47	0.61	1.06	0.74	0.78	0.00	1.70	2.62
time (sec)	N/A	0.132	0.207	0.111	0.512	1.263	0.000	2.041	9.436

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [33] had the largest ratio of [45]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	21	0.048
2	A	2	1	1.00	23	0.043
3	A	2	1	1.00	25	0.040
4	A	2	1	1.00	27	0.037
5	A	2	1	1.00	29	0.034
6	A	2	1	1.00	17	0.059
7	A	3	2	1.00	22	0.091
8	A	6	5	1.00	25	0.200
9	A	5	5	1.00	25	0.200
10	A	4	4	1.00	23	0.174
11	A	4	4	1.00	18	0.222
12	A	6	4	1.00	25	0.160
13	A	6	4	1.00	25	0.160
14	A	7	5	1.00	25	0.200
15	A	6	5	1.00	25	0.200
16	A	5	5	1.00	25	0.200
17	A	4	4	1.00	23	0.174
18	A	4	4	1.00	18	0.222
19	A	6	5	1.00	25	0.200
20	A	6	5	1.00	25	0.200
21	A	7	6	1.00	25	0.240
22	A	8	6	1.00	26	0.231
23	A	7	6	1.00	26	0.231
24	A	6	6	1.00	24	0.250
25	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	5	1.00	26	0.192
27	A	3	3	1.00	26	0.115
28	A	4	4	1.00	26	0.154
29	A	5	4	1.00	26	0.154
30	A	6	4	1.00	26	0.154
31	A	4	4	1.00	24	0.167
32	A	5	5	1.00	36	0.139
33	A	3	3	1.00	45	0.067
34	A	5	5	1.00	39	0.128
35	A	10	8	1.00	35	0.229
36	A	9	8	1.00	35	0.229
37	A	8	6	1.00	33	0.182
38	A	7	7	1.00	28	0.250
39	A	10	10	1.00	35	0.286
40	A	10	10	1.00	35	0.286
41	A	11	11	1.00	35	0.314
42	A	12	11	1.00	35	0.314
43	A	12	10	1.00	35	0.286
44	A	9	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	7	6	1.00	33	0.182
47	A	6	6	1.00	28	0.214
48	A	9	9	1.00	35	0.257
49	A	10	10	1.00	35	0.286
50	A	11	11	1.00	35	0.314
51	A	8	8	1.00	35	0.229
52	A	7	7	1.00	35	0.200
53	A	6	6	1.00	33	0.182
54	A	2	2	1.00	28	0.071
55	A	6	6	1.00	35	0.171
56	A	10	10	1.00	35	0.286
57	A	11	11	1.00	35	0.314
58	A	8	6	1.00	35	0.171
59	A	11	8	1.00	35	0.229
60	A	8	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	7	1.00	35	0.200
62	A	7	7	1.00	35	0.200
63	A	5	5	1.00	33	0.152
64	A	2	2	1.00	28	0.071
65	A	3	3	1.00	35	0.086
66	A	10	10	1.00	35	0.286
67	A	11	11	1.00	35	0.314
68	A	3	3	1.00	36	0.083
69	A	6	5	1.00	33	0.152
70	A	4	3	1.00	35	0.086
71	A	10	8	1.00	35	0.229
72	A	18	12	1.00	35	0.343
73	A	3	3	1.00	36	0.083
74	A	4	4	1.00	31	0.129
75	A	3	3	1.00	40	0.075
76	A	4	4	1.00	31	0.129
77	A	12	10	1.00	37	0.270
78	A	11	10	1.00	37	0.270
79	A	10	10	1.00	37	0.270
80	A	9	9	1.00	37	0.243
81	A	9	9	1.00	37	0.243
82	A	10	10	1.00	37	0.270
83	A	9	9	1.00	37	0.243
84	A	10	9	1.00	37	0.243
85	A	11	10	1.00	37	0.270
86	A	10	10	1.00	37	0.270
87	A	9	9	1.00	37	0.243
88	A	9	8	1.00	37	0.216
89	A	7	7	1.00	37	0.189
90	A	8	8	1.00	37	0.216
91	A	9	9	1.00	37	0.243
92	A	10	9	1.00	37	0.243
93	A	10	10	1.00	37	0.270
94	A	9	9	1.00	37	0.243
95	A	9	8	1.00	37	0.216

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	37	0.054
97	B	5	5	3.25	37	0.135
98	A	8	8	1.00	37	0.216
99	A	7	7	1.00	37	0.189
100	A	2	2	1.29	37	0.054
101	A	9	9	1.00	37	0.243
102	A	12	10	1.00	37	0.270
103	A	2	2	1.00	37	0.054
104	A	2	2	1.00	37	0.054
105	A	8	7	1.38	37	0.189
106	A	8	8	1.00	37	0.216
107	A	10	8	1.00	37	0.216
108	A	2	2	1.00	37	0.054
109	A	2	2	1.23	37	0.054
110	A	5	5	1.00	37	0.135
111	F	0	0	N/A	0.	N/A
112	A	8	3	1.00	25	0.120
113	A	6	3	1.00	25	0.120
114	A	4	2	1.00	25	0.080
115	A	3	2	1.00	23	0.087
116	A	3	2	1.00	22	0.091
117	A	5	3	1.00	25	0.120
118	A	6	3	1.00	25	0.120
119	A	2	1	1.00	23	0.043
120	A	2	2	1.01	25	0.080
121	A	3	2	1.00	27	0.074
122	A	5	2	1.00	29	0.069
123	A	6	3	1.00	25	0.120
124	A	3	3	1.00	25	0.120
125	A	3	3	1.00	29	0.103
126	A	3	3	1.00	27	0.111
127	A	3	3	1.00	29	0.103
128	A	3	3	1.01	29	0.103
129	A	3	3	1.00	29	0.103
130	A	3	3	1.00	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	29	0.103
132	A	10	9	0.99	31	0.290
133	A	9	8	0.99	31	0.258
134	A	3	3	0.99	29	0.103
135	A	3	3	0.99	24	0.125
136	A	5	5	1.07	27	0.185
137	A	5	5	1.00	29	0.172
138	A	7	4	1.00	29	0.138
139	A	31	5	1.00	29	0.172
140	A	15	5	1.00	29	0.172
141	A	7	4	1.00	27	0.148
142	A	3	3	1.00	22	0.136
143	A	0	0	0.00	0	0.000
144	A	7	4	1.00	33	0.121
145	A	7	4	1.00	34	0.118
146	A	5	5	1.00	34	0.147
147	A	3	3	0.99	34	0.088
148	A	4	3	1.00	34	0.088
149	A	4	4	1.00	31	0.129
150	A	4	4	1.00	30	0.133
151	A	7	7	1.00	33	0.212
152	A	7	7	1.00	33	0.212
153	A	6	6	1.00	33	0.182
154	A	5	5	1.74	30	0.167
155	B	5	5	2.60	29	0.172
156	B	8	8	2.45	32	0.250
157	B	8	8	2.45	32	0.250
158	A	6	6	1.55	32	0.188
159	A	7	7	1.47	32	0.219

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=112

$$acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2+\frac{1}{3}(b(deg+cfg+ceh)+a(dfg+deh+cfh))x^3+\frac{1}{4}(adfh+b(dfg+deh+cf$$

```
[Out] a*c*e*g*x+1/2*(b*c*e*g+a*(c*e*h+c*f*g+d*e*g))*x^2+1/3*(b*(c*e*h+c*f*g+d*e*g
)+a*(c*f*h+d*e*h+d*f*g))*x^3+1/4*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*x^4+1/5*b*
d*f*h*x^5
```

Rubi [A]
time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{147\}$

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)+acegx+\frac{1}{5}bdfhx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c*
f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g + d
*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \int (aceg+(bceg+a(deg+cfg+ceh))x+(b(deg+cfg+ceh)$$

$$= acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2+\frac{1}{3}(b(deg+cfg+ceh$$

Mathematica [A]
time = 0.03, size = 112, normalized size = 1.00

$$acegx+\frac{1}{2}(bceg+adeg+acfg+aceh)x^2+\frac{1}{3}(bdeg+bcfg+adfg+bceh+adeh+acfh)x^3+\frac{1}{4}(bdfg+bdeh+bcfh+adfh)x^4+\frac{1}{5}bdfhx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*d*e*g + a*c*f*g + a*c*e*h)*x^2)/2 + ((b*d*e*g + b
*c*f*g + a*d*f*g + b*c*e*h + a*d*e*h + a*c*f*h)*x^3)/3 + ((b*d*f*g + b*d*e*
h + b*c*f*h + a*d*f*h)*x^4)/4 + (b*d*f*h*x^5)/5
```

Maple [A]
time = 0.01, size = 109, normalized size = 0.97

method	result
default	$\frac{bdfh\,x^5}{5}+\frac{(((ad+bc)f+bde)h+bdfg)x^4}{4}+\frac{((acf+(ad+bc)e)h+((ad+bc)f+bde)g)x^3}{3}+\frac{(aceh+(acf+(ad+bc)e)g)x^2}{2}+acegx$
norman	$\frac{bdfh\,x^5}{5}+\left(\frac{1}{4}adfh+\frac{1}{4}bcfh+\frac{1}{4}bdeh+\frac{1}{4}bdfg\right)x^4+\left(\frac{1}{3}acfh+\frac{1}{3}adeh+\frac{1}{3}adfg+\frac{1}{3}bceh+\frac{1}{3}bcfg+\frac{1}{3}\right.$
gosper	$\frac{1}{5}bdfh\,x^5+\frac{1}{4}x^4adfh+\frac{1}{4}x^4bcfh+\frac{1}{4}x^4bdeh+\frac{1}{4}x^4bdfg+\frac{1}{3}x^3acfh+\frac{1}{3}x^3adeh+\frac{1}{3}x^3adfg+\frac{1}{3}x^3bce$
risch	$\frac{1}{5}bdfh\,x^5+\frac{1}{4}x^4adfh+\frac{1}{4}x^4bcfh+\frac{1}{4}x^4bdeh+\frac{1}{4}x^4bdfg+\frac{1}{3}x^3acfh+\frac{1}{3}x^3adeh+\frac{1}{3}x^3adfg+\frac{1}{3}x^3bce$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b*d*f*h*x^5+1/4*(((a*d+b*c)*f+b*d*e)*h+b*d*f*g)*x^4+1/3*((a*c*f+(a*d+b*
c)*e)*h+((a*d+b*c)*f+b*d*e)*g)*x^3+1/2*(a*c*e*h+(a*c*f+(a*d+b*c)*e)*g)*x^2+
a*c*e*g*x
```

Maxima [A]
time = 0.30, size = 114, normalized size = 1.02

$$\frac{1}{5}bdfhx^5+\frac{1}{4}(bdfg+(bde+(bc+ad)f)h)x^4+acgxe+\frac{1}{3}((bde+(bc+ad)f)g+(acf+bce+ade)h)x^3+\frac{1}{2}(ache+(acf+bce+ade)g)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/4*(b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^4 + a*c*g*x*e
 + 1/3*((b*d*e + (b*c + a*d)*f)*g + (a*c*f + b*c*e + a*d*e)*h)*x^3 + 1/2*(a
*c*h*e + (a*c*f + b*c*e + a*d*e)*g)*x^2
```

Fricas [A]
time = 0.91, size = 118, normalized size = 1.05

$$\frac{1}{5}bdfhx^5+\frac{1}{2}acfgx^2+\frac{1}{4}(bdfg+(bc+ad)fh)x^4+\frac{1}{3}(acfh+(bc+ad)fg)x^3+\frac{1}{12}(3bdhx^4+12acgx+4(bdg+(bc+ad)h)x^3+6(ach+(bc+ad)g)x^2)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/2*a*c*f*g*x^2 + 1/4*(b*d*f*g + (b*c + a*d)*f*h)*x^4 + 1
/3*(a*c*f*h + (b*c + a*d)*f*g)*x^3 + 1/12*(3*b*d*h*x^4 + 12*a*c*g*x + 4*(b*
d*g + (b*c + a*d)*h)*x^3 + 6*(a*c*h + (b*c + a*d)*g)*x^2)*e
```

Sympy [A]
time = 0.02, size = 148, normalized size = 1.32

$$acegx + \frac{bdfhx^5}{5} + x^4\left(\frac{adfh}{4} + \frac{bcfh}{4} + \frac{bdeh}{4} + \frac{bdfg}{4}\right) + x^3\left(\frac{acfh}{3} + \frac{adeh}{3} + \frac{adfg}{3} + \frac{bceh}{3} + \frac{bcfg}{3} + \frac{bdeg}{3}\right) + x^2\left(\frac{aceh}{2} + \frac{acfg}{2} + \frac{adeg}{2} + \frac{bceg}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] a*c*e*g*x + b*d*f*h*x**5/5 + x**4*(a*d*f*h/4 + b*c*f*h/4 + b*d*e*h/4 + b*d*
f*g/4) + x**3*(a*c*f*h/3 + a*d*e*h/3 + a*d*f*g/3 + b*c*e*h/3 + b*c*f*g/3 +
b*d*e*g/3) + x**2*(a*c*e*h/2 + a*c*f*g/2 + a*d*e*g/2 + b*c*e*g/2)
```

Giac [A]
time = 0.58, size = 150, normalized size = 1.34

$$\frac{1}{5}bdfhx^5 + \frac{1}{4}bdfgx^4 + \frac{1}{4}bcfhx^4 + \frac{1}{4}adfhx^4 + \frac{1}{4}bdhx^4e + \frac{1}{3}bcfgx^3 + \frac{1}{3}adfgx^3 + \frac{1}{3}acfhx^3 + \frac{1}{3}bdgx^3e + \frac{1}{3}bchx^3e + \frac{1}{3}adhx^3e + \frac{1}{2}acfgx^2 + \frac{1}{2}bcgx^2e + \frac{1}{2}adgx^2e + \frac{1}{2}achx^2e + acgxe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/4*b*d*f*g*x^4 + 1/4*b*c*f*h*x^4 + 1/4*a*d*f*h*x^4 + 1/4
*b*d*h*x^4*e + 1/3*b*c*f*g*x^3 + 1/3*a*d*f*g*x^3 + 1/3*a*c*f*h*x^3 + 1/3*b*
d*g*x^3*e + 1/3*b*c*h*x^3*e + 1/3*a*d*h*x^3*e + 1/2*a*c*f*g*x^2 + 1/2*b*c*g
*x^2*e + 1/2*a*d*g*x^2*e + 1/2*a*c*h*x^2*e + a*c*g*x*e
```

Mupad [B]
time = 2.38, size = 115, normalized size = 1.03

$$\frac{b\,d\,f\,h\,x^5}{5} + \left(\frac{a\,d\,f\,h}{4} + \frac{b\,c\,f\,h}{4} + \frac{b\,d\,e\,h}{4} + \frac{b\,d\,f\,g}{4}\right)\,x^4 + \left(\frac{a\,c\,f\,h}{3} + \frac{a\,d\,e\,h}{3} + \frac{a\,d\,f\,g}{3} + \frac{b\,c\,e\,h}{3} + \frac{b\,c\,f\,g}{3} + \frac{b\,d\,e\,g}{3}\right)\,x^3 + \left(\frac{a\,c\,e\,h}{2} + \frac{a\,c\,f\,g}{2} + \frac{a\,d\,e\,g}{2} + \frac{b\,c\,e\,g}{2}\right)\,x^2 + a\,c\,e\,g\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

```
[Out] x^3*((a*c*f*h)/3 + (a*d*e*h)/3 + (a*d*f*g)/3 + (b*c*e*h)/3 + (b*c*f*g)/3 +
(b*d*e*g)/3) + x^2*((a*c*e*h)/2 + (a*c*f*g)/2 + (a*d*e*g)/2 + (b*c*e*g)/2)
+ x^4*((a*d*f*h)/4 + (b*c*f*h)/4 + (b*d*e*h)/4 + (b*d*f*g)/4) + a*c*e*g*x +
 (b*d*f*h*x^5)/5
```

3.2 $\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=126

$$\frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2}+\frac{bdfx^3}{3h}-\frac{(bg-ah)(dg-c}{}$$

```
[Out] (b*(-c*h+d*g)*(-e*h+f*g)-a*h*(-c*f*h-d*e*h+d*f*g))*x/h^3+1/2*(a*d*f*h-b*(-c
*f*h-d*e*h+d*f*g))*x^2/h^2+1/3*b*d*f*x^3/h-(-a*h+b*g)*(-c*h+d*g)*(-e*h+f*g)
*ln(h*x+g)/h^4
```

Rubi [A]
time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{147\}$

$$-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}+\frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2}+\frac{bdfx^3}{3h}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*f
*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g -
a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \int \left(\frac{b(dg-ch)(fg-eh)-ah(dfg-deh-cfh)}{h^3}+\frac{(adfh-b(dfg-deh}{h^2}\right.$$
$$= \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3}+\frac{(adfh-b(dfg-deh-}{2h^2}$$

Mathematica [A]
time = 0.06, size = 123, normalized size = 0.98

$$\frac{hx(3ah(2cfh+d(-2fg+2eh+fhx))+b(3deh(-2g+hx)+3ch(-2fg+2eh+fhx)+df(6g^2-3ghx+2h^2x^2)))-6(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{6h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] (h*x*(3*a*h*(2*c*f*h + d*(-2*f*g + 2*e*h + f*h*x)) + b*(3*d*e*h*(-2*g + h*x
) + 3*c*h*(-2*f*g + 2*e*h + f*h*x) + d*f*(6*g^2 - 3*g*h*x + 2*h^2*x^2))) -
6*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/(6*h^4)
```

Maple [A]
time = 0.10, size = 194, normalized size = 1.54

method	result
norman	$\frac{(acf\,h^2+ade\,h^2-adfgh+bce\,h^2-bcfgh-bdegh+bdf\,g^2)x}{h^3}+\frac{(adfh+bcfh+bdeh-bdfg)x^2}{2h^2}+\frac{bdf\,x^3}{3h}+\frac{(ace\,h^3-acfg\,h^2-adeg\,h^2}{}$
default	$\frac{\frac{1}{3}bdf\,x^3h^2+\frac{1}{2}adf\,h^2x^2+\frac{1}{2}bcf\,h^2x^2+\frac{1}{2}bde\,h^2x^2-\frac{1}{2}bdfgh\,x^2+acf\,h^2x+ade\,h^2x-adfghx+bce\,h^2x-bcfghx-bdeghx+bdf\,g^2x}{h^3}+\frac{(ac}{}$
risch	$\frac{bdf\,x^3}{3h}+\frac{adf\,x^2}{2h}+\frac{bcf\,x^2}{2h}+\frac{bde\,x^2}{2h}-\frac{bdfg\,x^2}{2h^2}+\frac{acfx}{h}+\frac{adex}{h}-\frac{adfgx}{h^2}+\frac{bcex}{h}-\frac{bcfgx}{h^2}-\frac{bdegx}{h^2}+\frac{bdf\,g^2x}{h^3}+\frac{\ln(h}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^3*(1/3*b*d*f*x^3*h^2+1/2*a*d*f*h^2*x^2+1/2*b*c*f*h^2*x^2+1/2*b*d*e*h^2*
x^2-1/2*b*d*f*g*h*x^2+a*c*f*h^2*x+a*d*e*h^2*x-a*d*f*g*h*x+b*c*e*h^2*x-b*c*f
*g*h*x-b*d*e*g*h*x+b*d*f*g^2*x)+(a*c*e*h^3-a*c*f*g*h^2-a*d*e*g*h^2+a*d*f*g^
2*h-b*c*e*g*h^2+b*c*f*g^2*h+b*d*e*g^2*h-b*d*f*g^3)/h^4*ln(h*x+g)
```

Maxima [A]
time = 0.31, size = 168, normalized size = 1.33

$$\frac{2\,bdf h^2x^3 - 3\,(bdf gh - (bde + (bc + ad)f)h^2)x^2 + 6\,(bdf g^2 - (bde + (bc + ad)f)gh + (acf + bce + ade)h^2)x}{6\,h^3} - \frac{(bdf g^3 - ach^3e - (bde + (bc + ad)f)g^2h + (acf + bce + ade)gh^2)\log(hx + g)}{h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*(b*d*f*g*h - (b*d*e + (b*c + a*d)*f)*h^2)*x^2 + 6*
(b*d*f*g^2 - (b*d*e + (b*c + a*d)*f)*g*h + (a*c*f + b*c*e + a*d*e)*h^2)*x)/
h^3 - (b*d*f*g^3 - a*c*h^3*e - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + b*c
*e + a*d*e)*g*h^2)*log(h*x + g)/h^4
```

Fricas [A]
time = 0.84, size = 183, normalized size = 1.45

$$\frac{2\,bdf h^3x^3 - 3\,(bdf gh^2 - (bc + ad)fh^3)x^2 + 6\,(bdf g^2h + acfh^3 - (bc + ad)fgh^2)x + 3\,(bdh^3x^2 - 2\,(bdgh^2 - (bc + ad)h^3)x)e - 6\,(bdf g^3 + acfgh^2 - (bc + ad)fg^2h - (bdg^2h + ach^3 - (bc + ad)gh^2)e)\log(hx + g)}{6\,h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d*f*h^3*x^3 - 3*(b*d*f*g*h^2 - (b*c + a*d)*f*h^3)*x^2 + 6*(b*d*f*g
^2*h + a*c*f*h^3 - (b*c + a*d)*f*g*h^2)*x + 3*(b*d*h^3*x^2 - 2*(b*d*g*h^2 -
 (b*c + a*d)*h^3)*x)*e - 6*(b*d*f*g^3 + a*c*f*g*h^2 - (b*c + a*d)*f*g^2*h -
 (b*d*g^2*h + a*c*h^3 - (b*c + a*d)*g*h^2)*e)*log(h*x + g))/h^4
```

Sympy [A]
time = 0.28, size = 146, normalized size = 1.16

$$\frac{bdfx^3}{3h}+x^2\left(\frac{adf}{2h}+\frac{bcf}{2h}+\frac{bde}{2h}-\frac{bdfg}{2h^2}\right)+x\left(\frac{acf}{h}+\frac{ade}{h}-\frac{adfg}{h^2}+\frac{bce}{h}-\frac{bcfg}{h^2}-\frac{bdeg}{h^2}+\frac{bdfg^2}{h^3}\right)+\frac{(ah-bg)\,(ch-dg)\,(eh-fg)\log{(g+hx)}}{h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*f*x**3/(3*h) + x**2*(a*d*f/(2*h) + b*c*f/(2*h) + b*d*e/(2*h) - b*d*f*g/
(2*h**2)) + x*(a*c*f/h + a*d*e/h - a*d*f*g/h**2 + b*c*e/h - b*c*f*g/h**2 -
b*d*e*g/h**2 + b*d*f*g**2/h**3) + (a*h - b*g)*(c*h - d*g)*(e*h - f*g)*log(g
 + h*x)/h**4
```

Giac [A]
time = 0.64, size = 208, normalized size = 1.65

$$\frac{2\,bdfh^2x^3-3\,bdfghx^2+3\,bcfh^2x^2+3\,adfh^2x^2+3\,bdh^2x^2e+6\,bdfg^2x-6\,bcfghx-6\,adfghx+6\,acfh^2x-6\,bdghxe+6\,bch^2xe+6\,adh^2xe}{6\,h^3}-\frac{(bdfg^3-bcfg^2h-adfg^2h+acfgh^2-bdg^2he+bcgh^2e+adgh^2e-ach^3e)\log{(|hx+g|)}}{h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*b*d*f*g*h*x^2 + 3*b*c*f*h^2*x^2 + 3*a*d*f*h^2*x^2
+ 3*b*d*h^2*x^2*e + 6*b*d*f*g^2*x - 6*b*c*f*g*h*x - 6*a*d*f*g*h*x + 6*a*c*f
*h^2*x - 6*b*d*g*h*x*e + 6*b*c*h^2*x*e + 6*a*d*h^2*x*e)/h^3 - (b*d*f*g^3 -
b*c*f*g^2*h - a*d*f*g^2*h + a*c*f*g*h^2 - b*d*g^2*h*e + b*c*g*h^2*e + a*d*g
*h^2*e - a*c*h^3*e)*log(abs(h*x + g))/h^4
```

Mupad [B]
time = 2.54, size = 174, normalized size = 1.38

$$x\left(\frac{a\,c\,f+a\,d\,e+b\,c\,e}{h}-\frac{g\,\left(\frac{a\,d\,f+b\,c\,f+b\,d\,e}{h}-\frac{b\,d\,f\,g}{h^2}\right)}{h}\right)+x^2\,\left(\frac{a\,d\,f+b\,c\,f+b\,d\,e}{2\,h}-\frac{b\,d\,f\,g}{2\,h^2}\right)+\frac{\ln\left(g+h\,x\right)\,\left(a\,c\,e\,h^3-b\,d\,f\,g^3-a\,c\,f\,g\,h^2-a\,d\,e\,g\,h^2-b\,c\,e\,g\,h^2+a\,d\,f\,g^2\,h+b\,c\,f\,g^2\,h+b\,d\,e\,g^2\,h\right)}{h^4}+\frac{b\,d\,f\,x^3}{3\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x))/(g + h*x),x)
```

```
[Out] x*((a*c*f + a*d*e + b*c*e)/h - (g*((a*d*f + b*c*f + b*d*e)/h - (b*d*f*g)/h^
2))/h) + x^2*((a*d*f + b*c*f + b*d*e)/(2*h) - (b*d*f*g)/(2*h^2)) + (log(g +
 h*x)*(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e*g*h^2 - b*c*e*g*h^2 + a*
d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h))/h^4 + (b*d*f*x^3)/(3*h)
```

3.3 $\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx$

Optimal. Leaf size=84

$$\frac{bdx}{fh}+\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}$$

```
[Out] b*d*x/f/h+(-a*f+b*e)*(-c*f+d*e)*ln(f*x+e)/f^2/(-e*h+f*g)-(-a*h+b*g)*(-c*h+d
*g)*ln(h*x+g)/h^2/(-e*h+f*g)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{147\}$

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}+\frac{bdx}{fh}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] (b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx &= \int \left(\frac{bd}{fh}+\frac{(-be+af)(-de+cf)}{f(fg-eh)(e+fx)}+\frac{(-bg+ah)(-dg+ch)}{h(-fg+eh)(g+hx)}\right)dx\\ &= \frac{bdx}{fh}+\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 85, normalized size = 1.01

$$\frac{(be-af)(de-cf)h^2\log(e+fx)+f(bdh(fg-eh)x-f(bg-ah)(dg-ch)\log(g+hx))}{f^2h^2(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*h^2*Log[e + f*x] + f*(b*d*h*(f*g - e*h)*x - f*(b*g
 - a*h)*(d*g - c*h)*Log[g + h*x]))/(f^2*h^2*(f*g - e*h))
```

Maple [A]
time = 0.11, size = 102, normalized size = 1.21

method	result
default	$\frac{bdx}{fh}+\frac{\left(-ac\,f^{2}+adef+bcef-bd\,e^{2}\right)\ln\left(fx+e\right)}{f^{2}\left(eh-fg\right)}+\frac{\left(ac\,h^{2}-adgh-bcgh+bd\,g^{2}\right)\ln\left(hx+g\right)}{h^{2}\left(eh-fg\right)}$
norman	$\frac{bdx}{fh}+\frac{\left(ac\,h^{2}-adgh-bcgh+bd\,g^{2}\right)\ln\left(hx+g\right)}{h^{2}\left(eh-fg\right)}-\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)\ln\left(fx+e\right)}{\left(eh-fg\right)f^{2}}$
risch	$\frac{bdx}{fh}+\frac{\ln\left(-hx-g\right)ac}{eh-fg}-\frac{\ln\left(-hx-g\right)adg}{h\left(eh-fg\right)}-\frac{\ln\left(-hx-g\right)bcg}{h\left(eh-fg\right)}+\frac{\ln\left(-hx-g\right)bd\,g^{2}}{h^{2}\left(eh-fg\right)}-\frac{\ln\left(fx+e\right)ac}{eh-fg}+\frac{\ln\left(fx+e\right)ade}{\left(eh-fg\right)f}+\frac{\ln\left(fx+e\right)bce}{\left(eh-fg\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] b*d*x/f/h+(-a*c*f^2+a*d*e*f+b*c*e*f-b*d*e^2)/f^2/(e*h-f*g)*ln(f*x+e)+1/h^2*
(a*c*h^2-a*d*g*h-b*c*g*h+b*d*g^2)/(e*h-f*g)*ln(h*x+g)
```

Maxima [A]
time = 0.30, size = 109, normalized size = 1.30

$$\frac{bdx}{fh}+\frac{\left(acf^{2}+bde^{2}-(bce+ade)f\right)\log\left(fx+e\right)}{f^{3}g-f^{2}he}-\frac{\left(bdg^{2}+ach^{2}-(bc+ad)gh\right)\log\left(hx+g\right)}{fgh^{2}-h^{3}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b*d*x/(f*h) + (a*c*f^2 + b*d*e^2 - (b*c*e + a*d*e)*f)*log(f*x + e)/(f^3*g -
 f^2*h*e) - (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(h*x + g)/(f*g*h^2 - h
^3*e)
```

Fricas [A]
time = 1.12, size = 125, normalized size = 1.49

$$\frac{bdf^{2}ghx-bdfh^{2}xe+\left(acf^{2}h^{2}+bdh^{2}e^{2}-(bc+ad)fh^{2}e\right)\log\left(fx+e\right)-\left(bdf^{2}g^{2}+acf^{2}h^{2}-(bc+ad)f^{2}gh\right)\log\left(hx+g\right)}{f^{3}gh^{2}-f^{2}h^{3}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] (b*d*f^2*g*h*x - b*d*f*h^2*x*e + (a*c*f^2*h^2 + b*d*h^2*e^2 - (b*c + a*d)*f
*h^2*e)*log(f*x + e) - (b*d*f^2*g^2 + a*c*f^2*h^2 - (b*c + a*d)*f^2*g*h)*lo
g(h*x + g))/(f^3*g*h^2 - f^2*h^3*e)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(68) = 136$.
time = 196.58, size = 507, normalized size = 6.04

$$\frac{bdx}{fh} + \frac{(ah-bg)(ch-dg)\log\left(x + \frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2-\frac{e^2fh(ah-bg)(ch-dg)}{eh-fg}+\frac{2ef^2g(ah-bg)(ch-dg)}{eh-fg}-\frac{f^3g^2(ah-bg)(ch-dg)}{h(eh-fg)}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}\right)}{h^2(eh-fg)} - \frac{(af-be)(cf-de)\log\left(x + \frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2+\frac{e^2h^3(af-be)(cf-de)}{f(eh-fg)}-\frac{2egh^2(af-be)(cf-de)}{eh-fg}+\frac{fg^2h(af-be)(cf-de)}{eh-fg}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}\right)}{f^2(eh-fg)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*x/(f*h) + (a*h - b*g)*(c*h - d*g)*log(x + (a*c*e*f*h**2 + a*c*f**2*g*h
- 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h + b*d*e*f*g**2 - e**2*f*h*(a
*h - b*g)*(c*h - d*g)/(e*h - f*g) + 2*e*f**2*g*(a*h - b*g)*(c*h - d*g)/(e*h
 - f*g) - f**3*g**2*(a*h - b*g)*(c*h - d*g)/(h*(e*h - f*g)))/(2*a*c*f**2*h*
*2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**2 - b*c*f**2*g*h + b*d*e**2*h
**2 + b*d*f**2*g**2))/(h**2*(e*h - f*g)) - (a*f - b*e)*(c*f - d*e)*log(x +
(a*c*e*f*h**2 + a*c*f**2*g*h - 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h
 + b*d*e*f*g**2 + e**2*h**3*(a*f - b*e)*(c*f - d*e)/(f*(e*h - f*g)) - 2*e*g
*h**2*(a*f - b*e)*(c*f - d*e)/(e*h - f*g) + f*g**2*h*(a*f - b*e)*(c*f - d*e
)/(e*h - f*g))/(2*a*c*f**2*h**2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**
2 - b*c*f**2*g*h + b*d*e**2*h**2 + b*d*f**2*g**2))/(f**2*(e*h - f*g))
```

Giac [A]
time = 0.55, size = 112, normalized size = 1.33

$$\frac{bdx}{fh} + \frac{(acf^2 - bcfe - adfe + bde^2)\log(|fx+e|)}{f^3g - f^2he} - \frac{(bdg^2 - bcgh - adgh + ach^2)\log(|hx+g|)}{fgh^2 - h^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] b*d*x/(f*h) + (a*c*f^2 - b*c*f*e - a*d*f*e + b*d*e^2)*log(abs(f*x + e))/(f^
3*g - f^2*h*e) - (b*d*g^2 - b*c*g*h - a*d*g*h + a*c*h^2)*log(abs(h*x + g))/
(f*g*h^2 - h^3*e)
```

Mupad [B]
time = 2.97, size = 105, normalized size = 1.25

$$\frac{\ln(e+fx)\,(acf^2 - f(ade+bce) + bde^2)}{f^3g - ef^2h} + \frac{\ln(g+hx)\,(ach^2 - h(adg+bcg) + bdg^2)}{eh^3 - fgh^2} + \frac{bdx}{fh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

```
[Out] (log(e + f*x)*(a*c*f^2 - f*(a*d*e + b*c*e) + b*d*e^2))/(f^3*g - e*f^2*h) +
(log(g + h*x)*(a*c*h^2 - h*(a*d*g + b*c*g) + b*d*g^2))/(e*h^3 - f*g*h^2) +
(b*d*x)/(f*h)
```

3.4 $\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

```
[Out] -(-a*d+b*c)*ln(d*x+c)/(-c*f+d*e)/(-c*h+d*g)+(-a*f+b*e)*ln(f*x+e)/(-c*f+d*e)
/(-e*h+f*g)-(-a*h+b*g)*ln(h*x+g)/(-c*h+d*g)/(-e*h+f*g)
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {153}

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log[
e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g - c*
h)*(f*g - e*h))
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx &= \int \left(\frac{d(-bc+ad)}{(de-cf)(dg-ch)(c+dx)}+\frac{f(-be+af)}{(de-cf)(-fg+eh)(e+fx)}+\frac{\ldots}{(dg-\ldots}\right. \\ &= -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 102, normalized size = 0.94

$$\frac{(bc-ad)(fg-eh)\log(c+dx)-(be-af)(dg-ch)\log(e+fx)+(de-cf)(bg-ah)\log(g+hx)}{(de-cf)(dg-ch)(-fg+eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*c - a*d)*(f*g - e*h)*Log[c + d*x] - (b*e - a*f)*(d*g - c*h)*Log[e + f*x
] + (d*e - c*f)*(b*g - a*h)*Log[g + h*x])/((d*e - c*f)*(d*g - c*h)*(-(f*g)
+ e*h))
```

Maple [A]
time = 0.14, size = 108, normalized size = 1.00

method	result
default	$\frac{(ad-bc)\ln(dx+c)}{(cf-de)(ch-dg)}-\frac{(af-be)\ln(fx+e)}{(cf-de)(eh-fg)}+\frac{(ah-bg)\ln(hx+g)}{(ch-dg)(eh-fg)}$
norman	$\frac{(ah-bg)\ln(hx+g)}{ce\,h^{2}-cfgh-degh+df\,g^{2}}+\frac{(ad-bc)\ln(dx+c)}{(cf-de)(ch-dg)}-\frac{(af-be)\ln(fx+e)}{(cf-de)(eh-fg)}$
risch	$\frac{\ln(-hx-g)ah}{ce\,h^{2}-cfgh-degh+df\,g^{2}}-\frac{\ln(-hx-g)bg}{ce\,h^{2}-cfgh-degh+df\,g^{2}}+\frac{\ln(dx+c)ad}{c^{2}fh-cdeh-cdfg+d^{2}eg}-\frac{\ln(dx+c)bc}{c^{2}fh-cdeh-cdfg+d^{2}eg}-\frac{\ln(-fx-}{cefh-c\,f^{2}g-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] (a*d-b*c)/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)-(a*f-b*e)/(c*f-d*e)/(e*h-f*g)*ln(f*
x+e)+(a*h-b*g)/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)
```

Maxima [A]
time = 0.28, size = 140, normalized size = 1.30

$$\frac{(bc-ad)\log(dx+c)}{(cdf-d^{2}e)g-(c^{2}f-cde)h}+\frac{(af-be)\log(fx+e)}{(cf^{2}-dfe)g-(cfe-de^{2})h}-\frac{(bg-ah)\log(hx+g)}{dfg^{2}+ch^{2}e-(cf+de)gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*log(d*x + c)/((c*d*f - d^2*e)*g - (c^2*f - c*d*e)*h) + (a*f - b
*e)*log(f*x + e)/((c*f^2 - d*f*e)*g - (c*f*e - d*e^2)*h) - (b*g - a*h)*log(
h*x + g)/(d*f*g^2 + c*h^2*e - (c*f + d*e)*g*h)
```

Fricas [A]
time = 109.75, size = 163, normalized size = 1.51

$$\frac{((bc-ad)fg-(bc-ad)he)\log(dx+c)+(adfg-acfh-(bdg-bch)e)\log(fx+e)-(bcfg-acfh-(bdg-adh)e)\log(hx+g)}{cdf^{2}g^{2}-c^{2}f^{2}gh+(d^{2}gh-cdh^{2})e^{2}-(d^{2}fg^{2}-c^{2}fh^{2})e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] (((b*c - a*d)*f*g - (b*c - a*d)*h*e)*log(d*x + c) + (a*d*f*g - a*c*f*h - (b
*d*g - b*c*h)*e)*log(f*x + e) - (b*c*f*g - a*c*f*h - (b*d*g - a*d*h)*e)*log
```

```
(h*x + g))/(c*d*f^2*g^2 - c^2*f^2*g*h + (d^2*g*h - c*d*h^2)*e^2 - (d^2*f*g^
2 - c^2*f*h^2)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 162, normalized size = 1.50

$$\frac{(bcd-ad^2)\log(|dx+c|)}{cd^2fg-c^2dfh-d^3ge+cd^2he}+\frac{(af^2-bfe)\log(|fx+e|)}{cf^3g-df^2ge-cf^2he+dfhe^2}-\frac{(bgh-ah^2)\log(|hx+g|)}{dfg^2h-cfgh^2-dgh^2e+ch^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] (b*c*d - a*d^2)*log(abs(d*x + c))/(c*d^2*f*g - c^2*d*f*h - d^3*g*e + c*d^2*
h*e) + (a*f^2 - b*f*e)*log(abs(f*x + e))/(c*f^3*g - d*f^2*g*e - c*f^2*h*e +
 d*f*h*e^2) - (b*g*h - a*h^2)*log(abs(h*x + g))/(d*f*g^2*h - c*f*g*h^2 - d*
g*h^2*e + c*h^3*e)
```

Mupad [B]
time = 4.17, size = 127, normalized size = 1.18

$$\frac{\ln(e+f\,x)\,(a\,f-b\,e)}{c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g}+\frac{\ln(g+h\,x)\,(a\,h-b\,g)}{c\,e\,h^2+d\,f\,g^2-c\,f\,g\,h-d\,e\,g\,h}+\frac{\ln(c+d\,x)\,(a\,d-b\,c)}{d^2\,e\,g+c^2\,f\,h-c\,d\,e\,h-c\,d\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

```
[Out] (log(e + f*x)*(a*f - b*e))/(c*f^2*g + d*e^2*h - c*e*f*h - d*e*f*g) + (log(g
 + h*x)*(a*h - b*g))/(c*e*h^2 + d*f*g^2 - c*f*g*h - d*e*g*h) + (log(c + d*x
)*(a*d - b*c))/(d^2*e*g + c^2*f*h - c*d*e*h - c*d*f*g)
```

3.5 $\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=163

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(\cdots}{(bg-ah)(dg-\cdots}$$

```
[Out] b^2*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)-d^2*ln(d*x+c)/(-a*d+b*c)/(-c
*f+d*e)/(-c*h+d*g)+f^2*ln(f*x+e)/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)-h^2*ln(h*
x+g)/(-a*h+b*g)/(-c*h+d*g)/(-e*h+f*g)
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {186}

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f)*
(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(f*g
 - e*h))
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx &= \int \left(\frac{b^3}{(bc-ad)(be-af)(bg-ah)(a+bx)}-\frac{\cdots}{(bc-ad)(-de+c\cdots}\right.\\ &= \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\cdots\end{aligned}$$

Mathematica [A]
time = 0.15, size = 164, normalized size = 1.01

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(-de+cf)(-dg+ch)}-\frac{f^2\log(e+fx)}{(be-af)(de-cf)(-fg+eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) - (f^2*Log[e + f*x])/((b*e -
 a*f)*(d*e - c*f)*(-(f*g) + e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g -
c*h)*(f*g - e*h))
```

Maple [A]
time = 0.21, size = 164, normalized size = 1.01

method	result
default	$-\frac{b^{2} \ln(bx+a)}{(ad-bc)(af-be)(ah-bg)}+\frac{d^{2} \ln(dx+c)}{(ad-bc)(cf-de)(ch-dg)}-\frac{f^{2} \ln(fx+e)}{(af-be)(cf-de)(eh-fg)}+\frac{h^{2} \ln(hx+g)}{(ah-bg)(ch-dg)(eh-fg)}$
norman	$\frac{h^{2} \ln(hx+g)}{ace\,h^{3}-acfg\,h^{2}-adeg\,h^{2}+adf\,g^{2}h-bceg\,h^{2}+bcf\,g^{2}h+bde\,g^{2}h-bdf\,g^{3}}+\frac{d^{2} \ln(dx+c)}{(ad-bc)(cf-de)(ch-dg)}-\frac{f^{2} \ln(fx+e)}{(ac\,f^{2}-adef-bcef+bd\,e^{2})(e}$
risch	$-\frac{b^{2} \ln(-bx-a)}{a^{3}dfh-a^{2}bcfh-a^{2}bdeh-a^{2}bdfg+a\,b^{2}ceh+a\,b^{2}cfg+a\,b^{2}deg-b^{3}ceg}+\frac{h^{2} \ln(hx+g)}{ace\,h^{3}-acfg\,h^{2}-adeg\,h^{2}+adf\,g^{2}h-bceg\,h^{2}+bcf\,g^{2}h+bde\,g}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2/(a*d-b*c)/(a*f-b*e)/(a*h-b*g)*ln(b*x+a)+d^2/(a*d-b*c)/(c*f-d*e)/(c*h-d
*g)*ln(d*x+c)-f^2/(a*f-b*e)/(c*f-d*e)/(e*h-f*g)*ln(f*x+e)+h^2/(a*h-b*g)/(c*
h-d*g)/(e*h-f*g)*ln(h*x+g)
```

Maxima [A]
time = 0.30, size = 321, normalized size = 1.97

$$\frac{b^{2}\log\left(bx+a\right)}{(b^{3}ce-ab^{2}de-(ab^{2}c-a^{2}bd)f)g-(ab^{2}ce-a^{2}bde-(a^{2}bc-a^{3}d)f)h}-\frac{d^{2}\log\left(dx+c\right)}{(bcd^{2}e-ad^{3}e-(bc^{2}d-acd^{2})f)g-(bc^{2}de-acd^{2}e-(bc^{3}-ac^{2}d)f)h}+\frac{f^{2}\log\left(fx+e\right)}{(acf^{3}+bdfe^{2}-(bce+ade)f^{2})g-(acf^{2}e+bde^{3}-(bce^{2}+ade^{2})f)h}-\frac{h^{2}\log\left(hx+g\right)}{bdfg^{3}-ach^{3}e-(bde+(bc+ad)f)g^{2}h+(acf+bce+ade)gh^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/((b^3*c*e - a*b^2*d*e - (a*b^2*c - a^2*b*d)*f)*g - (a*b^2*
c*e - a^2*b*d*e - (a^2*b*c - a^3*d)*f)*h) - d^2*log(d*x + c)/((b*c*d^2*e -
a*d^3*e - (b*c^2*d - a*c*d^2)*f)*g - (b*c^2*d*e - a*c*d^2*e - (b*c^3 - a*c^
2*d)*f)*h) + f^2*log(f*x + e)/((a*c*f^3 + b*d*f*e^2 - (b*c*e + a*d*e)*f^2)*
g - (a*c*f^2*e + b*d*e^3 - (b*c*e^2 + a*d*e^2)*f)*h) - h^2*log(h*x + g)/(b*
d*f*g^3 - a*c*h^3*e - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + b*c*e + a*d*
e)*g*h^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(170) = 340.
time = 0.60, size = 363, normalized size = 2.23

$$-\frac{b^3\log\left(|bx+a|\right)}{ab^3cfg-a^2b^2dfg-a^2b^2cfh+a^3bdfh-b^4cge+ab^3dge+ab^3che-a^2b^2dhe}+\frac{d^3\log\left(|dx+c|\right)}{bc^2d^2fg-acd^3fg-bc^3dfh+ac^2d^2fh-bcd^3ge+ad^4ge+bc^2d^2he-acd^3he}+\frac{f^3\log\left(|fx+e|\right)}{acf^4g-bcf^3ge-adf^3ge-acf^3he+bdf^2ge^2+bcf^2he^2+adf^2he^2-bdfhe^3}-\frac{h^3\log\left(|hx+g|\right)}{bdfg^3h-bcfg^2h^2-adfg^2h^2+acfgh^3-bdg^2h^2e+bcgh^3e+adgh^3e-ach^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(b*x + a))/(a*b^3*c*f*g - a^2*b^2*d*f*g - a^2*b^2*c*f*h + a^3*b
*d*f*h - b^4*c*g*e + a*b^3*d*g*e + a*b^3*c*h*e - a^2*b^2*d*h*e) + d^3*log(a
bs(d*x + c))/(b*c^2*d^2*f*g - a*c*d^3*f*g - b*c^3*d*f*h + a*c^2*d^2*f*h - b
*c*d^3*g*e + a*d^4*g*e + b*c^2*d^2*h*e - a*c*d^3*h*e) + f^3*log(abs(f*x + e
))/(a*c*f^4*g - b*c*f^3*g*e - a*d*f^3*g*e - a*c*f^3*h*e + b*d*f^2*g*e^2 + b
*c*f^2*h*e^2 + a*d*f^2*h*e^2 - b*d*f*h*e^3) - h^3*log(abs(h*x + g))/(b*d*f*
g^3*h - b*c*f*g^2*h^2 - a*d*f*g^2*h^2 + a*c*f*g*h^3 - b*d*g^2*h^2*e + b*c*g
*h^3*e + a*d*g*h^3*e - a*c*h^4*e)
```

Mupad [**B**]
time = 6.62, size = 317, normalized size = 1.94

$$\frac{b^2\,\ln\left(a+b\,x\right)}{b^3\,c\,e\,g-a^3\,d\,f\,h-a\,b^2\,c\,e\,h-a\,b^2\,c\,f\,g-a\,b^2\,d\,e\,g+a^2\,b\,c\,f\,h+a^2\,b\,d\,e\,h+a^2\,b\,d\,f\,g}+\frac{d^2\,\ln\left(c+d\,x\right)}{a\,d^3\,e\,g-b\,c^3\,f\,h-a\,c\,d^2\,e\,h-a\,c\,d^2\,f\,g-b\,c\,d^2\,e\,g+a\,c^2\,d\,f\,h+b\,c^2\,d\,e\,h+b\,c^2\,d\,f\,g}+\frac{f^2\,\ln\left(e+f\,x\right)}{a\,c\,f^3\,g-b\,d\,e^3\,h-a\,c\,e\,f^2\,h-a\,d\,e\,f^2\,g-b\,c\,e\,f^2\,g+a\,d\,e^2\,f\,h+b\,c\,e^2\,f\,h+b\,d\,e^2\,f\,g}+\frac{h^2\,\ln\left(g+h\,x\right)}{a\,c\,e\,h^3-b\,d\,f\,g^3-a\,c\,f\,g\,h^2-a\,d\,e\,g\,h^2-b\,c\,e\,g\,h^2+a\,d\,f\,g^2\,h+b\,c\,f\,g^2\,h+b\,d\,e\,g^2\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x)),x)
```

```
[Out] (b^2*log(a + b*x))/(b^3*c*e*g - a^3*d*f*h - a*b^2*c*e*h - a*b^2*c*f*g - a*b
^2*d*e*g + a^2*b*c*f*h + a^2*b*d*e*h + a^2*b*d*f*g) + (d^2*log(c + d*x))/(a
*d^3*e*g - b*c^3*f*h - a*c*d^2*e*h - a*c*d^2*f*g - b*c*d^2*e*g + a*c^2*d*f*
h + b*c^2*d*e*h + b*c^2*d*f*g) + (f^2*log(e + f*x))/(a*c*f^3*g - b*d*e^3*h
- a*c*e*f^2*h - a*d*e*f^2*g - b*c*e*f^2*g + a*d*e^2*f*h + b*c*e^2*f*h + b*d
*e^2*f*g) + (h^2*log(g + h*x))/(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e
*g*h^2 - b*c*e*g*h^2 + a*d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h)
```

3.6 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {153}

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)} \, dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right) dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A]
time = 0.11, size = 20, normalized size = 0.87

method	result	size
default	$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$	20
norman	$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$	20
risch	$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Maxima [A]
time = 0.32, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log(x+3)+2\log(x+2)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Fricas [A]
time = 1.27, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log(x+3)+2\log(x+2)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.87

$$-\frac{\log(x+1)}{2}+2\log(x+2)-\frac{3\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Giac [A]
time = 0.58, size = 22, normalized size = 0.96

$$-\frac{3}{2}\log(|x+3|) + 2\log(|x+2|) - \frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

Mupad [B]
time = 0.08, size = 19, normalized size = 0.83

$$2\ln(x+2) - \frac{\ln(x+1)}{2} - \frac{3\ln(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

3.7 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx$

Optimal. Leaf size=43

$$-\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}$$

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+20/3993*ln(6-x)+1493/499125*ln(3+5*x)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1607, 153}

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3} \, dx\\ &= \int \left(\frac{20}{3993(-6+x)}+\frac{24}{275(3+5x)^3}-\frac{201}{3025(3+5x)^2}+\frac{1493}{99825(3+5x)}\right) dx\\ &= -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.77

$$\frac{\frac{99(157+335x)}{(3+5x)^2}+2500\log(-6+x)+1493\log(3+5x)}{499125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.79

method	result	size
risch	$\frac{\frac{201x}{3025}+\frac{471}{15125}}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$	30
norman	$\frac{-\frac{113}{3025}x-\frac{157}{1815}x^2}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$	33
default	$\frac{20\ln(-6+x)}{3993}-\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{1493\ln(3+5x)}{499125}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)/(-6+x)/(3+5*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 20/3993*ln(-6+x)-12/1375/(3+5*x)^2+201/15125/(3+5*x)+1493/499125*ln(3+5*x)
```

Maxima [A]
time = 0.31, size = 34, normalized size = 0.79

$$\frac{3\,(335\,x+157)}{15125\,(25\,x^2+30\,x+9)}+\frac{1493}{499125}\,\log\left(5\,x+3\right)+\frac{20}{3993}\,\log\left(x-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

Fricas [A]
time = 1.69, size = 53, normalized size = 1.23

$$\frac{1493\,(25\,x^2+30\,x+9)\log\left(5\,x+3\right)+2500\,(25\,x^2+30\,x+9)\log\left(x-6\right)+33165\,x+15543}{499125\,(25\,x^2+30\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A]
time = 0.06, size = 32, normalized size = 0.74

$$\frac{1005x + 471}{378125x^2 + 453750x + 136125} + \frac{20\log(x-6)}{3993} + \frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

Giac [A]
time = 0.76, size = 31, normalized size = 0.72

$$\frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2} + \frac{1493}{499125}\,\log(|5\,x+3|) + \frac{20}{3993}\,\log(|x-6|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

Mupad [B]
time = 0.12, size = 29, normalized size = 0.67

$$\frac{20\,\ln(x-6)}{3993} + \frac{1493\,\ln\left(x+\frac{3}{5}\right)}{499125} + \frac{\frac{201\,x}{75625}+\frac{471}{378125}}{x^2+\frac{6\,x}{5}+\frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

```
[Out] (20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/37
8125)/((6*x)/5 + x^2 + 9/25)
```

3.8 $\int \frac{(a+bx)^3\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=227

$$2a^3e\sqrt{c+dx}+\frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}+\frac{2(c+dx)^{3/2}\left(2(20a^3d^3\right.}{}$$

```
[Out] 2/21*(2*a*d*f-2*b*c*f+3*b*d*e)*(b*x+a)^2*(d*x+c)^(3/2)/d^2+2/9*f*(b*x+a)^3*
(d*x+c)^(3/2)/d+2/315*(d*x+c)^(3/2)*(40*a^3*d^3*f+6*a^2*b*d^2*(-16*c*f+45*d
*e)-18*a*b^2*c*d*(-4*c*f+7*d*e)+8*b^3*c^2*(-2*c*f+3*d*e)+3*b*d*(21*a*b*d^2*
e-4*(-a*d+b*c)*(2*a*d*f-2*b*c*f+3*b*d*e))*x)/d^4-2*a^3*e*arctanh((d*x+c)^(1
/2)/c^(1/2))*c^(1/2)+2*a^3*e*(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {158, 152, 52, 65, 214}

$$2a^3e\sqrt{c+dx}-2a^3\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2(c+dx)^{3/2}\left(2\left(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf)\right)+3bdx(21abd^2e-4(bc-ad)(2adf-2bcf+3bde))\right)}{315d^4}+\frac{2(a+bx)^2(c+dx)^{3/2}(2adf-2bcf+3bde)}{21d^2}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^3*e*Sqrt[c + d*x] + (2*(3*b*d*e - 2*b*c*f + 2*a*d*f)*(a + b*x)^2*(c + d
*x)^(3/2))/(21*d^2) + (2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + (2*(c + d*x
)^(3/2)*(2*(20*a^3*d^3*f + 3*a^2*b*d^2*(45*d*e - 16*c*f) - 9*a*b^2*c*d*(7*d
*e - 4*c*f) + 4*b^3*c^2*(3*d*e - 2*c*f)) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a
*d)*(3*b*d*e - 2*b*c*f + 2*a*d*f))*x))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[
Sqrt[c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^3\sqrt{c+dx}\,(e+fx)}{x}\,dx = \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2\int \frac{(a+bx)^2\sqrt{c+dx}\left(\frac{9ade}{2}+\frac{3}{2}(3bde-2bcf+2adf)x\right)}{x}dx}{9d}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} +$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} +$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+b}{}$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+b}{}$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+b}{}$$

Mathematica [A]

time = 0.32, size = 197, normalized size = 0.87

$$\frac{2\sqrt{c+dx}\,(105a^3d^3(3de+cf+dfx)+63a^2bd^2(c+dx)(5de-2cf+3dfx)+9ab^2d(c+dx)\left(8c^2f+3d^2x(7e+5fx)-2cd(7e+6fx)\right)-b^3(c+dx)\left(16c^3f-24c^2d(e+fx)+6cd^2x(6e+5fx)-5d^3x^2(9e+7fx)\right))}{315d^4} - 2a^3\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(105*a^3*d^3*(3*d*e + c*f + d*f*x) + 63*a^2*b*d^2*(c + d*x
)*(5*d*e - 2*c*f + 3*d*f*x) + 9*a*b^2*d*(c + d*x)*(8*c^2*f + 3*d^2*x*(7*e +
 5*f*x) - 2*c*d*(7*e + 6*f*x)) - b^3*(c + d*x)*(16*c^3*f - 24*c^2*d*(e + f*
x) + 6*c*d^2*x*(6*e + 5*f*x) - 5*d^3*x^2*(9*e + 7*f*x))))/(315*d^4) - 2*a^3
*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Maple [A]

time = 0.10, size = 301, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{2f\,b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6a\,b^2df(dx+c)^{\frac{7}{2}}}{7}-\frac{6b^3cf(dx+c)^{\frac{7}{2}}}{7}+\frac{2b^3de(dx+c)^{\frac{7}{2}}}{7}+\frac{6a^2b\,d^2f(dx+c)^{\frac{5}{2}}}{5}-\frac{12a\,b^2cdf(dx+c)^{\frac{5}{2}}}{5}+\frac{6a\,b^2d^2e(dx+c)^{\frac{5}{2}}}{5}+6}{}$
default	$\frac{\frac{2f\,b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6a\,b^2df(dx+c)^{\frac{7}{2}}}{7}-\frac{6b^3cf(dx+c)^{\frac{7}{2}}}{7}+\frac{2b^3de(dx+c)^{\frac{7}{2}}}{7}+\frac{6a^2b\,d^2f(dx+c)^{\frac{5}{2}}}{5}-\frac{12a\,b^2cdf(dx+c)^{\frac{5}{2}}}{5}+\frac{6a\,b^2d^2e(dx+c)^{\frac{5}{2}}}{5}+6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/9*f*b^3*(d*x+c)^(9/2)+3/7*a*b^2*d*f*(d*x+c)^(7/2)-3/7*b^3*c*f*(d*x
+c)^(7/2)+1/7*b^3*d*e*(d*x+c)^(7/2)+3/5*a^2*b*d^2*f*(d*x+c)^(5/2)-6/5*a*b^2
*c*d*f*(d*x+c)^(5/2)+3/5*a*b^2*d^2*e*(d*x+c)^(5/2)+3/5*b^3*c^2*f*(d*x+c)^(5
/2)-2/5*b^3*c*d*e*(d*x+c)^(5/2)+1/3*a^3*d^3*f*(d*x+c)^(3/2)-a^2*b*c*d^2*f*(
d*x+c)^(3/2)+a^2*b*d^3*e*(d*x+c)^(3/2)+a*b^2*c^2*d*f*(d*x+c)^(3/2)-a*b^2*c*
d^2*e*(d*x+c)^(3/2)-1/3*b^3*c^3*f*(d*x+c)^(3/2)+1/3*b^3*c^2*d*e*(d*x+c)^(3/
2)+a^3*d^4*e*(d*x+c)^(1/2)-a^3*c^(1/2)*d^4*e*arctanh((d*x+c)^(1/2)/c^(1/2))
)
```

Maxima [A]
time = 0.53, size = 246, normalized size = 1.08

$$a^{3} \sqrt{c} e \log\left(\frac{\sqrt{dx+c} - \sqrt{c}}{\sqrt{dx+c} + \sqrt{c}}\right) + \frac{2 \left(35 (dx+c)^{\frac{9}{2}} b^{3} f + 315 \sqrt{dx+c}\, a^{3} d^{4} e + 45 (b^{3} d e - 3 (b^{3} c - a b^{2} d) f)(dx+c)^{\frac{7}{2}} - 63 (2 b^{3} c d e - 3 a b^{2} d^{2} e - 3 (b^{3} c^{2} - 2 a b^{2} c d + a^{2} b d^{2}) f)(dx+c)^{\frac{5}{2}} + 105 (b^{3} c^{2} d e - 3 a b^{2} c d^{2} e + 3 a^{2} b d^{3} e - (b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}) f)(dx+c)^{\frac{3}{2}}\right)}{315 d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^3*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
315*(35*(d*x + c)^(9/2)*b^3*f + 315*sqrt(d*x + c)*a^3*d^4*e + 45*(b^3*d*e -
 3*(b^3*c - a*b^2*d)*f)*(d*x + c)^(7/2) - 63*(2*b^3*c*d*e - 3*a*b^2*d^2*e -
 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f)*(d*x + c)^(5/2) + 105*(b^3*c^2*d*
e - 3*a*b^2*c*d^2*e + 3*a^2*b*d^3*e - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*f)*(d*x + c)^(3/2))/d^4
```

Fricas [A]
time = 1.87, size = 643, normalized size = 2.83

$$\left[\frac{315 a^{3} \sqrt{c} d^{4} e \log\left(\frac{dx - 2 \sqrt{dx+c} \sqrt{c} + 2c}{x}\right) + 2 \left(35 b^{3} d^{4} f x^{4} + 5 (b^{3} c d^{3} + 27 a b^{2} d^{4}) f x^{3} - 3 (2 b^{3} c^{2} d^{2} - 9 a b^{2} c d^{3} - 63 a^{2} b d^{4}) f x^{2} + (8 b^{3} c^{3} d - 36 a b^{2} c^{2} d^{2} + 63 a^{2} b c d^{3} + 105 a^{3} d^{4}) f x - (16 b^{3} c^{4} - 72 a b^{2} c^{3} d + 126 a^{2} b c^{2} d^{2} - 105 a^{3} c d^{3}) f + 3 (15 b^{3} d^{4} x^{3} + 8 b^{3} c^{3} d - 42 a b^{2} c^{2} d^{2} + 105 a^{2} b c d^{3} + 105 a^{3} d^{4} + 3 (b^{3} c d^{3} + 21 a b^{2} d^{4}) x^{2} - (4 b^{3} c^{2} d^{2} - 21 a b^{2} c d^{3} - 105 a^{2} b d^{4}) x) e\right) \sqrt{dx+c}}{315 d^{4}}, \frac{2 \left(315 a^{3} \sqrt{-c} d^{4} \arctan\left(\frac{\sqrt{dx+c} \sqrt{-c}}{c}\right) e + \left(35 b^{3} d^{4} f x^{4} + 5 (b^{3} c d^{3} + 27 a b^{2} d^{4}) f x^{3} - 3 (2 b^{3} c^{2} d^{2} - 9 a b^{2} c d^{3} - 63 a^{2} b d^{4}) f x^{2} + (8 b^{3} c^{3} d - 36 a b^{2} c^{2} d^{2} + 63 a^{2} b c d^{3} + 105 a^{3} d^{4}) f x - (16 b^{3} c^{4} - 72 a b^{2} c^{3} d + 126 a^{2} b c^{2} d^{2} - 105 a^{3} c d^{3}) f + 3 (15 b^{3} d^{4} x^{3} + 8 b^{3} c^{3} d - 42 a b^{2} c^{2} d^{2} + 105 a^{2} b c d^{3} + 105 a^{3} d^{4} + 3 (b^{3} c d^{3} + 21 a b^{2} d^{4}) x^{2} - (4 b^{3} c^{2} d^{2} - 21 a b^{2} c d^{3} - 105 a^{2} b d^{4}) x) e\right) \sqrt{dx+c}\right)}{315 d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*a^3*sqrt(c)*d^4*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(35*b^3*d^4*f*x^4 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*f*x^3 - 3*(2*b^3*c^2*d
^2 - 9*a*b^2*c*d^3 - 63*a^2*b*d^4)*f*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2
+ 63*a^2*b*c*d^3 + 105*a^3*d^4)*f*x - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^
2*b*c^2*d^2 - 105*a^3*c*d^3)*f + 3*(15*b^3*d^4*x^3 + 8*b^3*c^3*d - 42*a*b^2
*c^2*d^2 + 105*a^2*b*c*d^3 + 105*a^3*d^4 + 3*(b^3*c*d^3 + 21*a*b^2*d^4)*x^2
 - (4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*x)*e)*sqrt(d*x + c))/d^
4, 2/315*(315*a^3*sqrt(-c)*d^4*arctan(sqrt(d*x + c)*sqrt(-c)/c)*e + (35*b^3
```

```
*d^4*f*x^4 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*f*x^3 - 3*(2*b^3*c^2*d^2 - 9*a*b^
2*c*d^3 - 63*a^2*b*d^4)*f*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*
c*d^3 + 105*a^3*d^4)*f*x - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2
 - 105*a^3*c*d^3)*f + 3*(15*b^3*d^4*x^3 + 8*b^3*c^3*d - 42*a*b^2*c^2*d^2 +
105*a^2*b*c*d^3 + 105*a^3*d^4 + 3*(b^3*c*d^3 + 21*a*b^2*d^4)*x^2 - (4*b^3*c
^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*x)*e)*sqrt(d*x + c))/d^4]
```

Sympy [A]
time = 15.11, size = 274, normalized size = 1.21

$$\frac{2a^3ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^3e\sqrt{c+dx}+\frac{2b^3f(c+dx)^{\frac{9}{2}}}{9d^4}+\frac{2(c+dx)^{\frac{7}{2}}\cdot(3ab^2df-3b^3cf+b^3de)}{7d^4}+\frac{2(c+dx)^{\frac{5}{2}}\cdot(3a^2bd^2f-6ab^2cdf+3ab^2d^2e+3b^3c^2f-2b^3cde)}{5d^4}+\frac{2(c+dx)^{\frac{3}{2}}\left(a^3d^3f-3a^2bcd^2f+3a^2bd^3e+3ab^2c^2df-3ab^2cd^2e-b^3c^3f+b^3c^2de\right)}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**3*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**3*e*sqrt(c + d*x) +
 2*b**3*f*(c + d*x)**(9/2)/(9*d**4) + 2*(c + d*x)**(7/2)*(3*a*b**2*d*f - 3*
b**3*c*f + b**3*d*e)/(7*d**4) + 2*(c + d*x)**(5/2)*(3*a**2*b*d**2*f - 6*a*b
**2*c*d*f + 3*a*b**2*d**2*e + 3*b**3*c**2*f - 2*b**3*c*d*e)/(5*d**4) + 2*(c
 + d*x)**(3/2)*(a**3*d**3*f - 3*a**2*b*c*d**2*f + 3*a**2*b*d**3*e + 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e - b**3*c**3*f + b**3*c**2*d*e)/(3*d**4)
```

Giac [A]
time = 0.63, size = 338, normalized size = 1.49

$$\frac{2a^3c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(35(dx+c)^{\frac{9}{2}}b^3d^{32}f-135(dx+c)^{\frac{7}{2}}b^3cd^{32}f+189(dx+c)^{\frac{5}{2}}b^3c^2d^{32}f-105(dx+c)^{\frac{3}{2}}b^3c^3d^{32}f+135(dx+c)^{\frac{7}{2}}ab^2d^{33}f-378(dx+c)^{\frac{5}{2}}ab^2cd^{33}f+315(dx+c)^{\frac{3}{2}}ab^2c^2d^{33}f+189(dx+c)^{\frac{5}{2}}a^2bd^{34}f-315(dx+c)^{\frac{3}{2}}a^2bcd^{34}f+105(dx+c)^{\frac{3}{2}}a^3d^{35}f+45(dx+c)^{\frac{7}{2}}b^3d^{33}e-126(dx+c)^{\frac{5}{2}}b^3cd^{33}e+105(dx+c)^{\frac{3}{2}}b^3c^2d^{33}e+189(dx+c)^{\frac{5}{2}}ab^2d^{34}e-315(dx+c)^{\frac{3}{2}}ab^2cd^{34}e+315(dx+c)^{\frac{3}{2}}a^2bd^{35}e+315\sqrt{dx+c}\,a^3d^{36}e\right)}{315d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/315*(35*(d*x + c)^(9/
2)*b^3*d^32*f - 135*(d*x + c)^(7/2)*b^3*c*d^32*f + 189*(d*x + c)^(5/2)*b^3*
c^2*d^32*f - 105*(d*x + c)^(3/2)*b^3*c^3*d^32*f + 135*(d*x + c)^(7/2)*a*b^2
*d^33*f - 378*(d*x + c)^(5/2)*a*b^2*c*d^33*f + 315*(d*x + c)^(3/2)*a*b^2*c^
2*d^33*f + 189*(d*x + c)^(5/2)*a^2*b*d^34*f - 315*(d*x + c)^(3/2)*a^2*b*c*d
^34*f + 105*(d*x + c)^(3/2)*a^3*d^35*f + 45*(d*x + c)^(7/2)*b^3*d^33*e - 12
6*(d*x + c)^(5/2)*b^3*c*d^33*e + 105*(d*x + c)^(3/2)*b^3*c^2*d^33*e + 189*(
d*x + c)^(5/2)*a*b^2*d^34*e - 315*(d*x + c)^(3/2)*a*b^2*c*d^34*e + 315*(d*x
 + c)^(3/2)*a^2*b*d^35*e + 315*sqrt(d*x + c)*a^3*d^36*e)/d^36
```

Mupad [B]
time = 0.16, size = 413, normalized size = 1.82

$$\left(c\left(c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b(ad-bc)(adf-2bcf+bde)}{d^4}\right)+\frac{2(ad-bc)^2(adf-4bcf+3bde)}{d^4}\right)-\frac{2(ad-bc)^3(cf-de)}{d^4}\right)\sqrt{c+dx}+\left(\frac{c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b(ad-bc)(adf-2bcf+bde)}{d^4}\right)}{3}+\frac{2(ad-bc)^2(adf-4bcf+3bde)}{3d^4}\right)(c+dx)^{3/2}+\left(\frac{2b^3de-8b^3cf+6ab^2df}{7d^4}+\frac{2b^3cf}{7d^4}\right)(c+dx)^{7/2}+\left(\frac{c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)}{5}+\frac{6b(ad-bc)(adf-2bcf+bde)}{5d^4}\right)(c+dx)^{5/2}+\frac{2b^3f(c+dx)^{9/2}}{9d^4}+a^3\sqrt{c}\,e\operatorname{atan}\left(\frac{\sqrt{c+dx}\,1\mathrm{i}}{\sqrt{c}}\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^3*(c + d*x)^(1/2))/x,x)

[Out] (c*(c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) + (6
*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4) + (2*(a*d - b*c)^2*(a*d*f -
4*b*c*f + 3*b*d*e))/d^4) - (2*(a*d - b*c)^3*(c*f - d*e))/d^4)*(c + d*x)^(1/
2) + ((c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) +
 (6*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4))/3 + (2*(a*d - b*c)^2*(a*
d*f - 4*b*c*f + 3*b*d*e))/(3*d^4))*(c + d*x)^(3/2) + ((2*b^3*d*e - 8*b^3*c*
f + 6*a*b^2*d*f)/(7*d^4) + (2*b^3*c*f)/(7*d^4))*(c + d*x)^(7/2) + ((c*((2*b
^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4))/5 + (6*b*(a*d - b
*c)*(a*d*f - 2*b*c*f + b*d*e))/(5*d^4))*(c + d*x)^(5/2) + a^3*c^(1/2)*e*ata
n(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*b^3*f*(c + d*x)^(9/2))/(9*d^4)
```

3.9 $\int \frac{(a+bx)^2\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=146

$$2a^2e\sqrt{c+dx}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}+\frac{2(c+dx)^{3/2}\left(2(10a^2d^2f-b^2c(7de-4cf)+7abd(5de-2cf))+3b\cdots\right.}{105d^3}$$

[Out] 2/7*f*(b*x+a)^2*(d*x+c)^(3/2)/d+2/105*(d*x+c)^(3/2)*(20*a^2*d^2*f-2*b^2*c*(-4*c*f+7*d*e)+14*a*b*d*(-2*c*f+5*d*e)+3*b*d*(4*a*d*f-4*b*c*f+7*b*d*e)*x)/d^3-2*a^2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*a^2*e*(d*x+c)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {158, 152, 52, 65, 214}

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}-2a^2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]

[Out] 2*a^2*e*Sqrt[c + d*x] + (2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + (2*(c + d*x)^(3/2)*(2*(10*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) + 7*a*b*d*(5*d*e - 2*c*f)) + 3*b*d*(7*b*d*e - 4*b*c*f + 4*a*d*f)*x))/(105*d^3) - 2*a^2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}\,(e+fx)}{x}\,dx = \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2\int \frac{(a+bx)\sqrt{c+dx}\,\left(\frac{7ade}{2}+\frac{1}{2}(7bde-4bcf+4adf)x\right)}{x}\,dx}{7d}$$

$$= \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f-b^2c(7de-4cf)+\right.}{1}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f-b\right.}{}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f-b\right.}{}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2(10a^2d^2f-b\right.}{}$$

Mathematica [A]

time = 0.21, size = 131, normalized size = 0.90

$$\frac{2\sqrt{c+dx}\,(35a^2d^2(3de+cf+dfx)+14abd(c+dx)(5de-2cf+3dfx)+b^2(c+dx)\,(8c^2f+3d^2x(7e+5fx)-2cd(7e+6fx)))}{105d^3}-2a^2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^2*d^2*(3*d*e + c*f + d*f*x) + 14*a*b*d*(c + d*x)*(5*
d*e - 2*c*f + 3*d*f*x) + b^2*(c + d*x)*(8*c^2*f + 3*d^2*x*(7*e + 5*f*x) - 2
*c*d*(7*e + 6*f*x))))/(105*d^3) - 2*a^2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqr
t[c]]
```

Maple [A]
time = 0.10, size = 176, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{2b^2f(dx+c)^{\frac{7}{2}}}{7}+\frac{4abdf(dx+c)^{\frac{5}{2}}}{5}-\frac{4b^2cf(dx+c)^{\frac{5}{2}}}{5}+\frac{2b^2de(dx+c)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(dx+c)^{\frac{3}{2}}}{3}-\frac{4abcdf(dx+c)^{\frac{3}{2}}}{3}+\frac{4ab\,d^2e(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c^2f(}{3}}{d^3}$
default	$\frac{\frac{2b^2f(dx+c)^{\frac{7}{2}}}{7}+\frac{4abdf(dx+c)^{\frac{5}{2}}}{5}-\frac{4b^2cf(dx+c)^{\frac{5}{2}}}{5}+\frac{2b^2de(dx+c)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(dx+c)^{\frac{3}{2}}}{3}-\frac{4abcdf(dx+c)^{\frac{3}{2}}}{3}+\frac{4ab\,d^2e(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c^2f(}{3}}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/7*b^2*f*(d*x+c)^(7/2)+2/5*a*b*d*f*(d*x+c)^(5/2)-2/5*b^2*c*f*(d*x+c
)^(5/2)+1/5*b^2*d*e*(d*x+c)^(5/2)+1/3*a^2*d^2*f*(d*x+c)^(3/2)-2/3*a*b*c*d*f
*(d*x+c)^(3/2)+2/3*a*b*d^2*e*(d*x+c)^(3/2)+1/3*b^2*c^2*f*(d*x+c)^(3/2)-1/3*
b^2*c*d*e*(d*x+c)^(3/2)+a^2*d^3*e*(d*x+c)^(1/2)-a^2*c^(1/2)*d^3*e*arctanh((
d*x+c)^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.50, size = 156, normalized size = 1.07

$$a^2\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^2f+105\,\sqrt{dx+c}\,a^2d^3e+21\,(b^2de-2\,(b^2c-abd)f)(dx+c)^{\frac{5}{2}}-35\,(b^2cde-2\,abd^2e-(b^2c^2-2\,abcd+a^2d^2)f)(dx+c)^{\frac{3}{2}}\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^2*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
105*(15*(d*x + c)^(7/2)*b^2*f + 105*sqrt(d*x + c)*a^2*d^3*e + 21*(b^2*d*e -
 2*(b^2*c - a*b*d)*f)*(d*x + c)^(5/2) - 35*(b^2*c*d*e - 2*a*b*d^2*e - (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(d*x + c)^(3/2))/d^3
```

Fricas [A]
time = 1.07, size = 403, normalized size = 2.76

$$\left[\frac{105\,a^{2}\sqrt{c}\,d^{3}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\left(15\,b^{2}d^{3}fx^{3}+3\left(b^{2}cd^{2}+14\,abd^{3}\right)fx^{2}-\left(4\,b^{2}c^{2}d-14\,abcd^{2}-35\,a^{2}d^{3}\right)fx+\left(8\,b^{2}c^{3}-28\,abc^{2}d+35\,a^{2}cd^{2}\right)f+7\left(3\,b^{2}d^{3}x^{2}-2\,b^{2}c^{2}d+10\,abcd^{2}+15\,a^{2}d^{3}+\left(b^{2}cd^{2}+10\,abd^{3}\right)x\right)e\right)\sqrt{dx+c}}{105\,d^{3}},\ \frac{2\left(105\,a^{2}\sqrt{-c}\,d^{3}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)e+\left(15\,b^{2}d^{3}fx^{3}+3\left(b^{2}cd^{2}+14\,abd^{3}\right)fx^{2}-\left(4\,b^{2}c^{2}d-14\,abcd^{2}-35\,a^{2}d^{3}\right)fx+\left(8\,b^{2}c^{3}-28\,abc^{2}d+35\,a^{2}cd^{2}\right)f+7\left(3\,b^{2}d^{3}x^{2}-2\,b^{2}c^{2}d+10\,abcd^{2}+15\,a^{2}d^{3}+\left(b^{2}cd^{2}+10\,abd^{3}\right)x\right)e\right)\sqrt{dx+c}\right)}{105\,d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^2*sqrt(c)*d^3*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(15*b^2*d^3*f*x^3 + 3*(b^2*c*d^2 + 14*a*b*d^3)*f*x^2 - (4*b^2*c^2*d - 1
4*a*b*c*d^2 - 35*a^2*d^3)*f*x + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f
 + 7*(3*b^2*d^3*x^2 - 2*b^2*c^2*d + 10*a*b*c*d^2 + 15*a^2*d^3 + (b^2*c*d^2
+ 10*a*b*d^3)*x)*e)*sqrt(d*x + c))/d^3, 2/105*(105*a^2*sqrt(-c)*d^3*arctan(
sqrt(d*x + c)*sqrt(-c)/c)*e + (15*b^2*d^3*f*x^3 + 3*(b^2*c*d^2 + 14*a*b*d^3
)*f*x^2 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f*x + (8*b^2*c^3 - 28*a
*b*c^2*d + 35*a^2*c*d^2)*f + 7*(3*b^2*d^3*x^2 - 2*b^2*c^2*d + 10*a*b*c*d^2
+ 15*a^2*d^3 + (b^2*c*d^2 + 10*a*b*d^3)*x)*e)*sqrt(d*x + c))/d^3]
```

Sympy [A]
time = 11.05, size = 167, normalized size = 1.14

$$\frac{2a^{2}ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^{2}e\sqrt{c+dx}+\frac{2b^{2}f(c+dx)^{\frac{7}{2}}}{7d^{3}}+\frac{2(c+dx)^{\frac{5}{2}}\cdot(2abdf-2b^{2}cf+b^{2}de)}{5d^{3}}+\frac{2(c+dx)^{\frac{3}{2}}\left(a^{2}d^{2}f-2abcdf+2abd^{2}e+b^{2}c^{2}f-b^{2}cde\right)}{3d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**2*e*sqrt(c + d*x) +
 2*b**2*f*(c + d*x)**(7/2)/(7*d**3) + 2*(c + d*x)**(5/2)*(2*a*b*d*f - 2*b**
2*c*f + b**2*d*e)/(5*d**3) + 2*(c + d*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
+ 2*a*b*d**2*e + b**2*c**2*f - b**2*c*d*e)/(3*d**3)
```

Giac [A]
time = 0.71, size = 201, normalized size = 1.38

$$\frac{2\,a^{2}c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^{2}d^{18}f-42\,(dx+c)^{\frac{5}{2}}b^{2}cd^{18}f+35\,(dx+c)^{\frac{3}{2}}b^{2}c^{2}d^{18}f+42\,(dx+c)^{\frac{5}{2}}abd^{19}f-70\,(dx+c)^{\frac{3}{2}}abcd^{19}f+35\,(dx+c)^{\frac{3}{2}}a^{2}d^{20}f+21\,(dx+c)^{\frac{5}{2}}b^{2}d^{19}e-35\,(dx+c)^{\frac{3}{2}}b^{2}cd^{19}e+70\,(dx+c)^{\frac{3}{2}}abd^{20}e+105\,\sqrt{dx+c}\,a^{2}d^{21}e\right)}{105\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/105*(15*(d*x + c)^(7/
2)*b^2*d^18*f - 42*(d*x + c)^(5/2)*b^2*c*d^18*f + 35*(d*x + c)^(3/2)*b^2*c^
2*d^18*f + 42*(d*x + c)^(5/2)*a*b*d^19*f - 70*(d*x + c)^(3/2)*a*b*c*d^19*f
+ 35*(d*x + c)^(3/2)*a^2*d^20*f + 21*(d*x + c)^(5/2)*b^2*d^19*e - 35*(d*x +
 c)^(3/2)*b^2*c*d^19*e + 70*(d*x + c)^(3/2)*a*b*d^20*e + 105*sqrt(d*x + c)*
a^2*d^21*e)/d^21
```

Mupad [B]
time = 2.62, size = 263, normalized size = 1.80

$$\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{5\,d^3}+\frac{2\,b^2\,c\,f}{5\,d^3}\right)\,{\left(c+d\,x\right)}^{5/2}+\left(c\,\left(c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)+\frac{2\,\left(a\,d-b\,c\right)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{d^3}\right)-\frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(c\,f-d\,e\right)}{d^3}\right)\,\sqrt{c+d\,x}+\left(\frac{c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)}{3}+\frac{2\,\left(a\,d-b\,c\right)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2}+\frac{2\,b^2\,f\,{\left(c+d\,x\right)}^{7/2}}{7\,d^3}+a^2\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^2*(c + d*x)^(1/2))/x,x)
```

```
[Out] ((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/(5*d^3) + (2*b^2*c*f)/(5*d^3))*(c + d*
x)^(5/2) + (c*(c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d^3 + (2*b^2*c*f)/d^3
) + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/d^3) - (2*(a*d - b*c)^2*(c*
f - d*e))/d^3)*(c + d*x)^(1/2) + ((c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d
^3 + (2*b^2*c*f)/d^3))/3 + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/(3*d
^3))*(c + d*x)^(3/2) + a^2*c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
+ (2*b^2*f*(c + d*x)^(7/2))/(7*d^3)
```

3.10 $\int \frac{(a+bx)\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] -2/15*(d*x+c)^(3/2)*(2*b*c*f-5*d*(a*f+b*e)-3*b*d*f*x)/d^2-2*a*e*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2)+2*a*e*(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {152, 52, 65, 214}

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a*e*Sqrt[c + d*x] - (2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d
*f*x))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}\,(e+fx)}{x}\,dx = -\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} + (ae)\int \frac{\sqrt{c+dx}}{x}\,dx$$
$$= 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} + (ace)\int \frac{}{x\sqrt{}}$$
$$= 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} + \frac{(2ace)\text{Subs}}{}$$
$$= 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} - 2a\sqrt{c}\,e\tan$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 1.05

$$\frac{2\sqrt{c+dx}\,(-b(c+dx)(-5de+2cf-3dfx)+5ad(3de+cf+dfx))}{15d^2} - 2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*(c + d*x)*(-5*d*e + 2*c*f - 3*d*f*x)) + 5*a*d*(3*d*e
+ c*f + d*f*x)))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Maple [A]
time = 0.10, size = 89, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{2fb(dx+c)^{\frac{5}{2}}}{5}+\frac{2adf(dx+c)^{\frac{3}{2}}}{3}-\frac{2bcf(dx+c)^{\frac{3}{2}}}{3}+\frac{2bde(dx+c)^{\frac{3}{2}}}{3}+2a\,d^{2}e\sqrt{dx+c}-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^{2}}$
default	$\frac{\frac{2fb(dx+c)^{\frac{5}{2}}}{5}+\frac{2adf(dx+c)^{\frac{3}{2}}}{3}-\frac{2bcf(dx+c)^{\frac{3}{2}}}{3}+\frac{2bde(dx+c)^{\frac{3}{2}}}{3}+2a\,d^{2}e\sqrt{dx+c}-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(1/5*f*b*(d*x+c)^(5/2)+1/3*a*d*f*(d*x+c)^(3/2)-1/3*b*c*f*(d*x+c)^(3/2
)+1/3*b*d*e*(d*x+c)^(3/2)+a*d^2*e*(d*x+c)^(1/2)-a*c^(1/2)*d^2*e*arctanh((d*
x+c)^(1/2)/c^(1/2)))
```

Maxima [A]

time = 0.48, size = 94, normalized size = 1.22

$$a\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}bf+15\,\sqrt{dx+c}\,ad^{2}e+5\,(bde-(bc-ad)f)(dx+c)^{\frac{3}{2}}\right)}{15\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/15
*(3*(d*x + c)^(5/2)*b*f + 15*sqrt(d*x + c)*a*d^2*e + 5*(b*d*e - (b*c - a*d)
*f)*(d*x + c)^(3/2))/d^2
```

Fricas [A]

time = 1.44, size = 217, normalized size = 2.82

$$\left[\frac{15\,a\sqrt{c}\,d^{2}e\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\,(3\,bd^{2}fx^{2}+(bcd+5\,ad^{2})fx-(2\,bc^{2}-5\,acd)f+5\,(bd^{2}x+bcd+3\,ad^{2})e)\sqrt{dx+c}}{15\,d^{2}},\ \frac{2\left(15\,a\sqrt{-c}\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)e+(3\,bd^{2}fx^{2}+(bcd+5\,ad^{2})fx-(2\,bc^{2}-5\,acd)f+5\,(bd^{2}x+bcd+3\,ad^{2})e)\sqrt{dx+c}\right)}{15\,d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a*sqrt(c)*d^2*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*
(3*b*d^2*f*x^2 + (b*c*d + 5*a*d^2)*f*x - (2*b*c^2 - 5*a*c*d)*f + 5*(b*d^2*x
 + b*c*d + 3*a*d^2)*e)*sqrt(d*x + c))/d^2, 2/15*(15*a*sqrt(-c)*d^2*arctan(s
qrt(d*x + c)*sqrt(-c)/c)*e + (3*b*d^2*f*x^2 + (b*c*d + 5*a*d^2)*f*x - (2*b*
c^2 - 5*a*c*d)*f + 5*(b*d^2*x + b*c*d + 3*a*d^2)*e)*sqrt(d*x + c))/d^2]
```

Sympy [A]

time = 11.00, size = 92, normalized size = 1.19

$$\frac{2ace\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2ae\sqrt{c+dx}+\frac{2bf(c+dx)^{\frac{5}{2}}}{5d^{2}}+\frac{2(c+dx)^{\frac{3}{2}}\,(adf-bcf+bde)}{3d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a*e*sqrt(c + d*x) + 2*b*f
*(c + d*x)**(5/2)/(5*d**2) + 2*(c + d*x)**(3/2)*(a*d*f - b*c*f + b*d*e)/(3*
d**2)
```

Giac [A]
time = 1.22, size = 105, normalized size = 1.36

$$\frac{2\,ac\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}bd^{8}f-5\,(dx+c)^{\frac{3}{2}}bcd^{8}f+5\,(dx+c)^{\frac{3}{2}}ad^{9}f+5\,(dx+c)^{\frac{3}{2}}bd^{9}e+15\,\sqrt{dx+c}\,ad^{10}e\right)}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/15*(3*(d*x + c)^(5/2)*b
*d^8*f - 5*(d*x + c)^(3/2)*b*c*d^8*f + 5*(d*x + c)^(3/2)*a*d^9*f + 5*(d*x +
 c)^(3/2)*b*d^9*e + 15*sqrt(d*x + c)*a*d^10*e)/d^10
```

Mupad [B]
time = 0.09, size = 136, normalized size = 1.77

$$\left(c\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{d^2}+\frac{2\,b\,c\,f}{d^2}\right)-\frac{2\,(a\,d-b\,c)\,(c\,f-d\,e)}{d^2}\right)\,\sqrt{c+d\,x}+\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{3\,d^2}+\frac{2\,b\,c\,f}{3\,d^2}\right)\,(c+d\,x)^{3/2}+\frac{2\,b\,f\,(c+d\,x)^{5/2}}{5\,d^2}+a\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*((2*a*d*f - 4*b*c*f + 2*b*d*e)/d^2 + (2*b*c*f)/d^2) - (2*(a*d - b*c)*(c*
f - d*e))/d^2)*(c + d*x)^(1/2) + ((2*a*d*f - 4*b*c*f + 2*b*d*e)/(3*d^2) + (
2*b*c*f)/(3*d^2))*(c + d*x)^(3/2) + (2*b*f*(c + d*x)^(5/2))/(5*d^2) + a*c^(
1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
```

3.11 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=54

$$2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] 2/3*f*(d*x+c)^(3/2)/d-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*e*(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {81, 52, 65, 214}

$$2e\sqrt{c+dx}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[c + d*x] + (2*f*(c + d*x)^(3/2))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}\,(e+fx)}{x}\,dx &= \frac{2f(c+dx)^{3/2}}{3d}+e\int \frac{\sqrt{c+dx}}{x}\,dx\\ &= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}+(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx\\ &= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}+\frac{(2ce)\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d}\\ &= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 0.98

$$\frac{2\sqrt{c+dx}\,(3de+cf+dfx)}{3d}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/x,x]
[Out] (2*Sqrt[c + d*x]*(3*d*e + c*f + d*f*x))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c
+ d*x]/Sqrt[c]]
```

Maple [A]
time = 0.12, size = 46, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{2f(dx+c)^{\frac{3}{2}}}{3}+2de\sqrt{dx+c}-2\sqrt{c}\,de\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d}$	46

default	$\frac{\frac{2f(dx+c)^{\frac{3}{2}}}{3}+2de\sqrt{dx+c}-2\sqrt{c}\,de\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*f*(d*x+c)^(3/2)+d*e*(d*x+c)^(1/2)-c^(1/2)*d*e*arctanh((d*x+c)^(1/2
)/c^(1/2)))
```

Maxima [A]
time = 0.56, size = 62, normalized size = 1.15

$$\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left((dx+c)^{\frac{3}{2}}f+3\sqrt{dx+c}\,de\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/3*((
d*x + c)^(3/2)*f + 3*sqrt(d*x + c)*d*e)/d
```

Fricas [A]
time = 1.70, size = 115, normalized size = 2.13

$$\left[\frac{3\sqrt{c}\,de\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\,(dfx+cf+3\,de)\sqrt{dx+c}}{3\,d},\frac{2\left(3\sqrt{-c}\,d\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)e+(dfx+cf+3\,de)\sqrt{dx+c}\right)}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*d*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(d*f*x
 + c*f + 3*d*e)*sqrt(d*x + c))/d, 2/3*(3*sqrt(-c)*d*arctan(sqrt(d*x + c)*sq
rt(-c)/c)*e + (d*f*x + c*f + 3*d*e)*sqrt(d*x + c))/d]
```

Sympy [A]
time = 2.52, size = 54, normalized size = 1.00

$$\frac{2ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2e\sqrt{c+dx}+\frac{2f(c+dx)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*e*sqrt(c + d*x) + 2*f*(c +
d*x)**(3/2)/(3*d)
```

Giac [A]
time = 1.08, size = 57, normalized size = 1.06

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left((dx+c)^{\frac{3}{2}}d^2f+3\,\sqrt{dx+c}\,d^3e\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/3*((d*x + c)^(3/2)*d^2*f
+ 3*sqrt(d*x + c)*d^3*e)/d^3
```

Mupad [B]
time = 0.07, size = 45, normalized size = 0.83

$$2\,e\,\sqrt{c+d\,x}+\frac{2\,f\,{\left(c+d\,x\right)}^{3/2}}{3\,d}+\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] 2*e*(c + d*x)^(1/2) + c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*
f*(c + d*x)^(3/2))/(3*d)
```

3.12 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)}\,dx$

Optimal. Leaf size=101

$$\frac{2f\sqrt{c+dx}}{b}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a+2*(-a*f+b*e)*arctanh(b^(1/2)*
(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(3/2)+2*f*(d*x+c)^(1/2
)/b
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {159, 162, 65, 214}

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)}\,dx &= \frac{2f\sqrt{c+dx}}{b}+\frac{2\int \frac{\frac{bce}{2}+\frac{1}{2}(bde+bcf-adf)x}{x(a+bx)\sqrt{c+dx}}\,dx}{b}\\ &= \frac{2f\sqrt{c+dx}}{b}+\frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a}-\frac{((bc-ad)(be-af))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{ab}\\ &= \frac{2f\sqrt{c+dx}}{b}+\frac{(2ce)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{ad}-\frac{(2(bc-ad)(be-af))\text{S}}{}\\ &= \frac{2f\sqrt{c+dx}}{b}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{ab^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 101, normalized size = 1.00

$$\frac{2f\sqrt{c+dx}}{b}+\frac{2\sqrt{-bc+ad}\,(be-af)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*ArcTan[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(a*b^(3/2)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]])/a
```

Maple [A]
time = 0.12, size = 103, normalized size = 1.02

method	result
derivativedivides	$\frac{2f\sqrt{dx+c}}{b}-\frac{2e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{a}+\frac{2(-a^{2}df+abcf+abde-b^{2}ce)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{ab\sqrt{(ad-bc)\,b}}$
default	$\frac{2f\sqrt{dx+c}}{b}-\frac{2e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{a}+\frac{2(-a^{2}df+abcf+abde-b^{2}ce)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{ab\sqrt{(ad-bc)\,b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(d*x+c)^(1/2)/b-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a+2*(-a^2*d*
f+a*b*c*f+a*b*d*e-b^2*c*e)/a/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/(
(a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.42, size = 458, normalized size = 4.53

$$\left[\frac{b\sqrt{c}\,e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}\,af-(af-be)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad+2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab},\ \frac{b\sqrt{c}\,e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}\,af-2(af-be)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)}{ab},\ \frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)e+2\sqrt{dx+c}\,af-(af-be)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad+2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab},\ \frac{2\left(b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)e+\sqrt{dx+c}\,af-(af-be)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c
)*a*f - (a*f - b*e)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(d
*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), (b*sqrt(c)*e*log((d*x - 2
*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c)*a*f - 2*(a*f - b*e)*sqrt
```

```
(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))
/(a*b), (2*b*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c)*e + 2*sqrt(d*x + c)*
a*f - (a*f - b*e)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(d*x
 + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), 2*(b*sqrt(-c)*arctan(sqrt(d
*x + c)*sqrt(-c)/c)*e + sqrt(d*x + c)*a*f - (a*f - b*e)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [A]
time = 9.97, size = 97, normalized size = 0.96

$$\frac{2f\sqrt{c+dx}}{b} + \frac{2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2(ad-bc)(af-be)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*sqrt(c + d*x)/b + 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/(a*sqrt(-c)) - 2*(
a*d - b*c)*(a*f - b*e)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt
((a*d - b*c)/b))
```

Giac [A]
time = 1.18, size = 112, normalized size = 1.11

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a\sqrt{-c}} + \frac{2\,\sqrt{dx+c}\,f}{b} + \frac{2\,(abcf-a^2df-b^2ce+abde)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a*sqrt(-c)) + 2*sqrt(d*x + c)*f/b + 2
*(a*b*c*f - a^2*d*f - b^2*c*e + a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B]
time = 2.87, size = 2368, normalized size = 23.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)),x)
```

```
[Out] (2*f*(c + d*x)^(1/2))/b - (c^(1/2)*e*atan(((c^(1/2)*e*((8*(c + d*x)^(1/2)*(
a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*
f + 4*a^2*b^2*c*d^3*e*f))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2
*d^2*f))/b + (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/
(a*b)))/a)*1i)/a + (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^
4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f)
)/b - (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*
e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a)*1i)/a)/((16*(
b^3*c^2*d^3*e^3 - a*b^2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3
*a*b^2*c^2*d^3*e^2*f - a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*
b*c*d^4*e^2*f))/b - (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d
^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f
))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b + (8*c^(1/2)
*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a + (c^(1/2
)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2
- 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3
*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (c^(1/2)*e*((8*(a^3*
b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4
*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a))*2i)/a - (atan(((((8*(c + d*x)^(1/2
)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2
*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f)
)/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/
2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))
*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3) + (((8*(c + d*x)^(1/2)*(a
^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^
2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f
 + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b
- (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*(
c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))*(a*
f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3))/((16*(b^3*c^2*d^3*e^3 - a*b^
2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3*a*b^2*c^2*d^3*e^2*f -
 a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*b*c*d^4*e^2*f))/b - ((
(8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a
^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2
- 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a
^2*b^3*c^2*d^2*f))/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3
*(a*d - b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c)
)^(1/2))/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3) + (((8*(c +
 d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^
4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b
^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*
```

```
c^2*d^2*f))/b - (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d -
 b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)
)/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3)))*(a*f - b*e)*(-b^
3*(a*d - b*c))^(1/2)*2i)/(a*b^3)
```

3.13 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^2}\,dx$

Optimal. Leaf size=127

$$\frac{(be-af)\sqrt{c+dx}}{ab(a+bx)}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{(2b^2ce-ad(be+af))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+(2*b^2*c*e-a*d*(a*f+b*e))*a
rctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(3/2)/(-a*d+b*c)^(1/2)
+(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {154, 162, 65, 214}

$$\frac{(2b^2ce-ad(af+be))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}\,(be-af)}{ab(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]])/a^2 + ((2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[b*c - a*d]])/(a^2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^2}\,dx = \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{\int \frac{-bce-\frac{1}{2}d(be+af)x}{x(a+bx)\sqrt{c+dx}}\,dx}{ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^2} + \frac{\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\int \frac{1}{(a+bx)\sqrt{c}}}{a^2b}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(2ce)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^2d} + \frac{\left(2\left(-b^2ce+\frac{1}{2}ad\right.\right.}{}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{(2b^2ce-ad(be+af))\tanh^{}}{a^2b^{3/2}\sqrt{bc-}}$$

Mathematica [A]
time = 0.49, size = 123, normalized size = 0.97

$$\frac{\frac{a(be-af)\sqrt{c+dx}}{b(a+bx)} + \frac{(-2b^2ce+abde+a^2df)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}} - 2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((a*(b*e - a*f)*Sqrt[c + d*x])/(b*(a + b*x)) + ((-2*b^2*c*e + a*b*d*e + a^2
*d*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(3/2)*Sqrt[-(b
*c) + a*d]) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a^2
```

Maple [A]
time = 0.12, size = 134, normalized size = 1.06

method	result
derivativedivides	$2d\left(-\frac{\sqrt{c}\, e \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d\,a^{2}}+\frac{-\frac{ad(af-be)\sqrt{dx+c}}{2b(b(dx+c)+ad-bc)}+\frac{\left(a^{2}df+abde-2b^{2}ce\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right.}{2b\sqrt{(ad-bc)\,b}}}{a^{2}d}\right.$
default	$2d\left(-\frac{\sqrt{c}\, e \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d\,a^{2}}+\frac{-\frac{ad(af-be)\sqrt{dx+c}}{2b(b(dx+c)+ad-bc)}+\frac{\left(a^{2}df+abde-2b^{2}ce\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right.}{2b\sqrt{(ad-bc)\,b}}}{a^{2}d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-c^(1/2)/d*e/a^2*arctanh((d*x+c)^(1/2)/c^(1/2))+1/a^2/d*(-1/2*a*d*(a*f
-b*e)/b*(d*x+c)^(1/2)/(b*(d*x+c)+a*d-b*c)+1/2*(a^2*d*f+a*b*d*e-2*b^2*c*e)/b
/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(114) = 228.
time = 1.87, size = 1008, normalized size = 7.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)*sqrt(c)*e*log((d*x - 2*
sqrt(d*x + c)*sqrt(c) + 2*c)/x) + (a^2*b*d*f*x + a^3*d*f - (2*a*b^2*c - a^2
*b*d + (2*b^3*c - a*b^2*d)*x)*e)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a
*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*((a^2*b^2*c - a^3*
b*d)*f - (a*b^3*c - a^2*b^2*d)*e)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2*d + (
a^2*b^4*c - a^3*b^3*d)*x), ((a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)*sqr
t(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + (a^2*b*d*f*x + a^3*d*
f - (2*a*b^2*c - a^2*b*d + (2*b^3*c - a*b^2*d)*x)*e)*sqrt(-b^2*c + a*b*d)*a
rctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - ((a^2*b^2*c - a^3
*b*d)*f - (a*b^3*c - a^2*b^2*d)*e)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2*d +
(a^2*b^4*c - a^3*b^3*d)*x), 1/2*(4*(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)
*x)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c)*e + (a^2*b*d*f*x + a^3*d*f -
(2*a*b^2*c - a^2*b*d + (2*b^3*c - a*b^2*d)*x)*e)*sqrt(b^2*c - a*b*d)*log((b
*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*((
a^2*b^2*c - a^3*b*d)*f - (a*b^3*c - a^2*b^2*d)*e)*sqrt(d*x + c))/(a^3*b^3*c
 - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), (2*(a*b^3*c - a^2*b^2*d + (b^4*c
 - a*b^3*d)*x)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c)*e + (a^2*b*d*f*x +
 a^3*d*f - (2*a*b^2*c - a^2*b*d + (2*b^3*c - a*b^2*d)*x)*e)*sqrt(-b^2*c + a
*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - ((a^2*b^2*
c - a^3*b*d)*f - (a*b^3*c - a^2*b^2*d)*e)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b
^2*d + (a^2*b^4*c - a^3*b^3*d)*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(112) = 224$.
time = 35.38, size = 1204, normalized size = 9.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*f*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x -
 2*b**3*c*d*x) + a*d**2*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*f*sqrt(-1/(b*(a*
d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) - 2*b*c*d*e*sqrt(c + d*x)/(2*a**3*d**2 - 2*a**2*b*c*d + 2*a**2*b*d
**2*x - 2*a*b**2*c*d*x) - c*d*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c*d*f*sqrt(-1/(b*(a*d
 - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/
(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x)
)/2 + 2*c*d*f*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**
2*c*d*x) - d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*
```

```
d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(
b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*
log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c
)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*d**2*
e*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2
*d*f*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + b*c*d*e*s
qrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a
*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3))
+ sqrt(c + d*x))/(2*a) - b*c*d*e*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*a) - 2*c*e*atan(sqrt(c
 + d*x)/sqrt(a*d/b - c))/(a**2*sqrt(a*d/b - c)) + 2*c*e*atan(sqrt(c + d*x)/
sqrt(-c))/(a**2*sqrt(-c))
```

Giac [A]
time = 0.96, size = 142, normalized size = 1.12

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^2\sqrt{-c}}+\frac{(a^2df-2\,b^2ce+abde)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2b}-\frac{\sqrt{dx+c}\,adf-\sqrt{dx+c}\,bde}{((dx+c)b-bc+ad)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^2*sqrt(-c)) + (a^2*d*f - 2*b^2*c*e
+ a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2*b) - (sqrt(d*x + c)*a*d*f - sqrt(d*x + c)*b*d*e)/(((d*x + c)*b - b*c
 + a*d)*a*b)
```

Mupad [B]
time = 0.60, size = 1827, normalized size = 14.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^2),x)
```

```
[Out] (atan((((((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) + ((4*a^5*b^
3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f
- 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))
^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) + (2*(c
 + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*
d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b
*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d))
- (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) - ((4*a^5*b^3*d^3
 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b
```

```
^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2
)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) - (2*(c + d*
x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e
*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^
(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d)))/((4*
(a*b^2*c*d^4*e^3 - 2*b^3*c^2*d^3*e^3 + a^3*c*d^4*e*f^2 - 2*a*b^2*c^2*d^3*e^
2*f + 2*a^2*b*c*d^4*e^2*f))/(a^3*b) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2
*c*d^3*f))/(a^3*b) + ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^
(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c -
a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(
a^2*b^4*c - a^3*b^3*d)) + (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2
 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^
3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/
(2*(a^2*b^4*c - a^3*b^3*d)) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f
))/(a^3*b) - ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c
 + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*
d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*
c - a^3*b^3*d)) - (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4
*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/
(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*
b^4*c - a^3*b^3*d))))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d
*e)*1i)/(a^2*b^4*c - a^3*b^3*d) - (2*c^(1/2)*e*atanh((4*c^(1/2)*d^4*e*f^2*(
c + d*x)^(1/2))/(4*c*d^4*e*f^2 + (4*b^2*c*d^4*e^3)/a^2 - (16*b^2*c^2*d^3*e^
2*f)/a^2 + (8*b*c*d^4*e^2*f)/a) + (8*c^(1/2)*d^4*e^2*f*(c + d*x)^(1/2))/(8*
c*d^4*e^2*f + (4*b*c*d^4*e^3)/a - (16*b*c^2*d^3*e^2*f)/a + (4*a*c*d^4*e*f^2
)/b) + (4*b*c^(1/2)*d^4*e^3*(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2
*f - 16*b*c^2*d^3*e^2*f + (4*a^2*c*d^4*e*f^2)/b) - (16*b*c^(3/2)*d^3*e^2*f*
(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2*f - 16*b*c^2*d^3*e^2*f + (4
*a^2*c*d^4*e*f^2)/b)))/a^2 - ((a*d*f - b*d*e)*(c + d*x)^(1/2))/(a*b*(a*d -
b*c + b*(c + d*x)))
```

3.14 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=208

$$\frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2}+\frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}+\frac{(8b^3c^2e-12ab^2cde+}{}$$

```
[Out] 1/4*(a^3*d^2*f+3*a^2*b*d^2*e-12*a*b^2*c*d*e+8*b^3*c^2*e)*arctanh(b^(1/2)*(d
*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/b^(3/2)/(-a*d+b*c)^(3/2)-2*e*arctanh((d*x
+c)^(1/2)/c^(1/2))*c^(1/2)/a^3+1/2*(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)^2+1
/4*(-a^2*d*f-3*a*b*d*e+4*b^2*c*e)*(d*x+c)^(1/2)/a^2/b/(-a*d+b*c)/(b*x+a)
```

Rubi [A]
time = 0.18, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {154, 156, 162, 65, 214}

$$-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}+\frac{\sqrt{c+dx}\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}+\frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx}\,(be-af)}{2ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + ((4*b^2*c*e - 3*a*b*d*e -
 a^2*d*f)*Sqrt[c + d*x])/(4*a^2*b*(b*c - a*d)*(a + b*x)) - (2*Sqrt[c]*e*Arc
Tanh[Sqrt[c + d*x]/Sqrt[c]])/a^3 + ((8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b
*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*a^
3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^3}\,dx = \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} - \frac{\int \frac{-2bce-\frac{1}{2}d(3be+af)x}{x(a+bx)^2\sqrt{c+dx}}\,dx}{2ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{\int \frac{-2bc(bc-ad)e-\frac{1}{4}d\left(4b^2ce-\right.}{x(a+bx)\sqrt{c+}}}{2a^2b(bc-a}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^3} -$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(2ce)\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{a^3d}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+}}{\sqrt{c}}\right.}{a^3}$$

Mathematica [A]
time = 1.00, size = 195, normalized size = 0.94

$$\frac{\frac{a\sqrt{c+dx}\,(a^3df+4b^3cex+3ab^2e(2c-dx)-a^2b(5de+2cf+dfx))}{b(bc-ad)(a+bx)^2}+\frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}(-bc+ad)^{3/2}}-8\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((a*Sqrt[c + d*x]*(a^3*d*f + 4*b^3*c*e*x + 3*a*b^2*e*(2*c - d*x) - a^2*b*(5
*d*e + 2*c*f + d*f*x)))/(b*(b*c - a*d)*(a + b*x)^2) + ((8*b^3*c^2*e - 12*a*
b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[
-(b*c) + a*d]])/(b^(3/2)*(-(b*c) + a*d)^(3/2)) - 8*Sqrt[c]*e*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(4*a^3)
```

Maple [A]
time = 0.11, size = 219, normalized size = 1.05

method	result
derivativedivides	$2d^2\left(-\frac{\sqrt{c}\;e\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^2a^3}+\frac{\frac{\frac{ad\left(a^2df+3abde-4b^2ce\right)(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(a^2df-5abde+4b^2ce\right)ad\sqrt{dx+c}}{8b}}{(b(dx+c)+ad-bc)^2}+\cdots}{a^3d^2}\right.$
default	$2d^2\left(-\frac{\sqrt{c}\;e\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{d^2a^3}+\frac{\frac{\frac{ad\left(a^2df+3abde-4b^2ce\right)(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(a^2df-5abde+4b^2ce\right)ad\sqrt{dx+c}}{8b}}{(b(dx+c)+ad-bc)^2}+\cdots}{a^3d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-c^(1/2)/d^2*e/a^3*arctanh((d*x+c)^(1/2)/c^(1/2))+1/a^3/d^2*((1/8*a*
d*(a^2*d*f+3*a*b*d*e-4*b^2*c*e)/(a*d-b*c)*(d*x+c)^(3/2)-1/8*(a^2*d*f-5*a*b*
d*e+4*b^2*c*e)*a*d/b*(d*x+c)^(1/2))/(b*(d*x+c)+a*d-b*c)^2+1/8*(a^3*d^2*f+3*
a^2*b*d^2*e-12*a*b^2*c*d*e+8*b^3*c^2*e)/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(188) = 376$.
time = 2.41, size = 2199, normalized size = 10.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d
 + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x)*sqrt(c
)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - (a^3*b^2*d^2*f*x^2 + 2*a
^4*b*d^2*f*x + a^5*d^2*f + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2 +
(8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*x^2 + 2*(8*a*b^4*c^2 - 12*a^2*b^
3*c*d + 3*a^3*b^2*d^2)*x)*e)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d -
 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*((a^3*b^3*c*d - a^4*b^
2*d^2)*f*x + (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f - (6*a^2*b^4*c^2
 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2 + (4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^
3*d^2)*x)*e)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a
^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*
c*d + a^6*b^3*d^2)*x), 1/4*(4*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 +
(b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d +
a^3*b^3*d^2)*x)*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - (a
^3*b^2*d^2*f*x^2 + 2*a^4*b*d^2*f*x + a^5*d^2*f + (8*a^2*b^3*c^2 - 12*a^3*b^
2*c*d + 3*a^4*b*d^2 + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*x^2 + 2*(8
*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x)*e)*sqrt(-b^2*c + a*b*d)*arc
tan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - ((a^3*b^3*c*d - a^4
*b^2*d^2)*f*x + (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f - (6*a^2*b^4*
c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2 + (4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3
*b^3*d^2)*x)*e)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 +
 (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b
^4*c*d + a^6*b^3*d^2)*x), 1/8*(16*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^
2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*
d + a^3*b^3*d^2)*x)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c)*e - (a^3*b^2*
d^2*f*x^2 + 2*a^4*b*d^2*f*x + a^5*d^2*f + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d +
 3*a^4*b*d^2 + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*x^2 + 2*(8*a*b^4*
c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x)*e)*sqrt(b^2*c - a*b*d)*log((b*d*x
```

```
+ 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*((a^3*b
^3*c*d - a^4*b^2*d^2)*f*x + (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f -
 (6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2 + (4*a*b^5*c^2 - 7*a^2*b^4
*c*d + 3*a^3*b^3*d^2)*x)*e)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a
^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c
^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), 1/4*(8*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2
*a^2*b^4*c*d + a^3*b^3*d^2)*x)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c)*e
- (a^3*b^2*d^2*f*x^2 + 2*a^4*b*d^2*f*x + a^5*d^2*f + (8*a^2*b^3*c^2 - 12*a^
3*b^2*c*d + 3*a^4*b*d^2 + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*x^2 +
2*(8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x)*e)*sqrt(-b^2*c + a*b*d)
*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - ((a^3*b^3*c*d -
 a^4*b^2*d^2)*f*x + (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f - (6*a^2*
b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2 + (4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3
*a^3*b^3*d^2)*x)*e)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d
^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a
^5*b^4*c*d + a^6*b^3*d^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.86, size = 300, normalized size = 1.44

$$-\frac{(a^3d^2f + 8\,b^3c^2e - 12\,ab^2cde + 3\,a^2bd^2e)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^3b^2c - a^4bd)\sqrt{-b^2c+abd}} + \frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^3\sqrt{-c}} - \frac{(dx+c)^{\frac{3}{2}}a^2bd^2f + \sqrt{dx+c}\,a^2bcd^2f - \sqrt{dx+c}\,a^3d^3f - 4\,(dx+c)^{\frac{3}{2}}b^3cde + 4\,\sqrt{dx+c}\,b^3c^2de + 3\,(dx+c)^{\frac{3}{2}}ab^2d^2e - 9\,\sqrt{dx+c}\,ab^2cd^2e + 5\,\sqrt{dx+c}\,a^2bd^3e}{4\,(a^2b^2c - a^3bd)((dx+c)b - bc + ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*d^2*f + 8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e)*arctan(sqrt
(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c - a^4*b*d)*sqrt(-b^2*c + a*b*
d)) + 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^3*sqrt(-c)) - 1/4*((d*x + c)^
(3/2)*a^2*b*d^2*f + sqrt(d*x + c)*a^2*b*c*d^2*f - sqrt(d*x + c)*a^3*d^3*f -
 4*(d*x + c)^(3/2)*b^3*c*d*e + 4*sqrt(d*x + c)*b^3*c^2*d*e + 3*(d*x + c)^(3
/2)*a*b^2*d^2*e - 9*sqrt(d*x + c)*a*b^2*c*d^2*e + 5*sqrt(d*x + c)*a^2*b*d^3
*e)/((a^2*b^2*c - a^3*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [**B**]
time = 4.54, size = 2500, normalized size = 12.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^3),x)
```

```
[Out] (c^(1/2)*e*atan(((c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*
e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320
*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4
*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + (c^(1/2)*e
*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^
2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c
^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*
c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2
*c*d))))/a^3)*1i)/a^3 + (c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b
^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^
2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f -
 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - (c^
(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7
*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*
d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a
^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*
a^5*b^2*c*d))))/a^3)*1i)/a^3)/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^
3 - 8*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*
a^2*b^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/
(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(((c + d*x)^(1/2)*(a
^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 25
6*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a
^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2
*a^5*b^2*c*d)) + (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a
^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^
2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3 - (c^(1/2)*e*(((c + d*x)^(1/2
)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f
+ 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 +
16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2
 - 2*a^5*b^2*c*d)) - (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a
^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^
6*b^3*c^2 - 2*a^7*b^2*c*d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 2
56*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*
b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3))*2i)/a^3 - (((c + d*x)^(
1/2)*(a^2*d^2*f - 5*a*b*d^2*e + 4*b^2*c*d*e))/(4*a^2*b) - ((c + d*x)^(3/2)*
(a^2*d^2*f + 3*a*b*d^2*e - 4*b^2*c*d*e))/(4*a^2*(a*d - b*c)))/(b^2*(c + d*x
)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (ata
n((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d
^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
```

```
320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*
a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*
(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*
d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 -
 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e
+ a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4
*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b
^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/
(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(
1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e
^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72
*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^
6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a
^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*
e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(
a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*
e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d
^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a
^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*
e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*
d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2
*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^
2*d + 3*a^5*b^4*c*d^2)))/(((a^5*c*d^6*e*f^2)/4 ...
```

3.15 $\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$

Optimal. Leaf size=226

$$2c^3e\sqrt{a+bx}+\frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}-\frac{2(a+bx)^{3/2}\left(2(8a^3d^3f\right.}{}$$

```
[Out] 2/21*(-2*a*d*f+2*b*c*f+3*b*d*e)*(b*x+a)^(3/2)*(d*x+c)^2/b^2+2/9*f*(b*x+a)^(
3/2)*(d*x+c)^3/b-2/315*(b*x+a)^(3/2)*(16*a^3*d^3*f-24*a^2*b*d^2*(3*c*f+d*e)
-10*b^3*c^2*(4*c*f+27*d*e)+6*a*b^2*c*d*(16*c*f+21*d*e)-3*b*d*(21*b^2*c*d*e+
4*(-a*d+b*c)*(-2*a*d*f+2*b*c*f+3*b*d*e))*x)/b^4-2*c^3*e*arctanh((b*x+a)^(1/
2)/a^(1/2))*a^(1/2)+2*c^3*e*(b*x+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {158, 152, 52, 65, 214}

$$-\frac{2(a+bx)^{3/2}\left(2\left(8a^3d^3f-12a^2bd^2(3cf+de)+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(-2adf+2bcf+3bde)+21b^2cde\right)\right)}{315b^4}+\frac{2(a+bx)^{3/2}(c+dx)^2(-2adf+2bcf+3bde)}{21b^2}+2c^3e\sqrt{a+bx}-2\sqrt{a}\,c^3e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] 2*c^3*e*Sqrt[a + b*x] + (2*(3*b*d*e + 2*b*c*f - 2*a*d*f)*(a + b*x)^(3/2)*(c
 + d*x)^2)/(21*b^2) + (2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) - (2*(a + b*x
)^(3/2)*(2*(8*a^3*d^3*f - 12*a^2*b*d^2*(d*e + 3*c*f) - 5*b^3*c^2*(27*d*e +
4*c*f) + 3*a*b^2*c*d*(21*d*e + 16*c*f)) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*
d)*(3*b*d*e + 2*b*c*f - 2*a*d*f))*x))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)^3(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{2\int \frac{\sqrt{a+bx}\,(c+dx)^2\left(\frac{9bce}{2}+\frac{3}{2}(3bde+2bcf-2adf)x\right)}{x}dx}{9b}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} +$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} -$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+b}{}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+b}{}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+b}{}$$

Mathematica [A]
time = 0.31, size = 236, normalized size = 1.04

$$\frac{2\sqrt{a+bx}\left(-16a^4d^3f+8a^3bd^2(3de+9cf+dfx)-6a^2b^2d(21c^2f+d^2x(2e+fx)+3cd(7e+2fx))+ab^3(105c^3f+63c^2d(5e+fx)+9cd^2x(7e+3fx)+d^3x^2(9e+5fx))+b^4(105c^3(3e+fx)+63c^2dx(5e+3fx)+27cd^2x^2(7e+5fx)+5d^3x^3(9e+7fx))\right)}{315b^4}-2\sqrt{a}\,c^3e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^4*d^3*f + 8*a^3*b*d^2*(3*d*e + 9*c*f + d*f*x) - 6*a
^2*b^2*d*(21*c^2*f + d^2*x*(2*e + f*x) + 3*c*d*(7*e + 2*f*x)) + a*b^3*(105*
c^3*f + 63*c^2*d*(5*e + f*x) + 9*c*d^2*x*(7*e + 3*f*x) + d^3*x^2*(9*e + 5*f
*x)) + b^4*(105*c^3*(3*e + f*x) + 63*c^2*d*x*(5*e + 3*f*x) + 27*c*d^2*x^2*(
7*e + 5*f*x) + 5*d^3*x^3*(9*e + 7*f*x))))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.11, size = 301, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{2f\,d^3(bx+a)^{\frac{9}{2}}}{9}-\frac{6a\,d^3f(bx+a)^{\frac{7}{2}}}{7}+\frac{6bc\,d^2f(bx+a)^{\frac{7}{2}}}{7}+\frac{2b\,d^3e(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2d^3f(bx+a)^{\frac{5}{2}}}{5}-\frac{12abc\,d^2f(bx+a)^{\frac{5}{2}}}{5}-\frac{4ab\,d^3e(bx+a)^{\frac{5}{2}}}{5}+6}{}$
default	$\frac{\frac{2f\,d^3(bx+a)^{\frac{9}{2}}}{9}-\frac{6a\,d^3f(bx+a)^{\frac{7}{2}}}{7}+\frac{6bc\,d^2f(bx+a)^{\frac{7}{2}}}{7}+\frac{2b\,d^3e(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2d^3f(bx+a)^{\frac{5}{2}}}{5}-\frac{12abc\,d^2f(bx+a)^{\frac{5}{2}}}{5}-\frac{4ab\,d^3e(bx+a)^{\frac{5}{2}}}{5}+6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/9*f*d^3*(b*x+a)^(9/2)-3/7*a*d^3*f*(b*x+a)^(7/2)+3/7*b*c*d^2*f*(b*x
+a)^(7/2)+1/7*b*d^3*e*(b*x+a)^(7/2)+3/5*a^2*d^3*f*(b*x+a)^(5/2)-6/5*a*b*c*d
^2*f*(b*x+a)^(5/2)-2/5*a*b*d^3*e*(b*x+a)^(5/2)+3/5*b^2*c^2*d*f*(b*x+a)^(5/2
)+3/5*b^2*c*d^2*e*(b*x+a)^(5/2)-1/3*a^3*d^3*f*(b*x+a)^(3/2)+a^2*b*c*d^2*f*(
b*x+a)^(3/2)+1/3*a^2*b*d^3*e*(b*x+a)^(3/2)-a*b^2*c^2*d*f*(b*x+a)^(3/2)-a*b^
2*c*d^2*e*(b*x+a)^(3/2)+1/3*b^3*c^3*f*(b*x+a)^(3/2)+b^3*c^2*d*e*(b*x+a)^(3/
2)+b^4*c^3*e*(b*x+a)^(1/2)-a^(1/2)*b^4*c^3*e*arctanh((b*x+a)^(1/2)/a^(1/2))
)
```

Maxima [A]
time = 0.50, size = 245, normalized size = 1.08

$$\sqrt{a} c^{3} e \log\left(\frac{\sqrt{bx + a} - \sqrt{a}}{\sqrt{bx + a} + \sqrt{a}}\right) + \frac{2 \left(35 (bx + a)^{\frac{9}{2}} d^{3} f + 315 \sqrt{bx + a}\, b^{4} c^{3} e + 45 \left(b d^{3} e + 3 \left(b c d^{2} - a d^{3}\right) f\right) (bx + a)^{\frac{7}{2}} + 63 \left(3 b^{2} c d^{2} e - 2 a b d^{3} e + 3 \left(b^{2} c^{2} d - 2 a b c d^{2} + a^{2} d^{3}\right) f\right) (bx + a)^{\frac{5}{2}} + 105 \left(3 b^{3} c^{2} d e - 3 a b^{2} c d^{2} e + a^{2} b d^{3} e + \left(b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}\right) f\right) (bx + a)^{\frac{3}{2}}\right)}{315 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^3*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
315*(35*(b*x + a)^(9/2)*d^3*f + 315*sqrt(b*x + a)*b^4*c^3*e + 45*(b*d^3*e +
 3*(b*c*d^2 - a*d^3)*f)*(b*x + a)^(7/2) + 63*(3*b^2*c*d^2*e - 2*a*b*d^3*e +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f)*(b*x + a)^(5/2) + 105*(3*b^3*c^2*
d*e - 3*a*b^2*c*d^2*e + a^2*b*d^3*e + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*f)*(b*x + a)^(3/2))/b^4
```

Fricas [A]
time = 1.51, size = 639, normalized size = 2.83

$$\left[\frac{315 \sqrt{a} b^{4} c^{3} e \log\left(\frac{bx - 2 \sqrt{bx + a} \sqrt{a} + 2 a}{x}\right) + 2 \left(35 b^{4} d^{3} f x^{4} + 5 \left(27 b^{4} c d^{2} + a b^{3} d^{3}\right) f x^{3} + 3 \left(63 b^{4} c^{2} d + 9 a b^{3} c d^{2} - 2 a^{2} b^{2} d^{3}\right) f x^{2} + \left(105 b^{4} c^{3} + 63 a b^{3} c^{2} d - 36 a^{2} b^{2} c d^{2} + 8 a^{3} b d^{3}\right) f x + \left(105 a b^{3} c^{3} - 126 a^{2} b^{2} c^{2} d + 72 a^{3} b c d^{2} - 16 a^{4} d^{3}\right) f + 3 \left(15 b^{4} d^{3} x^{3} + 105 b^{4} c^{3} + 105 a b^{3} c^{2} d - 42 a^{2} b^{2} c d^{2} + 8 a^{3} b d^{3} + 3 \left(21 b^{4} c d^{2} + a b^{3} d^{3}\right) x^{2} + \left(105 b^{4} c^{2} d + 21 a b^{3} c d^{2} - 4 a^{2} b^{2} d^{3}\right) x\right) e\right) \sqrt{bx + a}}{315 b^{4}}, \frac{2 \left(315 \sqrt{-a} b^{4} c^{3} \arctan\left(\frac{\sqrt{bx + a} \sqrt{-a}}{a}\right) e + \left(35 b^{4} d^{3} f x^{4} + 5 \left(27 b^{4} c d^{2} + a b^{3} d^{3}\right) f x^{3} + 3 \left(63 b^{4} c^{2} d + 9 a b^{3} c d^{2} - 2 a^{2} b^{2} d^{3}\right) f x^{2} + \left(105 b^{4} c^{3} + 63 a b^{3} c^{2} d - 36 a^{2} b^{2} c d^{2} + 8 a^{3} b d^{3}\right) f x + \left(105 a b^{3} c^{3} - 126 a^{2} b^{2} c^{2} d + 72 a^{3} b c d^{2} - 16 a^{4} d^{3}\right) f + 3 \left(15 b^{4} d^{3} x^{3} + 105 b^{4} c^{3} + 105 a b^{3} c^{2} d - 42 a^{2} b^{2} c d^{2} + 8 a^{3} b d^{3} + 3 \left(21 b^{4} c d^{2} + a b^{3} d^{3}\right) x^{2} + \left(105 b^{4} c^{2} d + 21 a b^{3} c d^{2} - 4 a^{2} b^{2} d^{3}\right) x\right) e\right) \sqrt{bx + a}\right)}{315 b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(a)*b^4*c^3*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(35*b^4*d^3*f*x^4 + 5*(27*b^4*c*d^2 + a*b^3*d^3)*f*x^3 + 3*(63*b^4*c^2*
d + 9*a*b^3*c*d^2 - 2*a^2*b^2*d^3)*f*x^2 + (105*b^4*c^3 + 63*a*b^3*c^2*d -
36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f*x + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d +
72*a^3*b*c*d^2 - 16*a^4*d^3)*f + 3*(15*b^4*d^3*x^3 + 105*b^4*c^3 + 105*a*b^
3*c^2*d - 42*a^2*b^2*c*d^2 + 8*a^3*b*d^3 + 3*(21*b^4*c*d^2 + a*b^3*d^3)*x^2
 + (105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*x)*e)*sqrt(b*x + a))/b^
4, 2/315*(315*sqrt(-a)*b^4*c^3*arctan(sqrt(b*x + a)*sqrt(-a)/a)*e + (35*b^4
```

```
*d^3*f*x^4 + 5*(27*b^4*c*d^2 + a*b^3*d^3)*f*x^3 + 3*(63*b^4*c^2*d + 9*a*b^3
*c*d^2 - 2*a^2*b^2*d^3)*f*x^2 + (105*b^4*c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*
c*d^2 + 8*a^3*b*d^3)*f*x + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*
d^2 - 16*a^4*d^3)*f + 3*(15*b^4*d^3*x^3 + 105*b^4*c^3 + 105*a*b^3*c^2*d - 4
2*a^2*b^2*c*d^2 + 8*a^3*b*d^3 + 3*(21*b^4*c*d^2 + a*b^3*d^3)*x^2 + (105*b^4
*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*x)*e)*sqrt(b*x + a))/b^4]
```

Sympy [A]
time = 15.12, size = 274, normalized size = 1.21

$$\frac{2ac^3e\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^3e\sqrt{a+bx}+\frac{2d^3f(a+bx)^{\frac{9}{2}}}{9b^4}+\frac{2(a+bx)^{\frac{7}{2}}\left(-3ad^3f+3bcd^2f+bd^3e\right)}{7b^4}+\frac{2(a+bx)^{\frac{5}{2}}\cdot\left(3a^2d^3f-6abcd^2f-2abd^3e+3b^2c^2df+3b^2cd^2e\right)}{5b^4}+\frac{2(a+bx)^{\frac{3}{2}}\left(-a^3d^3f+3a^2bcd^2f+a^2bd^3e-3ab^2c^2df-3ab^2cd^2e+b^3c^3f+3b^3c^2de\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**3*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**3*e*sqrt(a + b*x) +
 2*d**3*f*(a + b*x)**(9/2)/(9*b**4) + 2*(a + b*x)**(7/2)*(-3*a*d**3*f + 3*b
*c*d**2*f + b*d**3*e)/(7*b**4) + 2*(a + b*x)**(5/2)*(3*a**2*d**3*f - 6*a*b*
c*d**2*f - 2*a*b*d**3*e + 3*b**2*c**2*d*f + 3*b**2*c*d**2*e)/(5*b**4) + 2*(
a + b*x)**(3/2)*(-a**3*d**3*f + 3*a**2*b*c*d**2*f + a**2*b*d**3*e - 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e + b**3*c**3*f + 3*b**3*c**2*d*e)/(3*b**4)
```

Giac [A]
time = 1.21, size = 338, normalized size = 1.50

$$\frac{2\,ac^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(105\,(bx+a)^{\frac{3}{2}}b^{35}c^3f+189\,(bx+a)^{\frac{5}{2}}b^{34}c^2df-315\,(bx+a)^{\frac{3}{2}}ab^{34}c^2df+135\,(bx+a)^{\frac{7}{2}}b^{33}cd^2f-378\,(bx+a)^{\frac{5}{2}}ab^{33}cd^2f+315\,(bx+a)^{\frac{3}{2}}a^2b^{33}cd^2f+35\,(bx+a)^{\frac{9}{2}}b^{32}d^3f-135\,(bx+a)^{\frac{7}{2}}ab^{32}d^3f+189\,(bx+a)^{\frac{5}{2}}a^2b^{32}d^3f-105\,(bx+a)^{\frac{3}{2}}a^3b^{32}d^3f+315\,\sqrt{bx+a}\,b^{36}c^3e+315\,(bx+a)^{\frac{3}{2}}b^{35}c^2de+189\,(bx+a)^{\frac{5}{2}}b^{34}cd^2e-315\,(bx+a)^{\frac{3}{2}}ab^{34}cd^2e+45\,(bx+a)^{\frac{7}{2}}b^{33}d^3e-126\,(bx+a)^{\frac{5}{2}}ab^{33}d^3e+105\,(bx+a)^{\frac{3}{2}}a^2b^{33}d^3e\right)}{315\,b^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^3*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/315*(105*(b*x + a)^(3
/2)*b^35*c^3*f + 189*(b*x + a)^(5/2)*b^34*c^2*d*f - 315*(b*x + a)^(3/2)*a*b
^34*c^2*d*f + 135*(b*x + a)^(7/2)*b^33*c*d^2*f - 378*(b*x + a)^(5/2)*a*b^33
*c*d^2*f + 315*(b*x + a)^(3/2)*a^2*b^33*c*d^2*f + 35*(b*x + a)^(9/2)*b^32*d
^3*f - 135*(b*x + a)^(7/2)*a*b^32*d^3*f + 189*(b*x + a)^(5/2)*a^2*b^32*d^3*
f - 105*(b*x + a)^(3/2)*a^3*b^32*d^3*f + 315*sqrt(b*x + a)*b^36*c^3*e + 315
*(b*x + a)^(3/2)*b^35*c^2*d*e + 189*(b*x + a)^(5/2)*b^34*c*d^2*e - 315*(b*x
 + a)^(3/2)*a*b^34*c*d^2*e + 45*(b*x + a)^(7/2)*b^33*d^3*e - 126*(b*x + a)^
(5/2)*a*b^33*d^3*e + 105*(b*x + a)^(3/2)*a^2*b^33*d^3*e)/b^36
```

Mupad [B]
time = 2.53, size = 413, normalized size = 1.83

$$\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{7\,b^4}+\frac{2\,a\,d^3\,f}{7\,b^4}\right)\,{\left(a+b\,x\right)}^{7/2}+\left(\frac{a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)}{5}-\frac{6\,d\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{5\,b^4}\right)\,{\left(a+b\,x\right)}^{5/2}+\left(a\,\left(a\,\left(a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)-\frac{6\,d\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{b^4}\right)+\frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(b\,c\,f-4\,a\,d\,f+3\,b\,d\,e\right)}{b^4}\right)+\frac{2\,{\left(a\,d-b\,c\right)}^3\,\left(a\,f-b\,e\right)}{b^4}\right)\,\sqrt{a+b\,x}+\left(\frac{a\,\left(a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)-\frac{6\,d\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{b^4}\right)}{3}+\frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(b\,c\,f-4\,a\,d\,f+3\,b\,d\,e\right)}{3\,b^4}\right)\,{\left(a+b\,x\right)}^{3/2}+\frac{2\,d^3\,f\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}+\sqrt{a}\,c^3\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^3)/x,x)
```

```
[Out] ((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/(7*b^4) + (2*a*d^3*f)/(7*b^4))*(a +
b*x)^(7/2) + ((a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4))/5 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/(5*b^4))*(a + b*x)^(5/
2) + (a*(a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b^4)
 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4) + (2*(a*d - b*c)^2*(b*c
*f - 4*a*d*f + 3*b*d*e))/b^4) + (2*(a*d - b*c)^3*(a*f - b*e))/b^4)*(a + b*x
)^(1/2) + ((a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4) - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4))/3 + (2*(a*d - b*c)^
2*(b*c*f - 4*a*d*f + 3*b*d*e))/(3*b^4))*(a + b*x)^(3/2) + a^(1/2)*c^3*e*ata
n(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*d^3*f*(a + b*x)^(9/2))/(9*b^4)
```

3.16 $\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x} \, dx$

Optimal. Leaf size=145

$$2c^2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}+\frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(de+2cf)+5b^2c(7de+2cf))+3bd(\right.}{105b^3}$$

```
[Out] 2/7*f*(b*x+a)^(3/2)*(d*x+c)^2/b+2/105*(b*x+a)^(3/2)*(8*a^2*d^2*f-14*a*b*d*(
2*c*f+d*e)+10*b^2*c*(2*c*f+7*d*e)+3*b*d*(-4*a*d*f+4*b*c*f+7*b*d*e)*x)/b^3-2
*c^2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*c^2*e*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {158, 152, 52, 65, 214}

$$\frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de))+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}\,c^2e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] 2*c^2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + (2*(a + b
*x)^(3/2)*(2*(4*a^2*d^2*f - 7*a*b*d*(d*e + 2*c*f) + 5*b^2*c*(7*d*e + 2*c*f)
) + 3*b*d*(7*b*d*e + 4*b*c*f - 4*a*d*f)*x))/(105*b^3) - 2*Sqrt[a]*c^2*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)^2(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2\int \frac{\sqrt{a+bx}\,(c+dx)\left(\frac{7bce}{2}+\frac{1}{2}(7bde+4bcf-4adf)x\right)}{x}\,dx}{7b}$$

$$= \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7abd(de+2cf)+5\right.}{1\ldots}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7a\right.}{\ldots}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7a\right.}{\ldots}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2(4a^2d^2f-7a\right.}{\ldots}$$

Mathematica [A]

time = 0.21, size = 157, normalized size = 1.08

$$\frac{2\sqrt{a+bx}\,(8a^3d^2f-2a^2bd(7de+14cf+2dfx)+ab^2(35c^2f+14cd(5e+fx)+d^2x(7e+3fx))+b^3(35c^2(3e+fx)+14cdx(5e+3fx)+3d^2x^2(7e+5fx)))}{105b^3}-2\sqrt{a}\,c^2e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^3*d^2*f - 2*a^2*b*d*(7*d*e + 14*c*f + 2*d*f*x) + a*b^
2*(35*c^2*f + 14*c*d*(5*e + f*x) + d^2*x*(7*e + 3*f*x)) + b^3*(35*c^2*(3*e
+ f*x) + 14*c*d*x*(5*e + 3*f*x) + 3*d^2*x^2*(7*e + 5*f*x))))/(105*b^3) - 2*
Sqrt[a]*c^2*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.10, size = 176, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{2d^2f(bx+a)^{\frac{7}{2}}}{7}-\frac{4a\,d^2f(bx+a)^{\frac{5}{2}}}{5}+\frac{4bcdf(bx+a)^{\frac{5}{2}}}{5}+\frac{2b\,d^2e(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(bx+a)^{\frac{3}{2}}}{3}-\frac{4abcdf(bx+a)^{\frac{3}{2}}}{3}-\frac{2ab\,d^2e(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^2c^2f}{}}{b^3}$
default	$\frac{\frac{2d^2f(bx+a)^{\frac{7}{2}}}{7}-\frac{4a\,d^2f(bx+a)^{\frac{5}{2}}}{5}+\frac{4bcdf(bx+a)^{\frac{5}{2}}}{5}+\frac{2b\,d^2e(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(bx+a)^{\frac{3}{2}}}{3}-\frac{4abcdf(bx+a)^{\frac{3}{2}}}{3}-\frac{2ab\,d^2e(bx+a)^{\frac{3}{2}}}{3}+\frac{2b^2c^2f}{}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/7*d^2*f*(b*x+a)^(7/2)-2/5*a*d^2*f*(b*x+a)^(5/2)+2/5*b*c*d*f*(b*x+a
)^(5/2)+1/5*b*d^2*e*(b*x+a)^(5/2)+1/3*a^2*d^2*f*(b*x+a)^(3/2)-2/3*a*b*c*d*f
*(b*x+a)^(3/2)-1/3*a*b*d^2*e*(b*x+a)^(3/2)+1/3*b^2*c^2*f*(b*x+a)^(3/2)+2/3*
b^2*c*d*e*(b*x+a)^(3/2)+b^3*c^2*e*(b*x+a)^(1/2)-a^(1/2)*b^3*c^2*e*arctanh((
b*x+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.55, size = 156, normalized size = 1.08

$$\sqrt{a}\,c^2e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}d^2f+105\,\sqrt{bx+a}\,b^3c^2e+21\,(bd^2e+2\,(bcd-ad^2)f)(bx+a)^{\frac{5}{2}}+35\,(2\,b^2cde-abd^2e+(b^2c^2-2\,abcd+a^2d^2)f)(bx+a)^{\frac{3}{2}}\right)}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^2*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
105*(15*(b*x + a)^(7/2)*d^2*f + 105*sqrt(b*x + a)*b^3*c^2*e + 21*(b*d^2*e +
 2*(b*c*d - a*d^2)*f)*(b*x + a)^(5/2) + 35*(2*b^2*c*d*e - a*b*d^2*e + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(b*x + a)^(3/2))/b^3
```

Fricas [A]
time = 2.16, size = 401, normalized size = 2.77

$$\left[\frac{105\,\sqrt{a}b^{3}c^{2}e\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,\left(15\,b^{3}d^{2}fx^{3}+3\,\left(14\,b^{3}cd+ab^{2}d^{2}\right)fx^{2}+\left(35\,b^{3}c^{2}+14\,ab^{2}cd-4\,a^{2}bd^{2}\right)fx+\left(35\,ab^{2}c^{2}-28\,a^{2}bcd+8\,a^{3}d^{2}\right)f+7\,\left(3\,b^{3}d^{2}x^{2}+15\,b^{3}c^{2}+10\,ab^{2}cd-2\,a^{2}bd^{2}+\left(10\,b^{3}cd+ab^{2}d^{2}\right)x\right)e\right)\sqrt{bx+a}}{105\,b^{3}},\ \frac{2\,\left(105\,\sqrt{-a}b^{3}c^{2}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)e+\left(15\,b^{3}d^{2}fx^{3}+3\,\left(14\,b^{3}cd+ab^{2}d^{2}\right)fx^{2}+\left(35\,b^{3}c^{2}+14\,ab^{2}cd-4\,a^{2}bd^{2}\right)fx+\left(35\,ab^{2}c^{2}-28\,a^{2}bcd+8\,a^{3}d^{2}\right)f+7\,\left(3\,b^{3}d^{2}x^{2}+15\,b^{3}c^{2}+10\,ab^{2}cd-2\,a^{2}bd^{2}+\left(10\,b^{3}cd+ab^{2}d^{2}\right)x\right)e\right)\sqrt{bx+a}\right)}{105\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(a)*b^3*c^2*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(15*b^3*d^2*f*x^3 + 3*(14*b^3*c*d + a*b^2*d^2)*f*x^2 + (35*b^3*c^2 + 14
*a*b^2*c*d - 4*a^2*b*d^2)*f*x + (35*a*b^2*c^2 - 28*a^2*b*c*d + 8*a^3*d^2)*f
 + 7*(3*b^3*d^2*x^2 + 15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2 + (10*b^3*c*d
 + a*b^2*d^2)*x)*e)*sqrt(b*x + a))/b^3, 2/105*(105*sqrt(-a)*b^3*c^2*arctan(
sqrt(b*x + a)*sqrt(-a)/a)*e + (15*b^3*d^2*f*x^3 + 3*(14*b^3*c*d + a*b^2*d^2
)*f*x^2 + (35*b^3*c^2 + 14*a*b^2*c*d - 4*a^2*b*d^2)*f*x + (35*a*b^2*c^2 - 2
8*a^2*b*c*d + 8*a^3*d^2)*f + 7*(3*b^3*d^2*x^2 + 15*b^3*c^2 + 10*a*b^2*c*d -
 2*a^2*b*d^2 + (10*b^3*c*d + a*b^2*d^2)*x)*e)*sqrt(b*x + a))/b^3]
```

Sympy [A]
time = 11.02, size = 167, normalized size = 1.15

$$\frac{2ac^{2}e\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^{2}e\sqrt{a+bx}+\frac{2d^{2}f(a+bx)^{\frac{7}{2}}}{7b^{3}}+\frac{2(a+bx)^{\frac{5}{2}}\left(-2ad^{2}f+2bcdf+bd^{2}e\right)}{5b^{3}}+\frac{2(a+bx)^{\frac{3}{2}}\left(a^{2}d^{2}f-2abcdf-abd^{2}e+b^{2}c^{2}f+2b^{2}cde\right)}{3b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**2*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**2*e*sqrt(a + b*x) +
 2*d**2*f*(a + b*x)**(7/2)/(7*b**3) + 2*(a + b*x)**(5/2)*(-2*a*d**2*f + 2*b
*c*d*f + b*d**2*e)/(5*b**3) + 2*(a + b*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
 - a*b*d**2*e + b**2*c**2*f + 2*b**2*c*d*e)/(3*b**3)
```

Giac [A]
time = 1.20, size = 201, normalized size = 1.39

$$\frac{2\,ac^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\,\left(35\,(bx+a)^{\frac{3}{2}}b^{20}c^{2}f+42\,(bx+a)^{\frac{5}{2}}b^{19}cdf-70\,(bx+a)^{\frac{3}{2}}ab^{19}cdf+15\,(bx+a)^{\frac{7}{2}}b^{18}d^{2}f-42\,(bx+a)^{\frac{5}{2}}ab^{18}d^{2}f+35\,(bx+a)^{\frac{3}{2}}a^{2}b^{18}d^{2}f+105\,\sqrt{bx+a}\,b^{21}c^{2}e+70\,(bx+a)^{\frac{3}{2}}b^{20}cde+21\,(bx+a)^{\frac{5}{2}}b^{19}d^{2}e-35\,(bx+a)^{\frac{3}{2}}ab^{19}d^{2}e\right)}{105\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^2*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/105*(35*(b*x + a)^(3/
2)*b^20*c^2*f + 42*(b*x + a)^(5/2)*b^19*c*d*f - 70*(b*x + a)^(3/2)*a*b^19*c
*d*f + 15*(b*x + a)^(7/2)*b^18*d^2*f - 42*(b*x + a)^(5/2)*a*b^18*d^2*f + 35
*(b*x + a)^(3/2)*a^2*b^18*d^2*f + 105*sqrt(b*x + a)*b^21*c^2*e + 70*(b*x +
a)^(3/2)*b^20*c*d*e + 21*(b*x + a)^(5/2)*b^19*d^2*e - 35*(b*x + a)^(3/2)*a*
b^19*d^2*e)/b^21
```

Mupad [B]
time = 0.09, size = 263, normalized size = 1.81

$$\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{5\,b^3}+\frac{2\,a\,d^2\,f}{5\,b^3}\right)\,{\left(a+b\,x\right)}^{5/2}+\left(a\,\left(a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{b^3}\right)-\frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(a\,f-b\,e\right)}{b^3}\right)\,\sqrt{a+b\,x}+\left(\frac{a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)}{3}-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{3\,b^3}\right)\,{\left(a+b\,x\right)}^{3/2}+\frac{2\,d^2\,f\,{\left(a+b\,x\right)}^{7/2}}{7\,b^3}+\sqrt{a}\,c^2\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^2)/x,x)
```

```
[Out] ((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/(5*b^3) + (2*a*d^2*f)/(5*b^3))*(a + b*
x)^(5/2) + (a*(a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b^3 + (2*a*d^2*f)/b^3
) - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/b^3) - (2*(a*d - b*c)^2*(a*
f - b*e))/b^3)*(a + b*x)^(1/2) + ((a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b
^3 + (2*a*d^2*f)/b^3))/3 - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/(3*b
^3))*(a + b*x)^(3/2) + a^(1/2)*c^2*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
+ (2*d^2*f*(a + b*x)^(7/2))/(7*b^3)
```

3.17 $\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2/15*(b*x+a)^(3/2)*(2*a*d*f-5*b*(c*f+d*e)-3*b*d*f*x)/b^2-2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2))*a^(1/2)+2*c*e*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {152, 52, 65, 214}

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] - (2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d
*f*x))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)(e+fx)}{x}\,dx = -\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+(ce)\int \frac{\sqrt{a+bx}}{x}\,dx$$

$$= 2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+(ace)\int \frac{}{x\sqrt{}}$$

$$= 2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}+\frac{(2ace)\text{Subst}}{}$$

$$= 2ce\sqrt{a+bx}-\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2}-2\sqrt{a}\,ce\tanh$$

Mathematica [A]
time = 0.15, size = 91, normalized size = 1.18

$$\frac{2\sqrt{a+bx}\,(15b^2ce+5bde(a+bx)+5bcf(a+bx)-5adf(a+bx)+3df(a+bx)^2)}{15b^2}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(15*b^2*c*e + 5*b*d*e*(a + b*x) + 5*b*c*f*(a + b*x) - 5*a*
d*f*(a + b*x) + 3*d*f*(a + b*x)^2))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.10, size = 89, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{2df(bx+a)^{\frac{5}{2}}}{5}-\frac{2adf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bcf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bde(bx+a)^{\frac{3}{2}}}{3}+2b^2ce\sqrt{bx+a}-2\sqrt{a}\,b^2ce\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2}$
default	$\frac{\frac{2df(bx+a)^{\frac{5}{2}}}{5}-\frac{2adf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bcf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bde(bx+a)^{\frac{3}{2}}}{3}+2b^2ce\sqrt{bx+a}-2\sqrt{a}\,b^2ce\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/5*d*f*(b*x+a)^(5/2)-1/3*a*d*f*(b*x+a)^(3/2)+1/3*b*c*f*(b*x+a)^(3/2
)+1/3*b*d*e*(b*x+a)^(3/2)+b^2*c*e*(b*x+a)^(1/2)-a^(1/2)*b^2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 93, normalized size = 1.21

$$\sqrt{a}\,ce\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}df+15\,\sqrt{bx+a}\,b^2ce+5\,(bde+(bc-ad)f)(bx+a)^{\frac{3}{2}}\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/15
*(3*(b*x + a)^(5/2)*d*f + 15*sqrt(b*x + a)*b^2*c*e + 5*(b*d*e + (b*c - a*d)
*f)*(b*x + a)^(3/2))/b^2
```

Fricas [A]
time = 1.62, size = 215, normalized size = 2.79

$$\left[\frac{15\,\sqrt{a}\,b^2ce\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(3\,b^2dfx^2+(5\,b^2c+abd)fx+(5\,abc-2\,a^2d)f+5\,(b^2dx+3\,b^2c+abd)e)\sqrt{bx+a}}{15\,b^2},\ \frac{2\left(15\,\sqrt{-a}\,b^2c\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)e+(3\,b^2dfx^2+(5\,b^2c+abd)fx+(5\,abc-2\,a^2d)f+5\,(b^2dx+3\,b^2c+abd)e)\sqrt{bx+a}\right)}{15\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(a)*b^2*c*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*b^2*d*f*x^2 + (5*b^2*c + a*b*d)*f*x + (5*a*b*c - 2*a^2*d)*f + 5*(b^2*d*x
 + 3*b^2*c + a*b*d)*e)*sqrt(b*x + a))/b^2, 2/15*(15*sqrt(-a)*b^2*c*arctan(s
qrt(b*x + a)*sqrt(-a)/a)*e + (3*b^2*d*f*x^2 + (5*b^2*c + a*b*d)*f*x + (5*a*
b*c - 2*a^2*d)*f + 5*(b^2*d*x + 3*b^2*c + a*b*d)*e)*sqrt(b*x + a))/b^2]
```

Sympy [A]
time = 10.76, size = 92, normalized size = 1.19

$$\frac{2ace\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2ce\sqrt{a+bx}+\frac{2df(a+bx)^{\frac{5}{2}}}{5b^2}+\frac{2(a+bx)^{\frac{3}{2}}\,(-adf+bcf+bde)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c*e*sqrt(a + b*x) + 2*d*f
*(a + b*x)**(5/2)/(5*b**2) + 2*(a + b*x)**(3/2)*(-a*d*f + b*c*f + b*d*e)/(3
*b**2)
```

Giac [A]
time = 1.05, size = 105, normalized size = 1.36

$$\frac{2\,ac\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(5\,(bx+a)^{\frac{3}{2}}b^{9}cf+3\,(bx+a)^{\frac{5}{2}}b^{8}df-5\,(bx+a)^{\frac{3}{2}}ab^{8}df+15\,\sqrt{bx+a}\,b^{10}ce+5\,(bx+a)^{\frac{3}{2}}b^{9}de\right)}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/15*(5*(b*x + a)^(3/2)*b
^9*c*f + 3*(b*x + a)^(5/2)*b^8*d*f - 5*(b*x + a)^(3/2)*a*b^8*d*f + 15*sqrt(
b*x + a)*b^10*c*e + 5*(b*x + a)^(3/2)*b^9*d*e)/b^10
```

Mupad [B]
time = 2.49, size = 136, normalized size = 1.77

$$\left(a\left(\frac{2\,b\,c\,f-4\,a\,d\,f+2\,b\,d\,e}{b^2}+\frac{2\,a\,d\,f}{b^2}\right)+\frac{2\,(a\,d-b\,c)\,(a\,f-b\,e)}{b^2}\right)\,\sqrt{a+b\,x}+\left(\frac{2\,b\,c\,f-4\,a\,d\,f+2\,b\,d\,e}{3\,b^2}+\frac{2\,a\,d\,f}{3\,b^2}\right)\,(a+b\,x)^{3/2}+\frac{2\,d\,f\,(a+b\,x)^{5/2}}{5\,b^2}+\sqrt{a}\,c\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x))/x,x)
```

```
[Out] (a*((2*b*c*f - 4*a*d*f + 2*b*d*e)/b^2 + (2*a*d*f)/b^2) + (2*(a*d - b*c)*(a*
f - b*e))/b^2)*(a + b*x)^(1/2) + ((2*b*c*f - 4*a*d*f + 2*b*d*e)/(3*b^2) + (
2*a*d*f)/(3*b^2))*(a + b*x)^(3/2) + (2*d*f*(a + b*x)^(5/2))/(5*b^2) + a^(1/
2)*c*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

3.18 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=54

$$2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*f*(b*x+a)^(3/2)/b-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*e*(b*x+a
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {81, 52, 65, 214}

$$2e\sqrt{a+bx} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}\,(e+fx)}{x}\,dx &= \frac{2f(a+bx)^{3/2}}{3b}+e\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}}{3b}+(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}}{3b}+\frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}}{3b}-2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.98

$$\frac{2\sqrt{a+bx}\,(3be+af+bfx)}{3b}-2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(3*b*e + a*f + b*f*x))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]]
```

Maple [A]
time = 0.10, size = 46, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{2f(bx+a)^{\frac{3}{2}}}{3}+2be\sqrt{bx+a}-2\sqrt{a}\,be\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46

default	$\frac{\frac{2f(bx+a)^{\frac{3}{2}}}{3}+2be\sqrt{bx+a}-2\sqrt{a}\,be\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(1/3*f*(b*x+a)^(3/2)+b*e*(b*x+a)^(1/2)-a^(1/2)*b*e*arctanh((b*x+a)^(1/2
)/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 62, normalized size = 1.15

$$\sqrt{a}\,e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left((bx+a)^{\frac{3}{2}}f+3\sqrt{bx+a}\,be\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*((
b*x + a)^(3/2)*f + 3*sqrt(b*x + a)*b*e)/b
```

Fricas [A]
time = 1.22, size = 115, normalized size = 2.13

$$\left[\frac{3\sqrt{a}\,be\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(bfx+af+3\,be)\sqrt{bx+a}}{3\,b},\ \frac{2\left(3\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)e+(bfx+af+3\,be)\sqrt{bx+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(a)*b*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(b*f*x
 + a*f + 3*b*e)*sqrt(b*x + a))/b, 2/3*(3*sqrt(-a)*b*arctan(sqrt(b*x + a)*sq
rt(-a)/a)*e + (b*f*x + a*f + 3*b*e)*sqrt(b*x + a))/b]
```

Sympy [A]
time = 2.46, size = 54, normalized size = 1.00

$$\frac{2ae\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2e\sqrt{a+bx}+\frac{2f(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*e*sqrt(a + b*x) + 2*f*(a +
b*x)**(3/2)/(3*b)
```

Giac [A]
time = 1.06, size = 57, normalized size = 1.06

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left((bx+a)^{\frac{3}{2}}b^{2}f+3\sqrt{bx+a}\,b^{3}e\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/3*((b*x + a)^(3/2)*b^2*f
+ 3*sqrt(b*x + a)*b^3*e)/b^3
```

Mupad [B]
time = 0.07, size = 45, normalized size = 0.83

$$2\,e\,\sqrt{a+b\,x}+\frac{2\,f\,{\left(a+b\,x\right)}^{3/2}}{3\,b}+\sqrt{a}\;e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\;1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/x,x)
```

```
[Out] 2*e*(a + b*x)^(1/2) + a^(1/2)*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*
f*(a + b*x)^(3/2))/(3*b)
```

3.19 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2f\sqrt{a+bx}}{d}+\frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

```
[Out] -2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c+2*(-c*f+d*e)*arctan(d^(1/2)*(
b*x+a)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c/d^(3/2)+2*f*(b*x+a)^(1/2)
/d
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {159, 162, 65, 214, 211}

$$\frac{2\sqrt{bc-ad}\,(de-cf)\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d + (2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)}\,dx = \frac{2f\sqrt{a+bx}}{d} + \frac{2\int \frac{\frac{ade}{2}+\frac{1}{2}(bde-bcf+adf)x}{x\sqrt{a+bx}\,(c+dx)}\,dx}{d}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c} + \frac{((bc-ad)(de-cf))\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx}{cd}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc} + \frac{(2(bc-ad)(de-cf))\text{S}}{}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\right.}{c}$$

Mathematica [A]
time = 0.21, size = 101, normalized size = 1.00

$$\frac{2f\sqrt{a+bx}}{d} - \frac{2\sqrt{bc-ad}\,(-de+cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d]*S
qrt[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/c
```

Maple [A]
time = 0.11, size = 103, normalized size = 1.02

method	result
derivativedivides	$\frac{2 f \sqrt{b x +a}}{d}-\frac{2 e \,\mathrm{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{a}}{c}-\frac{2 \left(a c d f -a \,d^{2} e -c^{2} f b +b c d e \right) \mathrm{arctanh}\left(\frac{d \sqrt{b x +a}}{\sqrt{\left(a d -b c \right) d}}\right)}{d c \sqrt{\left(a d -b c \right) d}}$
default	$\frac{2 f \sqrt{b x +a}}{d}-\frac{2 e \,\mathrm{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) \sqrt{a}}{c}-\frac{2 \left(a c d f -a \,d^{2} e -c^{2} f b +b c d e \right) \mathrm{arctanh}\left(\frac{d \sqrt{b x +a}}{\sqrt{\left(a d -b c \right) d}}\right)}{d c \sqrt{\left(a d -b c \right) d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(b*x+a)^(1/2)/d-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c-2/d*(a*c*d
*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/((a*d-b*c)*d)^(1/2)*arctanh(d*(b*x+a)^(1/2)/(
(a*d-b*c)*d)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.90, size = 457, normalized size = 4.52

$$\left[\frac{\sqrt{a}\, d e \log\left(\frac{b x - 2\sqrt{b x + a}\sqrt{a} + 2 a}{x}\right) + 2\sqrt{b x + a}\, c f - (c f - d e)\sqrt{-\frac{b c - a d}{d}} \log\left(\frac{b d x - b c + 2 a d + 2\sqrt{b x + a}\, d\sqrt{-\frac{b c - a d}{d}}}{d x + c}\right)}{c d}, \frac{\sqrt{a}\, d e \log\left(\frac{b x - 2\sqrt{b x + a}\sqrt{a} + 2 a}{x}\right) + 2\sqrt{b x + a}\, c f + 2 (c f - d e)\sqrt{\frac{b c - a d}{d}} \arctan\left(-\frac{\sqrt{b x + a}\, d\sqrt{\frac{b c - a d}{d}}}{b c - a d}\right)}{c d}, \frac{2\sqrt{-a}\, d \arctan\left(\frac{\sqrt{b x + a}\sqrt{-a}}{a}\right) e + 2\sqrt{b x + a}\, c f - (c f - d e)\sqrt{-\frac{b c - a d}{d}} \log\left(\frac{b d x - b c + 2 a d + 2\sqrt{b x + a}\, d\sqrt{-\frac{b c - a d}{d}}}{d x + c}\right)}{c d}, \frac{2\left(\sqrt{-a}\, d \arctan\left(\frac{\sqrt{b x + a}\sqrt{-a}}{a}\right) e + \sqrt{b x + a}\, c f + (c f - d e)\sqrt{\frac{b c - a d}{d}} \arctan\left(-\frac{\sqrt{b x + a}\, d\sqrt{\frac{b c - a d}{d}}}{b c - a d}\right)\right)}{c d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a
)*c*f - (c*f - d*e)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d + 2*sqrt(
```

```
b*x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), (sqrt(a)*d*e*log((b*x -
 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*c*f + 2*(c*f - d*e)*sq
rt((b*c - a*d)/d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))
/(c*d), (2*sqrt(-a)*d*arctan(sqrt(b*x + a)*sqrt(-a)/a)*e + 2*sqrt(b*x + a)*
c*f - (c*f - d*e)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d + 2*sqrt(b*
x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), 2*(sqrt(-a)*d*arctan(sqrt
(b*x + a)*sqrt(-a)/a)*e + sqrt(b*x + a)*c*f + (c*f - d*e)*sqrt((b*c - a*d)/
d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d)]
```

Sympy [A]
time = 8.74, size = 100, normalized size = 0.99

$$\frac{2ae\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{c\sqrt{-a}}+\frac{2f\sqrt{a+bx}}{d}+\frac{2(ad-bc)\,(cf-de)\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-\frac{ad-bc}{d}}}\right)}{cd^2\sqrt{-\frac{ad-bc}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c),x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/(c*sqrt(-a)) + 2*f*sqrt(a + b*x)/d + 2*(
a*d - b*c)*(c*f - d*e)*atan(sqrt(a + b*x)/sqrt(-(a*d - b*c)/d))/(c*d**2*sqr
t(-(a*d - b*c)/d))
```

Giac [A]
time = 1.55, size = 112, normalized size = 1.11

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c}+\frac{2\,\sqrt{bx+a}\,f}{d}-\frac{2\,(bc^2f-acdf-bcde+ad^2e)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c) + 2*sqrt(b*x + a)*f/d - 2
*(b*c^2*f - a*c*d*f - b*c*d*e + a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d
- a*d^2))/(sqrt(b*c*d - a*d^2)*c*d)
```

Mupad [B]
time = 2.82, size = 2355, normalized size = 23.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)),x)

[Out] (2*f*(a + b*x)^(1/2))/d - (a^(1/2)*e*atan(((a^(1/2)*e*((8*(a + b*x)^(1/2)*(
b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*
f - 2*a^2*b^2*c*d^3*e*f))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2
*d^3*f))/d + (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/
(c*d)))/c)*1i)/c + (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*
d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f)
)/d - (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*
e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c)*1i)/c)/((16*(
a^2*b^3*d^3*e^3 - a*b^4*c*d^2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3
*a^2*b^3*c*d^2*e^2*f + 2*a^2*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^
4*c^2*d*e^2*f))/d - (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2
*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f
))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d + (8*a^(1/2)
*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c + (a^(1/2
)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2
- 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d
*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (a^(1/2)*e*((8*(a*b^
3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c
^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c))*2i)/c - (atan(((((8*(a + b*x)^(1/2
)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2
*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f)
)/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2
)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(
c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3) + (((8*(a + b*x)^(1/2)*(b^4*
c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c
^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f -
2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (
8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*(a +
b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(c*f - d
*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3))/((16*(a^2*b^3*d^3*e^3 - a*b^4*c*d^
2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3*a^2*b^3*c*d^2*e^2*f + 2*a^2
*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^4*c^2*d*e^2*f))/d - (((8*(a
+ b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3
*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*
b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2
*c^2*d^3*f))/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d -
 b*c))^(1/2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))
/(c*d^3))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3) + (((8*(a + b*x)^(1/
2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a
```

```
^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^
2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f
))/d - (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/
2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*
(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3)))*(c*f - d*e)*(d^3*(a*d - b*c)
)^(1/2)*2i)/(c*d^3)
```

3.20 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^2}\,dx$

Optimal. Leaf size=128

$$\frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)}-\frac{(2ad^2e-bc(de+cf))\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

```
[Out] -2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c^2-(2*a*d^2*e-b*c*(c*f+d*e))*a
rctan(d^(1/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(3/2)/(-a*d+b*c)^(1/2)+
(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {154, 162, 65, 214, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)(2ad^2e-bc(cf+de))}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}\,(de-cf)}{cd(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) - ((2*a*d^2*e - b*c*(d*e + c*f)
)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)*Sqrt[b*c -
a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^2}\,dx = \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\int \frac{-ade-\frac{1}{2}b(de+cf)x}{x\sqrt{a+bx}\,(c+dx)}\,dx}{cd}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^2} - \frac{(2ad^2e-bc(de+cf))\int \frac{1}{\sqrt{a+bx}\,(\cdots}}{2c^2d}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^2} - \frac{(2ad^2e-bc(de\cdots}{\cdots}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{(2ad^2e-bc(de+cf))\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}\,e\,\text{ta}\cdots}{\cdots}$$

Mathematica [A]
time = 0.49, size = 122, normalized size = 0.95

$$\frac{\frac{c(de-cf)\sqrt{a+bx}}{d(c+dx)} + \frac{(-2ad^2e+bc(de+cf))\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((c*(d*e - c*f)*Sqrt[a + b*x])/(d*(c + d*x)) + ((-2*a*d^2*e + b*c*(d*e + c*
f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c - a*
d]) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Maple [A]
time = 0.11, size = 137, normalized size = 1.07

method	result
derivativedivides	$2b\left(\frac{\frac{bc(cf-de)\sqrt{bx+a}}{2d(-d(bx+a)+ad-bc)}+\frac{(2a\,d^{2}e-c^{2}fb-bcde)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)\,d}}\right)}{2d\sqrt{(ad-bc)\,d}}}{c^{2}b}-\frac{e\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^{2}}\right.$
default	$2b\left(\frac{\frac{bc(cf-de)\sqrt{bx+a}}{2d(-d(bx+a)+ad-bc)}+\frac{(2a\,d^{2}e-c^{2}fb-bcde)\,\mathrm{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)\,d}}\right)}{2d\sqrt{(ad-bc)\,d}}}{c^{2}b}-\frac{e\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/c^2/b*(1/2*b*c*(c*f-d*e)/d*(b*x+a)^(1/2)/(-d*(b*x+a)+a*d-b*c)+1/2*(2
*a*d^2*e-b*c^2*f-b*c*d*e)/d/((a*d-b*c)*d)^(1/2)*arctanh(d*(b*x+a)^(1/2)/((a
*d-b*c)*d)^(1/2)))-1/b*e*a^(1/2)/c^2*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(115) = 230.
time = 0.97, size = 998, normalized size = 7.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2*d^2 - a*c*d^3 + (b*c*d^3 - a*d^4)*x)*sqrt(a)*e*log((b*x - 2*
sqrt(b*x + a)*sqrt(a) + 2*a)/x) + (b*c^2*d*f*x + b*c^3*f + (b*c^2*d - 2*a*c
*d^2 + (b*c*d^2 - 2*a*d^3)*x)*e)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*
a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x + c)) - 2*((b*c^3*d - a*c^
2*d^2)*f - (b*c^2*d^2 - a*c*d^3)*e)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 +
 (b*c^3*d^3 - a*c^2*d^4)*x), 1/2*(4*(b*c^2*d^2 - a*c*d^3 + (b*c*d^3 - a*d^4
)*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)*e + (b*c^2*d*f*x + b*c^3*f +
 (b*c^2*d - 2*a*c*d^2 + (b*c*d^2 - 2*a*d^3)*x)*e)*sqrt(-b*c*d + a*d^2)*log(
(b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x + c)) - 2
*((b*c^3*d - a*c^2*d^2)*f - (b*c^2*d^2 - a*c*d^3)*e)*sqrt(b*x + a))/(b*c^4*
d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), ((b*c^2*d^2 - a*c*d^3 + (b*c*
d^3 - a*d^4)*x)*sqrt(a)*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - (b
*c^2*d*f*x + b*c^3*f + (b*c^2*d - 2*a*c*d^2 + (b*c*d^2 - 2*a*d^3)*x)*e)*sqr
t(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) -
((b*c^3*d - a*c^2*d^2)*f - (b*c^2*d^2 - a*c*d^3)*e)*sqrt(b*x + a))/(b*c^4*d
^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), (2*(b*c^2*d^2 - a*c*d^3 + (b*c
*d^3 - a*d^4)*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)*e - (b*c^2*d*f*x
 + b*c^3*f + (b*c^2*d - 2*a*c*d^2 + (b*c*d^2 - 2*a*d^3)*x)*e)*sqrt(b*c*d -
a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - ((b*c^3*d
- a*c^2*d^2)*f - (b*c^2*d^2 - a*c*d^3)*e)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^3
*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. $2(112) = 224$.
time = 35.60, size = 1149, normalized size = 8.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**2,x)
```

```
[Out] 2*a*b*d*e*sqrt(a + b*x)/(2*a*b*c**2*d + 2*a*b*c*d**2*x - 2*b**2*c**3 - 2*b*
*2*c**2*d*x) - a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a
*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d
*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log
(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)
) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2*a*b*f*sqrt(
a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*x) + a*b*d*e*
sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*
b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + s
qrt(a + b*x))/(2*c) - a*b*d*e*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt
(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*s
```

```
qrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*c) - 2*a*e*atan(sqrt(a + b*x)
/sqrt(-a + b*c/d))/(c**2*sqrt(-a + b*c/d)) + 2*a*e*atan(sqrt(a + b*x)/sqrt(
-a))/(c**2*sqrt(-a)) + 2*b**2*c*f*sqrt(a + b*x)/(2*a*b*c*d**2 + 2*a*b*d**3*
x - 2*b**2*c**2*d - 2*b**2*c*d**2*x) + b**2*c*f*sqrt(1/(d*(a*d - b*c)**3))*
log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)
**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*d) - b**2*
c*f*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2
*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3))
+ sqrt(a + b*x))/(2*d) - b**2*e*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*s
qrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**
2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + b**2*e*sqrt(1/(d*(a*d - b
*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*
d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2
*b**2*e*sqrt(a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*
x) + 2*b*f*atan(sqrt(a + b*x)/sqrt(-a + b*c/d))/(d**2*sqrt(-a + b*c/d))
```

Giac [A]
time = 1.26, size = 142, normalized size = 1.11

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c^2}+\frac{(bc^2f+bcde-2\,ad^2e)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^2d}-\frac{\sqrt{bx+a}\,bcf-\sqrt{bx+a}\,bde}{(bc+(bx+a)d-ad)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^2) + (b*c^2*f + b*c*d*e -
2*a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)
*c^2*d) - (sqrt(b*x + a)*b*c*f - sqrt(b*x + a)*b*d*e)/((b*c + (b*x + a)*d -
 a*d)*c*d)
```

Mupad [B]
time = 2.95, size = 1814, normalized size = 14.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^2),x)
```

```
[Out] (atan((((((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) + ((4*b^3*c^
5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f -
 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(
1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) + (2*(a +
 b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*
d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c)
)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)) - (
```

```
((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) - ((4*b^3*c^5*d^3 -
8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2
*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(1/2)*(b*
c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) - (2*(a + b*x)^(1
/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f -
4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c))^(1/2)*
(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)))/((4*(a*b^4
*c*d^2*e^3 - 2*a^2*b^3*d^3*e^3 + a*b^4*c^3*e*f^2 - 2*a^2*b^3*c*d^2*e^2*f +
2*a*b^4*c^2*d*e^2*f))/(c^3*d) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2
*f))/(c^3*d) + ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(
a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d
^3)))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^
4 - b*c^3*d^3)) + (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4
*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/
(c^2*d))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2
*d^4 - b*c^3*d^3)) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d
) - ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1
/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*
(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d
^3)) - (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^
2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d
^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^
3*d^3))))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(a*c^
2*d^4 - b*c^3*d^3) - (2*a^(1/2)*e*atanh((4*a^(1/2)*b^4*e*f^2*(a + b*x)^(1/2
))/(4*a*b^4*e*f^2 + (4*a*b^4*d^2*e^3)/c^2 - (16*a^2*b^3*d^2*e^2*f)/c^2 + (8
*a*b^4*d*e^2*f)/c) + (8*a^(1/2)*b^4*e^2*f*(a + b*x)^(1/2))/(8*a*b^4*e^2*f +
 (4*a*b^4*d*e^3)/c - (16*a^2*b^3*d*e^2*f)/c + (4*a*b^4*c*e*f^2)/d) + (4*a^(
1/2)*b^4*d*e^3*(a + b*x)^(1/2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b
^3*d*e^2*f + (4*a*b^4*c^2*e*f^2)/d) - (16*a^(3/2)*b^3*d*e^2*f*(a + b*x)^(1/
2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b^3*d*e^2*f + (4*a*b^4*c^2*e*
f^2)/d)))/c^2 - ((b*c*f - b*d*e)*(a + b*x)^(1/2))/(c*d*(b*c - a*d + d*(a +
b*x)))
```

3.21 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^3}\,dx$

Optimal. Leaf size=205

$$\frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2}-\frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)}-\frac{\left(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf)\right)\tan^{-1}\left(\frac{\sqrt{d}\ldots}{\sqrt{\ldots}}\right.}{4c^3d^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*(12*a*b*c*d^2*e-8*a^2*d^3*e-b^2*c^2*(c*f+3*d*e))*arctan(d^(1/2)*(b*x+a
)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)/(-a*d+b*c)^(3/2)-2*e*arctanh((b*x+a)^
(1/2)/a^(1/2))*a^(1/2)/c^3+1/2*(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)^2-1/4*(
4*a*d^2*e-b*c*(c*f+3*d*e))*(b*x+a)^(1/2)/c^2/d/(-a*d+b*c)/(d*x+c)
```

Rubi [A]
time = 0.19, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {154, 156, 162, 65, 214, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+bx}\,\left(4ad^2e-bc(cf+3de)\right)}{4c^2d(c+dx)(bc-ad)}+\frac{\sqrt{a+bx}\,(de-cf)}{2cd(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) - ((4*a*d^2*e - b*c*(3*d*e
+ c*f))*Sqrt[a + b*x])/(4*c^2*d*(b*c - a*d)*(c + d*x)) - ((12*a*b*c*d^2*e -
 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
```

```
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^3}\,dx = \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\int \frac{-2ade-\frac{1}{2}b(3de+cf)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx}{2cd}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{\int \frac{2ad(bc-ad)e-\frac{1}{4}b(4ad^2e-}{x\sqrt{a+bx}\,(c}}{2c^2d(bc-}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^3}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{bc^3}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{(12abcd^2e-8a^2d^3e-}{}$$

Mathematica [A]
time = 1.10, size = 194, normalized size = 0.95

$$\frac{\frac{c\sqrt{a+bx}\,(2ad(3cde-c^2f+2d^2ex)+bc(c^2f-3d^2ex-cd(5e+fx)))}{d(-bc+ad)(c+dx)^2} + \frac{(-12abcd^2e+8a^2d^3e+b^2c^2(3de+cf))\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}} - 8\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((c*Sqrt[a + b*x]*(2*a*d*(3*c*d*e - c^2*f + 2*d^2*e*x) + b*c*(c^2*f - 3*d^2
*e*x - c*d*(5*e + f*x))))/(d*(-(b*c) + a*d)*(c + d*x)^2) + ((-12*a*b*c*d^2*
e + 8*a^2*d^3*e + b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[b*c - a*d]])/(d^(3/2)*(b*c - a*d)^(3/2)) - 8*Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/(4*c^3)
```

Maple [A]
time = 0.11, size = 221, normalized size = 1.08

method	result

derivativedivides	$2b^2$	$\left(\frac{\frac{\frac{bc\left(4a\,d^2e-c^2fb-3bcde\right)(bx+a)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(4a\,d^2e+c^2fb-5bcde\right)bc\sqrt{bx+a}}{8d}}{(-d(bx+a)+ad-bc)^2}+\frac{\left(8a^2d^3e-12abc\,d^2e+b^2c^3f+3b^2c^2de\right)\operatorname{arctanh}\left(\right.}{8(ad-bc)d\sqrt{(ad-bc}}}{c^3b^2}\right.$
default	$2b^2$	$\left(\frac{\frac{\frac{bc\left(4a\,d^2e-c^2fb-3bcde\right)(bx+a)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(4a\,d^2e+c^2fb-5bcde\right)bc\sqrt{bx+a}}{8d}}{(-d(bx+a)+ad-bc)^2}+\frac{\left(8a^2d^3e-12abc\,d^2e+b^2c^3f+3b^2c^2de\right)\operatorname{arctanh}\left(\right.}{8(ad-bc)d\sqrt{(ad-bc}}}{c^3b^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/c^3/b^2*((1/8*b*c*(4*a*d^2*e-b*c^2*f-3*b*c*d*e)/(a*d-b*c)*(b*x+a)^
(3/2)-1/8*(4*a*d^2*e+b*c^2*f-5*b*c*d*e)*b*c/d*(b*x+a)^(1/2))/(-d*(b*x+a)+a*
d-b*c)^2+1/8*(8*a^2*d^3*e-12*a*b*c*d^2*e+b^2*c^3*f+3*b^2*c^2*d*e)/(a*d-b*c)
/d/((a*d-b*c)*d)^(1/2)*arctanh(d*(b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2)))-1/b^2*
e*a^(1/2)/c^3*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(186) = 372$.
time = 1.45, size = 2195, normalized size = 10.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4 + (b^2*c^2*d^4 - 2*a*b*c
*d^5 + a^2*d^6)*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x)*sqrt(a
)*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + (b^2*c^3*d^2*f*x^2 + 2*b
^2*c^4*d*f*x + b^2*c^5*f + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3 +
(3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*x^2 + 2*(3*b^2*c^3*d^2 - 12*a*b*
c^2*d^3 + 8*a^2*c*d^4)*x)*e)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d
+ 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x + c)) + 2*((b^2*c^4*d^2 - a*b*
c^3*d^3)*f*x - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + (5*b^2*c^4*d
^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4 + (3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^
2*c*d^5)*x)*e)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 +
(b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^
5*d^4 + a^2*c^4*d^5)*x), 1/4*(4*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4
+ (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^
4 + a^2*c*d^5)*x)*sqrt(a)*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
(b^2*c^3*d^2*f*x^2 + 2*b^2*c^4*d*f*x + b^2*c^5*f + (3*b^2*c^4*d - 12*a*b*c^
3*d^2 + 8*a^2*c^2*d^3 + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*x^2 + 2*
(3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*x)*e)*sqrt(b*c*d - a*d^2)*ar
ctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) + ((b^2*c^4*d^2 - a*b
*c^3*d^3)*f*x - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + (5*b^2*c^4*
d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4 + (3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a
^2*c*d^5)*x)*e)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 +
 (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c
^5*d^4 + a^2*c^4*d^5)*x), 1/8*(16*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^
4 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*
d^4 + a^2*c*d^5)*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)*e + (b^2*c^3*
d^2*f*x^2 + 2*b^2*c^4*d*f*x + b^2*c^5*f + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8
*a^2*c^2*d^3 + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*x^2 + 2*(3*b^2*c^
3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*x)*e)*sqrt(-b*c*d + a*d^2)*log((b*d*x
 - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x + c)) + 2*((b^2
*c^4*d^2 - a*b*c^3*d^3)*f*x - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f
 + (5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4 + (3*b^2*c^3*d^3 - 7*a*b
*c^2*d^4 + 4*a^2*c*d^5)*x)*e)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 +
 a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6
*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), 1/4*(8*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
 + a^2*c^2*d^4 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^2 + 2*(b^2*c^3*d^3
 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)*
e - (b^2*c^3*d^2*f*x^2 + 2*b^2*c^4*d*f*x + b^2*c^5*f + (3*b^2*c^4*d - 12*a*
b*c^3*d^2 + 8*a^2*c^2*d^3 + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*x^2
+ 2*(3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*x)*e)*sqrt(b*c*d - a*d^2
)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) + ((b^2*c^4*d^2 -
 a*b*c^3*d^3)*f*x - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + (5*b^2*
c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4 + (3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 +
 4*a^2*c*d^5)*x)*e)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d
```

```
^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a
*b*c^5*d^4 + a^2*c^4*d^5)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.96, size = 301, normalized size = 1.47

$$\frac{\left(b^{2} c^{3} f + 3 \, b^{2} c^{2} d e - 12 \, a b c d^{2} e + 8 \, a^{2} d^{3} e\right) \arctan\left(\frac{\sqrt{b x + a} d}{\sqrt{b c d - a d^{2}}}\right)}{4 \, {\left(b c^{4} d - a c^{3} d^{2}\right)} \sqrt{b c d - a d^{2}}} + \frac{2 \, a \arctan\left(\frac{\sqrt{b x + a}}{\sqrt{-a}}\right) e}{\sqrt{-a} c^{3}} - \frac{\sqrt{b x + a} b^{3} c^{3} f - {\left(b x + a\right)}^{\frac{3}{2}} b^{2} c^{2} d f - \sqrt{b x + a} a b^{2} c^{2} d f - 5 \, \sqrt{b x + a} b^{3} c^{2} d e - 3 \, {\left(b x + a\right)}^{\frac{3}{2}} b^{2} c d^{2} e + 9 \, \sqrt{b x + a} a b^{2} c d^{2} e + 4 \, {\left(b x + a\right)}^{\frac{3}{2}} a b d^{3} e - 4 \, \sqrt{b x + a} a^{2} b d^{3} e}{4 \, {\left(b c^{3} d - a c^{2} d^{2}\right)} {\left(b c + {\left(b x + a\right)} d - a d\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e)*arctan(sqrt(
b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^4*d - a*c^3*d^2)*sqrt(b*c*d - a*d^2))
 + 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^3) - 1/4*(sqrt(b*x + a)
*b^3*c^3*f - (b*x + a)^(3/2)*b^2*c^2*d*f - sqrt(b*x + a)*a*b^2*c^2*d*f - 5*
sqrt(b*x + a)*b^3*c^2*d*e - 3*(b*x + a)^(3/2)*b^2*c*d^2*e + 9*sqrt(b*x + a)
*a*b^2*c*d^2*e + 4*(b*x + a)^(3/2)*a*b*d^3*e - 4*sqrt(b*x + a)*a^2*b*d^3*e)
/((b*c^3*d - a*c^2*d^2)*(b*c + (b*x + a)*d - a*d)^2)
```

Mupad [B]
time = 4.63, size = 2500, normalized size = 12.20

$$\text{Too large to display}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^3),x)
```

```
[Out] (atan((((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*
b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2
 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f -
24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - ((d^
3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e
 + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*
```

```
c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a
^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))
/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^
2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 -
 b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) + ((d^3*(a*d - b*c)^3)^(
1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e
^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320
*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^
2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*
b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e
 + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + ((d^3*(a*
d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e
- 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5
 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3
*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e
+ b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^
3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c
^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^
4 - 3*a^2*b*c^4*d^5)))/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a
^4*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5
*c^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^
8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)
^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d
*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e
^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)) - ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e -
a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d
^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2
)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e
)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c
^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c
^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*
b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*
d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*
c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5
)) + ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^
2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24
*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*
(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*
d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 -
 2*a*b*c^7*d^2) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e +
 b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^
8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2
```

```
*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(
8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*
d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*
c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5))))*(d^3*(a*d - b*c)^3)^(1/2)*(
8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(4*(a^3*c^3*d
^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) - (((a + b*x)^(1/2)*
(b^2*c^2*f + 4*a*b*d^2*e - 5*b^2*c*d*e))/(4*c^2*d) + ((a + b*x)^(3/2)*(b^2*
c^2*f - 4*a*b*d^2*e + 3*b^2*c*d*e))/(4*c^2*(a*d - b*c)))/(d^2*(a + b*x)^2 -
 (2*a*d^2 - 2*b*c*d)*(a + b*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (a^(1/2)*
e*atan(((a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9
*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3
*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^
2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5...
```

3.22 $\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=111

$$-\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4}-\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4}-\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4}-\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

```
[Out] 75/128*arcsin(2*a*x-1)/a^4-25/32*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^4-5/8*(a*x)^(
5/2)*(-a*x+1)^(1/2)/a^4-1/4*(a*x)^(7/2)*(-a*x+1)^(1/2)/a^4-75/64*(a*x)^(1/2
)*(-a*x+1)^(1/2)/a^4
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {16, 81, 52, 55, 633, 222}

$$-\frac{75\text{ArcSin}(1-2ax)}{128a^4}-\frac{\sqrt{1-ax}\,(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}\,(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}\,(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\,\sqrt{ax}}{64a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-75*Sqrt[a*x]*Sqrt[1 - a*x])/(64*a^4) - (25*(a*x)^(3/2)*Sqrt[1 - a*x])/(32
*a^4) - (5*(a*x)^(5/2)*Sqrt[1 - a*x])/(8*a^4) - ((a*x)^(7/2)*Sqrt[1 - a*x])
/(4*a^4) - (75*ArcSin[1 - 2*a*x])/(128*a^4)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\int \frac{(ax)^{5/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^3}$$

$$= -\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{15\int \frac{(ax)^{5/2}}{\sqrt{1-ax}}\,dx}{8a^3}$$

$$= -\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{25\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{16a^3}$$

$$= -\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,d}{64a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4}$$

Mathematica [A]
time = 0.14, size = 107, normalized size = 0.96

$$-\frac{\sqrt{-a}\,x(-75+25ax+10a^2x^2+24a^3x^3+16a^4x^4)-75\sqrt{x}\sqrt{1-ax}\log\left(-\sqrt{-a}\sqrt{x}+\sqrt{1-ax}\right)}{64(-a)^{7/2}\sqrt{-ax(-1+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -1/64*(Sqrt[-a]*x*(-75 + 25*a*x + 10*a^2*x^2 + 24*a^3*x^3 + 16*a^4*x^4) - 7
5*Sqrt[x]*Sqrt[1 - a*x]*Log[-(Sqrt[-a]*Sqrt[x]) + Sqrt[1 - a*x]])/((-a)^(7/
2)*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 132, normalized size = 1.19

method	result

default	$-\frac{\sqrt{-ax+1}\, x \left(32\,\mathrm{csgn}(a) a^3 x^3 \sqrt{-x\,(ax-1)\,a}+80\,\mathrm{csgn}(a) x^2 a^2 \sqrt{-x\,(ax-1)\,a}+100\,\mathrm{csgn}(a) \sqrt{-x\,(ax-1\right.}{128 a^3 \sqrt{ax}\, \sqrt{-x\,(ax-1)\,a}}$
risch	$\frac{(16a^3x^3+40a^2x^2+50ax+75)\,x(ax-1)\sqrt{ax\,(-ax+1)}}{64a^3\sqrt{-x\,(ax-1)\,a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{75\arctan\left(\frac{\sqrt{a^2}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax\,(-ax+1)}}{128a^3\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{9}{2}}\left(144a^3x^3+168a^2x^2+210ax+315\right)\sqrt{-ax+1}}{576a^4}+\frac{35\sqrt{\pi}\,(-a)^{\frac{9}{2}}\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{64a^{\frac{9}{2}}}\right)}{(-a)^{\frac{7}{2}}\sqrt{ax}\,\sqrt{\pi}}-\frac{\sqrt{x}\,\left(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(-a*x+1)^(1/2)*x*(32*csgn(a)*a^3*x^3*(-x*(a*x-1)*a)^(1/2)+80*csgn(a)
*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+100*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x+150*csgn
(a)*(-x*(a*x-1)*a)^(1/2)-75*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/
2)))*csgn(a)/a^3/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [A]
time = 0.55, size = 105, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+ax}\,x^3}{4\,a}-\frac{5\,\sqrt{-a^2x^2+ax}\,x^2}{8\,a^2}-\frac{25\,\sqrt{-a^2x^2+ax}\,x}{32\,a^3}-\frac{75\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{128\,a^4}-\frac{75\,\sqrt{-a^2x^2+ax}}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + a*x)*x^3/a - 5/8*sqrt(-a^2*x^2 + a*x)*x^2/a^2 - 25/32*
sqrt(-a^2*x^2 + a*x)*x/a^3 - 75/128*arcsin(-(2*a^2*x - a)/a)/a^4 - 75/64*sq
rt(-a^2*x^2 + a*x)/a^4
```

Fricas [A]
time = 0.93, size = 65, normalized size = 0.59

$$-\frac{(16\,a^3x^3+40\,a^2x^2+50\,ax+75)\sqrt{ax}\,\sqrt{-ax+1}+75\,\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*((16*a^3*x^3 + 40*a^2*x^2 + 50*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) + 7
5*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a^4
```

Sympy [C] Result contains complex when optimal does not.
time = 53.18, size = 484, normalized size = 4.36

$$a\left(\begin{cases} -\frac{35i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{64a^5} - \frac{ix^{\frac{9}{2}}}{4\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{7ix^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{ax-1}} - \frac{35ix^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{ax-1}} + \frac{35i\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{35\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{64a^5} + \frac{x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{7x^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{-ax+1}} + \frac{35x^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{-ax+1}} - \frac{35\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-35*I*acosh(sqrt(a)*sqrt(x))/(64*a**5) - I*x**(9/2)/(4*sqrt(a)
*sqrt(a*x - 1)) - I*x**(7/2)/(24*a**(3/2)*sqrt(a*x - 1)) - 7*I*x**(5/2)/(96
*a**(5/2)*sqrt(a*x - 1)) - 35*I*x**(3/2)/(192*a**(7/2)*sqrt(a*x - 1)) + 35*
I*sqrt(x)/(64*a**(9/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (35*asin(sqrt(a)*sqrt
(x))/(64*a**5) + x**(9/2)/(4*sqrt(a)*sqrt(-a*x + 1)) + x**(7/2)/(24*a**(3/2
)*sqrt(-a*x + 1)) + 7*x**(5/2)/(96*a**(5/2)*sqrt(-a*x + 1)) + 35*x**(3/2)/(
192*a**(7/2)*sqrt(-a*x + 1)) - 35*sqrt(x)/(64*a**(9/2)*sqrt(-a*x + 1)), Tru
e)) + Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(
a)*sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(
24*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*
x) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x
+ 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqr
t(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True))
```

Giac [A]
time = 1.16, size = 46, normalized size = 0.41

$$-\frac{(2\,(4\,(2\,ax+5)ax+25)ax+75)\sqrt{ax}\,\sqrt{-ax+1} - 75\,\arcsin\left(\sqrt{ax}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*((2*(4*(2*a*x + 5)*a*x + 25)*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) - 75*
arcsin(sqrt(a*x)))/a^4
```

Mupad [B]
time = 7.78, size = 345, normalized size = 3.11

$$\frac{75\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{32\,a^4} - \frac{\frac{5\,\sqrt{a\,x}}{4\,\left(\sqrt{1-a\,x}-1\right)} + \frac{85\,{\left(a\,x\right)}^{3/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^3} + \frac{33\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5} - \frac{33\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7} - \frac{85\,{\left(a\,x\right)}^{9/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^9} - \frac{5\,{\left(a\,x\right)}^{11/2}}{4\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}}{a^4\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^6} - \frac{\frac{35\,\sqrt{a\,x}}{32\,\left(\sqrt{1-a\,x}-1\right)} + \frac{805\,{\left(a\,x\right)}^{3/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^3} + \frac{2681\,{\left(a\,x\right)}^{5/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^5} + \frac{5053\,{\left(a\,x\right)}^{7/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^7} - \frac{5053\,{\left(a\,x\right)}^{9/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^9} - \frac{2681\,{\left(a\,x\right)}^{11/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{11}} - \frac{805\,{\left(a\,x\right)}^{13/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{13}} - \frac{35\,{\left(a\,x\right)}^{15/2}}{32\,{\left(\sqrt{1-a\,x}-1\right)}^{15}}}{a^4\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (75*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(32*a^4) - ((5*(a*x)^(1/2))/(4
*((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (
```

```
33*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x
)^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^
(11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
 1)^6) - ((35*(a*x)^(1/2))/(32*((1 - a*x)^(1/2) - 1)) + (805*(a*x)^(3/2))/(
96*((1 - a*x)^(1/2) - 1)^3) + (2681*(a*x)^(5/2))/(96*((1 - a*x)^(1/2) - 1)^
5) + (5053*(a*x)^(7/2))/(96*((1 - a*x)^(1/2) - 1)^7) - (5053*(a*x)^(9/2))/(
96*((1 - a*x)^(1/2) - 1)^9) - (2681*(a*x)^(11/2))/(96*((1 - a*x)^(1/2) - 1)
^11) - (805*(a*x)^(13/2))/(96*((1 - a*x)^(1/2) - 1)^13) - (35*(a*x)^(15/2))
/(32*((1 - a*x)^(1/2) - 1)^15))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 + 1)^8)
```

3.23 $\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=87

$$-\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

```
[Out] 11/16*arcsin(2*a*x-1)/a^3-11/12*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^3-1/3*(a*x)^(5
/2)*(-a*x+1)^(1/2)/a^3-11/8*(a*x)^(1/2)*(-a*x+1)^(1/2)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {16, 81, 52, 55, 633, 222}

$$-\frac{11\text{ArcSin}(1-2ax)}{16a^3}-\frac{\sqrt{1-ax}\,(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}\,(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\,\sqrt{ax}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-11*Sqrt[a*x]*Sqrt[1 - a*x])/(8*a^3) - (11*(a*x)^(3/2)*Sqrt[1 - a*x])/(12*
a^3) - ((a*x)^(5/2)*Sqrt[1 - a*x])/(3*a^3) - (11*ArcSin[1 - 2*a*x])/(16*a^3
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1+ax)}{\sqrt{ax}\,\sqrt{1-ax}}\,dx &= \frac{\int \frac{(ax)^{3/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^2}\\
&= -\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{6a^2}\\
&= -\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{8a^2}\\
&= -\frac{11\sqrt{ax}\,\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{1}{\sqrt{ax}\,\sqrt{1-a}}}{16a^2}\\
&= -\frac{11\sqrt{ax}\,\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{1}{\sqrt{ax-a^2x^2}}}{16a^2}\\
&= -\frac{11\sqrt{ax}\,\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.}{}\\
&= -\frac{11\sqrt{ax}\,\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 99, normalized size = 1.14

$$\frac{\sqrt{-a}\,x(-33+11ax+14a^2x^2+8a^3x^3)-33\sqrt{x}\,\sqrt{1-ax}\,\log\left(-\sqrt{-a}\,\sqrt{x}+\sqrt{1-ax}\right)}{24(-a)^{5/2}\sqrt{-ax(-1+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[-a]*x*(-33 + 11*a*x + 14*a^2*x^2 + 8*a^3*x^3) - 33*Sqrt[x]*Sqrt[1 - a
*x]*Log[-(Sqrt[-a]*Sqrt[x]) + Sqrt[1 - a*x]])/(24*(-a)^(5/2)*Sqrt[-(a*x*(-1
 + a*x))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 111, normalized size = 1.28

method	result
default	$-\frac{\sqrt{-ax+1}\,x\left(16\,\mathrm{csgn}(a)x^2a^2\sqrt{-x\,(ax-1)\,a}+44\,\mathrm{csgn}(a)\sqrt{-x\,(ax-1)\,a}\,ax+66\,\mathrm{csgn}(a)\sqrt{-x\,(ax-1)}\right.}{48a^2\sqrt{ax}\,\sqrt{-x\,(ax-1)\,a}}$
risch	$\frac{(8a^2x^2+22ax+33)x(ax-1)\sqrt{ax\,(-ax+1)}}{24a^2\sqrt{-x\,(ax-1)\,a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{11\arctan\left(\frac{\sqrt{a^2}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax\,(-ax+1)}}{16a^2\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{7}{2}}\left(56a^2x^2+70ax+105\right)\sqrt{-ax+1}}{168a^3}+\frac{5\sqrt{\pi}\,(-a)^{\frac{7}{2}}\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{8a^{\frac{7}{2}}}\right)}{(-a)^{\frac{5}{2}}\sqrt{ax}\,\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-a*x+1)^(1/2)*x*(16*csgn(a)*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+44*csgn(a)*
(-x*(a*x-1)*a)^(1/2)*a*x+66*csgn(a)*(-x*(a*x-1)*a)^(1/2)-33*arctan(1/2*csgn
(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/a^2/(a*x)^(1/2)/(-x*(a*x-1)*a)
^(1/2)
```

Maxima [A]
time = 0.54, size = 83, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+ax}\,x^2}{3\,a}-\frac{11\,\sqrt{-a^2x^2+ax}\,x}{12\,a^2}-\frac{11\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{16\,a^3}-\frac{11\,\sqrt{-a^2x^2+ax}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + a*x)*x^2/a - 11/12*sqrt(-a^2*x^2 + a*x)*x/a^2 - 11/16*
arcsin(-(2*a^2*x - a)/a)/a^3 - 11/8*sqrt(-a^2*x^2 + a*x)/a^3
```

Fricas [A]
time = 1.00, size = 57, normalized size = 0.66

$$-\frac{(8\,a^2x^2+22\,ax+33)\sqrt{ax}\,\sqrt{-ax+1}+33\,\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{24\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*((8*a^2*x^2 + 22*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) + 33*arctan(sqrt(
a*x)*sqrt(-a*x + 1)/(a*x)))/a^3
```

Sympy [C] Result contains complex when optimal does not.
time = 17.46, size = 393, normalized size = 4.52

$$a\left(\begin{cases} -\frac{5i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{8a^4}-\frac{ix^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{ax-1}}-\frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}}-\frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}}+\frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{5\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{8a^4}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-ax+1}}+\frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}}+\frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}}-\frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{3i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^3}-\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{ax-1}}-\frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}}+\frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for}\ |ax|>1 \\ \frac{3\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^3}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-ax+1}}+\frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}}-\frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(a)*s
qrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(24*a
**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*x) >
 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x + 1)
) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqrt(-a
*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True)) + Piecewise((-3*I*
acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*sqrt(a*x - 1)) - I*
x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(5/2)*sqrt(a*x - 1)
), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**(5/2)/(2*sqrt(a)*s
qrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3*sqrt(x)/(4*a**(5/
2)*sqrt(-a*x + 1)), True))
```

Giac [A]
time = 0.90, size = 40, normalized size = 0.46

$$-\frac{(2\,(4\,ax+11)ax+33)\sqrt{ax}\,\sqrt{-ax+1}-33\,\arcsin\left(\sqrt{ax}\right)}{24\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(4*a*x + 11)*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) - 33*arcsin(sqrt(
a*x)))/a^3
```

Mupad [B]
time = 5.92, size = 269, normalized size = 3.09

$$\frac{11\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{4\,a^3}-\frac{\frac{5\,\sqrt{a\,x}}{4\,\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,{\left(a\,x\right)}^{3/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^3}+\frac{33\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{33\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}-\frac{85\,{\left(a\,x\right)}^{9/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^9}-\frac{5\,{\left(a\,x\right)}^{11/2}}{4\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^6}-\frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)}+\frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3}-\frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (11*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(4*a^3) - ((5*(a*x)^(1/2))/(4*
((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (3
3*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x)
^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^(
11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
1)^6) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1)) + (11*(a*x)^(3/2))/(2*((
1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (3*
(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)
^2 + 1)^4)
```

3.24 $\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=63

$$-\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

```
[Out] 7/8*arcsin(2*a*x-1)/a^2-1/2*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^2-7/4*(a*x)^(1/2)*
(-a*x+1)^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {16, 81, 52, 55, 633, 222}

$$-\frac{7\text{ArcSin}(1-2ax)}{8a^2}-\frac{\sqrt{1-ax}\,(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\sqrt{ax}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-7*Sqrt[a*x]*Sqrt[1 - a*x])/(4*a^2) - ((a*x)^(3/2)*Sqrt[1 - a*x])/(2*a^2)
- (7*ArcSin[1 - 2*a*x])/(8*a^2)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{\sqrt{ax}\,(1+ax)}{\sqrt{1-ax}}\,dx}{a} \\
&= -\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{4a} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{8a} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{8a} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{8a^3} \\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\sin^{-1}(1-2ax)}{8a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 91, normalized size = 1.44

$$
-\frac{\sqrt{-a}\,x(-7+5ax+2a^2x^2) - 7\sqrt{x}\sqrt{1-ax}\,\log\left(-\sqrt{-a}\sqrt{x}+\sqrt{1-ax}\right)}{4(-a)^{3/2}\sqrt{-ax(-1+ax)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -1/4*(Sqrt[-a]*x*(-7 + 5*a*x + 2*a^2*x^2) - 7*Sqrt[x]*Sqrt[1 - a*x]*Log[-(S
qrt[-a]*Sqrt[x]) + Sqrt[1 - a*x]])/((-a)^(3/2)*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 90, normalized size = 1.43

method	result
default	$\frac{\sqrt{-ax+1}\, x \left(-4\,\mathrm{csgn}(a)\sqrt{-x\,(ax-1)\,a}\; ax-14\,\mathrm{csgn}(a)\sqrt{-x\,(ax-1)\,a}+7\arctan\left(\frac{\mathrm{csgn}(a)(2ax-1)}{2\sqrt{-x\,(ax-1)\,a}}\right)\right)}{8a\sqrt{ax}\;\sqrt{-x\,(ax-1)\,a}}$
risch	$\frac{(2ax+7)x(ax-1)\sqrt{ax\,(-ax+1)}}{4a\sqrt{-x\,(ax-1)\,a}\;\sqrt{ax}\;\sqrt{-ax+1}}+\frac{7\arctan\left(\frac{\sqrt{a^2}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax\,(-ax+1)}}{8a\sqrt{a^2}\;\sqrt{ax}\;\sqrt{-ax+1}}$
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{5}{2}}(10ax+15)\sqrt{-ax+1}}{20a^2}+\frac{3\sqrt{\pi}\,(-a)^{\frac{5}{2}}\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{\frac{5}{2}}}\right)}{(-a)^{\frac{3}{2}}\sqrt{ax}\;\sqrt{\pi}}-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}\sqrt{-\ldots}}{a}\right.}{\sqrt{-\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-a*x+1)^(1/2)*x*(-4*csgn(a)*(-x*(a*x-1)*a)^(1/2)*a*x-14*csgn(a)*(-x*(a
*x-1)*a)^(1/2)+7*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a
)/a/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [A]
time = 0.55, size = 61, normalized size = 0.97

$$-\frac{\sqrt{-a^2x^2+ax}\,x}{2\,a}-\frac{7\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{8\,a^2}-\frac{7\,\sqrt{-a^2x^2+ax}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + a*x)*x/a - 7/8*arcsin(-(2*a^2*x - a)/a)/a^2 - 7/4*sqrt
(-a^2*x^2 + a*x)/a^2
```

Fricas [A]
time = 0.79, size = 49, normalized size = 0.78

$$-\frac{(2\,ax+7)\sqrt{ax}\;\sqrt{-ax+1}\;+7\,\arctan\left(\frac{\sqrt{ax}\;\sqrt{-ax+1}}{ax}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) + 7*arctan(sqrt(a*x)*sqrt(-a*x +
 1)/(a*x)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 8.95, size = 269, normalized size = 4.27

$$a\left(\begin{cases} -\frac{3i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}} + \frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{3\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}} - \frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2} - \frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax| > 1 \\ \frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2} + \frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}} - \frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*s
qrt(a*x - 1)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(
5/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**
(5/2)/(2*sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3
*sqrt(x)/(4*a**(5/2)*sqrt(-a*x + 1)), True)) + Piecewise((-I*acosh(sqrt(a)*
sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3/2), Abs(a*x) > 1), (asin(sqrt
(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a*x + 1)) - sqrt(x)/(a**(3/2)*s
qrt(-a*x + 1)), True))
```

Giac [A]
time = 1.02, size = 34, normalized size = 0.54

$$-\frac{(2\,ax+7)\sqrt{ax}\,\sqrt{-ax+1} - 7\,\arcsin\left(\sqrt{ax}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) - 7*arcsin(sqrt(a*x)))/a^2
```

Mupad [B]
time = 4.53, size = 191, normalized size = 3.03

$$\frac{7\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{2\,a^2} - \frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1} - \frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2} - \frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)} + \frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3} - \frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5} - \frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (7*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(2*a^2) - ((2*(a*x)^(1/2))/((1
- a*x)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a^2*((a*x)/((
1 - a*x)^(1/2) - 1)^2 + 1)^2) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1))
+ (11*(a*x)^(3/2))/(2*((1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 -
a*x)^(1/2) - 1)^5) - (3*(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^2*((a*
x)/((1 - a*x)^(1/2) - 1)^2 + 1)^4)
```

3.25 $\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

```
[Out] 3/2*arcsin(2*a*x-1)/a-(a*x)^(1/2)*(-a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {81, 55, 633, 222}

$$-\frac{3\text{ArcSin}(1-2ax)}{2a}-\frac{\sqrt{ax}\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[1 - 2*a*x])/(2*a)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx &= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx\\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx\\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{2a^2}\\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.
time = 0.08, size = 79, normalized size = 2.14

$$
\frac{\sqrt{-a}\,x(-1+ax)-3\sqrt{x}\sqrt{1-ax}\,\log\left(-\sqrt{-a}\sqrt{x}+\sqrt{1-ax}\right)}{\sqrt{-a}\sqrt{-ax(-1+ax)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[-a]*x*(-1 + a*x) - 3*Sqrt[x]*Sqrt[1 - a*x]*Log[-(Sqrt[-a]*Sqrt[x]) +
Sqrt[1 - a*x]])/(Sqrt[-a]*Sqrt[-(a*x*(-1 + a*x))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 70, normalized size = 1.89

method	result	size
default	$-\frac{\sqrt{-ax+1}\,x\left(2\,\mathrm{csgn}(a)\sqrt{-x\,(ax-1)\,a}-3\arctan\left(\frac{\mathrm{csgn}(a)(2ax-1)}{2\sqrt{-x\,(ax-1)\,a}}\right)\right)\mathrm{csgn}(a)}{2\sqrt{ax}\,\sqrt{-x\,(ax-1)\,a}}$	70
meijerg	$-\frac{\sqrt{x}\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,(-a)^{\frac{3}{2}}\sqrt{-ax+1}}{a}+\frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{a^{\frac{3}{2}}}\right)}{\sqrt{-a}\,\sqrt{ax}\,\sqrt{\pi}}+\frac{2\sqrt{x}\,\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}\,\sqrt{ax}}$	86

risch	$\frac{x(ax-1)\sqrt{ax(-ax+1)}}{\sqrt{-x(ax-1)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax(-ax+1)}}{2\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-a*x+1)^(1/2)*x*(2*csgn(a)*(-x*(a*x-1)*a)^(1/2)-3*arctan(1/2*csgn(a)*
(2*a*x-1)/(-x*(a*x-1)*a)^(1/2)))*csgn(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [A]
time = 0.54, size = 41, normalized size = 1.11

$$-\frac{3\arcsin\left(-\frac{2a^2x-a}{a}\right)}{2a}-\frac{\sqrt{-a^2x^2+ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*arcsin(-(2*a^2*x - a)/a)/a - sqrt(-a^2*x^2 + a*x)/a
```

Fricas [A]
time = 1.06, size = 43, normalized size = 1.16

$$-\frac{\sqrt{ax}\,\sqrt{-ax+1}+3\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) + 3*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 4.88, size = 133, normalized size = 3.59

$$a\left(\begin{cases}-\frac{i\operatorname{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\,\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>1\\ \frac{\operatorname{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2i\operatorname{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\operatorname{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-I*acosh(sqrt(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3
/2), Abs(a*x) > 1), (asin(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a
```

```
*x + 1)) - sqrt(x)/(a**(3/2)*sqrt(-a*x + 1)), True)) + Piecewise((-2*I*acos
h(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*sqrt(x))/a, True))
```

Giac [A]
time = 1.29, size = 28, normalized size = 0.76

$$-\frac{\sqrt{ax}\,\sqrt{-ax+1}-3\arcsin\left(\sqrt{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) - 3*arcsin(sqrt(a*x)))/a
```

Mupad [B]
time = 3.45, size = 118, normalized size = 3.19

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{a}-\frac{4\,\mathrm{atan}\left(\frac{a\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1}-\frac{2\,(a\,x)^{3/2}}{\left(\sqrt{1-a\,x}-1\right)^3}}{a\left(\frac{a\,x}{\left(\sqrt{1-a\,x}-1\right)^2}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (2*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/a - (4*atan((a*((1 - a*x)^(1/2)
 - 1))/((a*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2) - ((2*(a*x)^(1/2))/((1 - a*x
)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a*((a*x)/((1 - a*x
)^(1/2) - 1)^2 + 1)^2)
```

3.26 $\int \frac{1+ax}{x\sqrt{ax}\,\sqrt{1-ax}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

```
[Out] arcsin(2*a*x-1)-2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {16, 79, 55, 633, 222}

$$-\text{ArcSin}(1-2ax)-\frac{2\sqrt{1-ax}}{\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - ArcSin[1 - 2*a*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx &= a\int \frac{1+ax}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}}+a\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}}+a\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{a}\\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.
time = 0.08, size = 69, normalized size = 2.38

$$\frac{2\left(-1+ax+\sqrt{-a}\sqrt{x}\sqrt{1-ax}\log\left(-\sqrt{-a}\sqrt{x}+\sqrt{1-ax}\right)\right)}{\sqrt{-ax(-1+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (2*(-1 + a*x + Sqrt[-a]*Sqrt[x]*Sqrt[1 - a*x]*Log[-(Sqrt[-a]*Sqrt[x]) + Sqr
t[1 - a*x]]))/Sqrt[-(a*x*(-1 + a*x))]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 69, normalized size = 2.38

method	result	size

meijerg	$\frac{2\sqrt{a}\,\sqrt{x}\,\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{ax}}-\frac{2\sqrt{-ax+1}}{\sqrt{ax}}$	38
default	$\frac{\left(\arctan\left(\frac{\mathrm{csgn}(a)(2ax-1)}{2\sqrt{-x\left(ax-1\right)a}}\right)ax-2\,\mathrm{csgn}(a)\sqrt{-x\left(ax-1\right)a}\right)\sqrt{-ax+1}\,\mathrm{csgn}(a)}{\sqrt{ax}\,\sqrt{-x\left(ax-1\right)a}}$	69
risch	$\frac{2(ax-1)\sqrt{ax\left(-ax+1\right)}}{\sqrt{-x\left(ax-1\right)a}\,\sqrt{ax}\,\sqrt{-ax+1}}+\frac{a\arctan\left(\frac{\sqrt{a^2}\left(x-\frac{1}{2a}\right)}{\sqrt{-a^2x^2+ax}}\right)\sqrt{ax\left(-ax+1\right)}}{\sqrt{a^2}\,\sqrt{ax}\,\sqrt{-ax+1}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/2))*a*x-2*csgn(a)*(-x*(a*x-1)*a)^(1/2))*(-a*x+1)^(1/2)*csgn(a)/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

Maxima [A]
time = 0.52, size = 41, normalized size = 1.41

$$-\frac{2\sqrt{-a^2x^2+ax}}{ax}-\arcsin\left(-\frac{2\,a^2x-a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + a*x)/(a*x) - arcsin(-(2*a^2*x - a)/a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.
time = 1.31, size = 47, normalized size = 1.62

$$-\frac{2\left(ax\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)+\sqrt{ax}\,\sqrt{-ax+1}\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(a*x*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)) + sqrt(a*x)*sqrt(-a*x + 1))/(a*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.88, size = 71, normalized size = 2.45

$$a\left(\begin{cases}-\frac{2i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}\right)+\begin{cases}-2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*I*acosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*
sqrt(x))/a, True)) + Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2
*I*sqrt(1 - 1/(a*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.
time = 1.46, size = 51, normalized size = 1.76

$$\frac{2\,a\arcsin\left(\sqrt{ax}\right) - \frac{a\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}} + \frac{\sqrt{ax}\,a}{\sqrt{-ax+1}-1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arcsin(sqrt(a*x)) - a*(sqrt(-a*x + 1) - 1)/sqrt(a*x) + sqrt(a*x)*a/(sq
rt(-a*x + 1) - 1))/a
```

Mupad [B]
time = 2.98, size = 47, normalized size = 1.62

$$-\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x}} - \frac{4\,a\,\mathrm{atan}\left(\frac{a\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] - (2*(1 - a*x)^(1/2))/(a*x)^(1/2) - (4*a*atan((a*((1 - a*x)^(1/2) - 1))/((a
*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2)
```

3.27 $\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=45

$$-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}$$

```
[Out] -2/3*a*(-a*x+1)^(1/2)/(a*x)^(3/2)-10/3*a*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {16, 79, 37}

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a*Sqrt[1 - a*x])/(3*(a*x)^(3/2)) - (10*a*Sqrt[1 - a*x])/(3*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx = a^2\int \frac{1+ax}{(ax)^{5/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}} + \frac{1}{3}(5a^2)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}} - \frac{10a\sqrt{1-ax}}{3\sqrt{ax}}$$

Mathematica [A]
time = 0.09, size = 29, normalized size = 0.64

$$-\frac{2\sqrt{-ax(-1+ax)}\,(1+5ax)}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 5*a*x))/(3*a*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.11, size = 29, normalized size = 0.64

method	result	size
gosper	$-\frac{2(5ax+1)\sqrt{-ax+1}}{3x\sqrt{ax}}$	25
default	$-\frac{2\sqrt{-ax+1}\,\mathrm{csgn}(a)^2(5ax+1)}{3x\sqrt{ax}}$	29
meijerg	$-\frac{2a\sqrt{-ax+1}}{\sqrt{ax}} - \frac{2(2ax+1)\sqrt{-ax+1}}{3\sqrt{ax}\,x}$	42
risch	$\frac{2\sqrt{ax(-ax+1)}\,(5a^2x^2-4ax-1)}{3\sqrt{ax}\sqrt{-ax+1}\,x\sqrt{-x(ax-1)a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-a*x+1)^(1/2)*csgn(a)^2/x*(5*a*x+1)/(a*x)^(1/2)
```

Maxima [A]
time = 0.57, size = 42, normalized size = 0.93

$$-\frac{10\sqrt{-a^2x^2+ax}}{3x} - \frac{2\sqrt{-a^2x^2+ax}}{3ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -10/3*sqrt(-a^2*x^2 + a*x)/x - 2/3*sqrt(-a^2*x^2 + a*x)/(a*x^2)
```

Fricas [A]
time = 1.00, size = 27, normalized size = 0.60

$$-\frac{2\,(5\,ax+1)\sqrt{ax}\,\sqrt{-ax+1}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2/3*(5*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.98, size = 107, normalized size = 2.38

$$a\left(\begin{cases} -2\sqrt{-1+\frac{1}{ax}} & \text{for}\ \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}\right)+\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**2/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
[Out] a*Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2*I*sqrt(1 - 1/(a*x)
), True)) + Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*
x), 1/Abs(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3
*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(33) = 66.
time = 1.25, size = 88, normalized size = 1.96

$$-\frac{\frac{a^2\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{21\,a^2\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^2+\frac{21\,a\left(\sqrt{-ax+1}-1\right)^2}{x}\right)(ax)^{\frac{3}{2}}}{\left(\sqrt{-ax+1}-1\right)^3}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(a^2*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 21*a^2*(sqrt(-a*x + 1) - 1)
/sqrt(a*x) - (a^2 + 21*a*(sqrt(-a*x + 1) - 1)^2/x)*(a*x)^(3/2)/(sqrt(-a*x +
 1) - 1)^3)/a
```

Mupad [B]
time = 2.75, size = 24, normalized size = 0.53

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{10\,a\,x}{3}+\frac{2}{3}\right)}{x\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((10*a*x)/3 + 2/3))/(x*(a*x)^(1/2))
```

3.28 $\int \frac{1+ax}{x^3\sqrt{ax}\,\sqrt{1-ax}}\,dx$

Optimal. Leaf size=73

$$-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}$$

```
[Out] -2/5*a^2*(-a*x+1)^(1/2)/(a*x)^(5/2)-6/5*a^2*(-a*x+1)^(1/2)/(a*x)^(3/2)-12/5
*a^2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 79, 47, 37}

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^2*Sqrt[1 - a*x])/(5*(a*x)^(5/2)) - (6*a^2*Sqrt[1 - a*x])/(5*(a*x)^(3/
2)) - (12*a^2*Sqrt[1 - a*x])/(5*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\begin{aligned}\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx &= a^3\int \frac{1+ax}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}+\frac{1}{5}\left(9a^3\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}+\frac{1}{5}\left(6a^3\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 37, normalized size = 0.51

$$-\frac{2\sqrt{-ax(-1+ax)}\left(1+3ax+6a^2x^2\right)}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 3*a*x + 6*a^2*x^2))/(5*a*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.13, size = 37, normalized size = 0.51

method	result	size
gosper	$-\frac{2(6a^2x^2+3ax+1)\sqrt{-ax+1}}{5x^2\sqrt{ax}}$	33
default	$-\frac{2\sqrt{-ax+1}\,\mathrm{csgn}(a)^2(6a^2x^2+3ax+1)}{5x^2\sqrt{ax}}$	37

meijerg	$-\frac{2a(2ax+1)\sqrt{-ax+1}}{3\sqrt{ax}\,x}-\frac{2\left(\frac{8}{3}a^2x^2+\frac{4}{3}ax+1\right)\sqrt{-ax+1}}{5\sqrt{ax}\,x^2}$	59
risch	$\frac{2\sqrt{ax\,(-ax+1)}\,(6a^3x^3-3a^2x^2-2ax-1)}{5\sqrt{ax}\,\sqrt{-ax+1}\,x^2\sqrt{-x\,(ax-1)\,a}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-a*x+1)^(1/2)*csgn(a)^2/x^2*(6*a^2*x^2+3*a*x+1)/(a*x)^(1/2)
```

Maxima [A]
time = 0.55, size = 62, normalized size = 0.85

$$-\frac{12\sqrt{-a^2x^2+ax}\,a}{5\,x}-\frac{6\sqrt{-a^2x^2+ax}}{5\,x^2}-\frac{2\sqrt{-a^2x^2+ax}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -12/5*sqrt(-a^2*x^2 + a*x)*a/x - 6/5*sqrt(-a^2*x^2 + a*x)/x^2 - 2/5*sqrt(-a
^2*x^2 + a*x)/(a*x^3)
```

Fricas [A]
time = 0.87, size = 35, normalized size = 0.48

$$-\frac{2\,(6\,a^2x^2+3\,ax+1)\sqrt{ax}\,\sqrt{-ax+1}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.04, size = 189, normalized size = 2.59

$$a\left(\begin{cases}-\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**3/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*x), 1/Abs(
a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3*x), True)
```

```
) + Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*
x) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1
/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(55) = 110.
time = 1.09, size = 130, normalized size = 1.78

$$-\frac{\frac{a^{3}\left(\sqrt{-ax+1}-1\right)^{5}}{(ax)^{\frac{5}{2}}}+\frac{15\,a^{3}\left(\sqrt{-ax+1}-1\right)^{3}}{(ax)^{\frac{3}{2}}}+\frac{110\,a^{3}\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^{3}+\frac{15\,a^{2}\left(\sqrt{-ax+1}-1\right)^{2}}{x}+\frac{110\,a\left(\sqrt{-ax+1}-1\right)^{4}}{x^{2}}\right)(ax)^{\frac{5}{2}}}{\left(\sqrt{-ax+1}-1\right)^{5}}}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80*(a^3*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 15*a^3*(sqrt(-a*x + 1) - 1)
^3/(a*x)^(3/2) + 110*a^3*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (a^3 + 15*a^2*(sq
rt(-a*x + 1) - 1)^2/x + 110*a*(sqrt(-a*x + 1) - 1)^4/x^2)*(a*x)^(5/2)/(sqrt
(-a*x + 1) - 1)^5)/a
```

Mupad [B]
time = 2.73, size = 32, normalized size = 0.44

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{12\,a^2\,x^2}{5}+\frac{6\,a\,x}{5}+\frac{2}{5}\right)}{x^2\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((6*a*x)/5 + (12*a^2*x^2)/5 + 2/5))/(x^2*(a*x)^(1/2))
```

3.29 $\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=97

$$-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}$$

```
[Out] -2/7*a^3*(-a*x+1)^(1/2)/(a*x)^(7/2)-26/35*a^3*(-a*x+1)^(1/2)/(a*x)^(5/2)-10
4/105*a^3*(-a*x+1)^(1/2)/(a*x)^(3/2)-208/105*a^3*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 79, 47, 37}

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^3*Sqrt[1 - a*x])/(7*(a*x)^(7/2)) - (26*a^3*Sqrt[1 - a*x])/(35*(a*x)^(
5/2)) - (104*a^3*Sqrt[1 - a*x])/(105*(a*x)^(3/2)) - (208*a^3*Sqrt[1 - a*x])
/(105*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\begin{aligned}\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx &= a^4\int \frac{1+ax}{(ax)^{9/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}+\frac{1}{7}\left(13a^4\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}+\frac{1}{35}\left(52a^4\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}+\frac{1}{105}\left(104a^4\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 45, normalized size = 0.46

$$-\frac{2\sqrt{-ax(-1+ax)}\left(15+39ax+52a^2x^2+104a^3x^3\right)}{105ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*
a*x^4)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.11, size = 45, normalized size = 0.46

method	result	size
gosper	$-\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{-ax+1}}{105x^3\sqrt{ax}}$	41
default	$-\frac{2\sqrt{-ax+1}\,\operatorname{csgn}(a)^2(104a^3x^3+52a^2x^2+39ax+15)}{105x^3\sqrt{ax}}$	45
risch	$\frac{2\sqrt{ax(-ax+1)}\,(104a^4x^4-52a^3x^3-13a^2x^2-24ax-15)}{105\sqrt{ax}\,\sqrt{-ax+1}\,x^3\sqrt{-x(ax-1)a}}$	71
meijerg	$-\frac{2a(\frac{8}{3}a^2x^2+\frac{4}{3}ax+1)\sqrt{-ax+1}}{5\sqrt{ax}\,x^2}-\frac{2(\frac{16}{5}a^3x^3+\frac{8}{5}a^2x^2+\frac{6}{5}ax+1)\sqrt{-ax+1}}{7\sqrt{ax}\,x^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(-a*x+1)^(1/2)*csgn(a)^2/x^3*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/(a*x
)^(1/2)
```

Maxima [A]
time = 0.49, size = 84, normalized size = 0.87

$$-\frac{208\sqrt{-a^2x^2+ax}\,a^2}{105\,x}-\frac{104\sqrt{-a^2x^2+ax}\,a}{105\,x^2}-\frac{26\sqrt{-a^2x^2+ax}}{35\,x^3}-\frac{2\sqrt{-a^2x^2+ax}}{7\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -208/105*sqrt(-a^2*x^2 + a*x)*a^2/x - 104/105*sqrt(-a^2*x^2 + a*x)*a/x^2 -
26/35*sqrt(-a^2*x^2 + a*x)/x^3 - 2/7*sqrt(-a^2*x^2 + a*x)/(a*x^4)
```

Fricas [A]
time = 0.89, size = 43, normalized size = 0.44

$$-\frac{2\,(104\,a^3x^3+52\,a^2x^2+39\,ax+15)\sqrt{ax}\,\sqrt{-ax+1}}{105\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt(a*x)*sqrt(-a*x + 1)/(a
*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.02, size = 274, normalized size = 2.82

$$a\left(\begin{cases}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**4/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*x)
 - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1/(
a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2),
 True)) + Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(
a*x))/(35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*
x**3), 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1
 - 1/(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a
*x))/(7*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(73) =$ 146.
time = 1.09, size = 175, normalized size = 1.80

$$-\frac{\frac{15\,a^4\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{231\,a^4\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{1435\,a^4\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{7875\,a^4\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(15\,a^4+\frac{231\,a^3\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{1435\,a^2\left(\sqrt{-ax+1}-1\right)^4}{x^2}+\frac{7875\,a\left(\sqrt{-ax+1}-1\right)^6}{x^3}\right)(ax)^{\frac{7}{2}}}{\left(\sqrt{-ax+1}-1\right)^7}}{6720\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6720*(15*a^4*(sqrt(-a*x + 1) - 1)^7/(a*x)^(7/2) + 231*a^4*(sqrt(-a*x + 1
) - 1)^5/(a*x)^(5/2) + 1435*a^4*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 7875*a
^4*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (15*a^4 + 231*a^3*(sqrt(-a*x + 1) - 1)^
2/x + 1435*a^2*(sqrt(-a*x + 1) - 1)^4/x^2 + 7875*a*(sqrt(-a*x + 1) - 1)^6/x
^3)*(a*x)^(7/2)/(sqrt(-a*x + 1) - 1)^7)/a
```

Mupad [B]
time = 2.77, size = 40, normalized size = 0.41

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{208\,a^3\,x^3}{105}+\frac{104\,a^2\,x^2}{105}+\frac{26\,a\,x}{35}+\frac{2}{7}\right)}{x^3\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((26*a*x)/35 + (104*a^2*x^2)/105 + (208*a^3*x^3)/105 + 2/
7))/(x^3*(a*x)^(1/2))
```

3.30 $\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=121

$$-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}$$

```
[Out] -2/9*a^4*(-a*x+1)^(1/2)/(a*x)^(9/2)-34/63*a^4*(-a*x+1)^(1/2)/(a*x)^(7/2)-68
/105*a^4*(-a*x+1)^(1/2)/(a*x)^(5/2)-272/315*a^4*(-a*x+1)^(1/2)/(a*x)^(3/2)-
544/315*a^4*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 79, 47, 37}

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^4*Sqrt[1 - a*x])/(9*(a*x)^(9/2)) - (34*a^4*Sqrt[1 - a*x])/(63*(a*x)^(
7/2)) - (68*a^4*Sqrt[1 - a*x])/(105*(a*x)^(5/2)) - (272*a^4*Sqrt[1 - a*x])/
(315*(a*x)^(3/2)) - (544*a^4*Sqrt[1 - a*x])/(315*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rubi steps

$$\begin{aligned}\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx &= a^5\int \frac{1+ax}{(ax)^{11/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}+\frac{1}{9}\left(17a^5\right)\int \frac{1}{(ax)^{9/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}+\frac{1}{21}\left(34a^5\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}+\frac{1}{105}\left(136a^5\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-}}\\ &= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}+\frac{1}{315}(272a\\ &= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{544a^4\sqrt{1}}{315\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 53, normalized size = 0.44

$$-\frac{2\sqrt{-ax(-1+ax)}\,(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*
a^4*x^4))/(315*a*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.11, size = 53, normalized size = 0.44

method	result	size
gosper	$-\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{-ax+1}}{315x^4\sqrt{ax}}$	49
default	$-\frac{2\sqrt{-ax+1}\,\mathrm{csgn}(a)^2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)}{315x^4\sqrt{ax}}$	53
risch	$\frac{2\sqrt{ax(-ax+1)}\,(272a^5x^5-136a^4x^4-34a^3x^3-17a^2x^2-50ax-35)}{315\sqrt{ax}\,\sqrt{-ax+1}\,x^4\sqrt{-x(ax-1)a}}$	79
meijerg	$-\frac{2a(\frac{16}{5}a^3x^3+\frac{8}{5}a^2x^2+\frac{6}{5}ax+1)\sqrt{-ax+1}}{7\sqrt{ax}\,x^3}-\frac{2(\frac{128}{35}a^4x^4+\frac{64}{35}a^3x^3+\frac{48}{35}a^2x^2+\frac{8}{7}ax+1)\sqrt{-ax+1}}{9\sqrt{ax}\,x^4}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(-a*x+1)^(1/2)*csgn(a)^2/x^4*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85
*a*x+35)/(a*x)^(1/2)
```

Maxima [A]
time = 0.49, size = 106, normalized size = 0.88

$$-\frac{544\sqrt{-a^2x^2+ax}\,a^3}{315\,x}-\frac{272\sqrt{-a^2x^2+ax}\,a^2}{315\,x^2}-\frac{68\sqrt{-a^2x^2+ax}\,a}{105\,x^3}-\frac{34\sqrt{-a^2x^2+ax}}{63\,x^4}-\frac{2\sqrt{-a^2x^2+ax}}{9\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -544/315*sqrt(-a^2*x^2 + a*x)*a^3/x - 272/315*sqrt(-a^2*x^2 + a*x)*a^2/x^2
- 68/105*sqrt(-a^2*x^2 + a*x)*a/x^3 - 34/63*sqrt(-a^2*x^2 + a*x)/x^4 - 2/9*
sqrt(-a^2*x^2 + a*x)/(a*x^5)
```

Fricas [A]
time = 0.92, size = 51, normalized size = 0.42

$$-\frac{2\,(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35)\sqrt{ax}\,\sqrt{-ax+1}}{315\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt(a*x)*sq
rt(-a*x + 1)/(a*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 17.24, size = 359, normalized size = 2.97

$$a\left(\begin{cases} -\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{256a^4\sqrt{-1+\frac{1}{ax}}}{315}-\frac{128a^3\sqrt{-1+\frac{1}{ax}}}{315x}-\frac{32a^2\sqrt{-1+\frac{1}{ax}}}{105x^2}-\frac{16a\sqrt{-1+\frac{1}{ax}}}{63x^3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{9x^4} & \text{for } \frac{1}{|ax|}>1 \\ -\frac{256ia^4\sqrt{1-\frac{1}{ax}}}{315}-\frac{128ia^3\sqrt{1-\frac{1}{ax}}}{315x}-\frac{32ia^2\sqrt{1-\frac{1}{ax}}}{105x^2}-\frac{16ia\sqrt{1-\frac{1}{ax}}}{63x^3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{9x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**5/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(a*x))/(3
5*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*x**3), 1
/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1 - 1/(a*
x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a*x))/(7*
x**3), True)) + Piecewise((-256*a**4*sqrt(-1 + 1/(a*x))/315 - 128*a**3*sqrt
(-1 + 1/(a*x))/(315*x) - 32*a**2*sqrt(-1 + 1/(a*x))/(105*x**2) - 16*a*sqrt(
-1 + 1/(a*x))/(63*x**3) - 2*sqrt(-1 + 1/(a*x))/(9*x**4), 1/Abs(a*x) > 1), (
-256*I*a**4*sqrt(1 - 1/(a*x))/315 - 128*I*a**3*sqrt(1 - 1/(a*x))/(315*x) -
32*I*a**2*sqrt(1 - 1/(a*x))/(105*x**2) - 16*I*a*sqrt(1 - 1/(a*x))/(63*x**3)
 - 2*I*sqrt(1 - 1/(a*x))/(9*x**4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(91) = 182$.
time = 0.85, size = 217, normalized size = 1.79

$$-\frac{\frac{35\,a^5\left(\sqrt{-ax+1}-1\right)^9}{(ax)^{\frac{9}{2}}}+\frac{585\,a^5\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{4032\,a^5\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{17640\,a^5\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{83790\,a^5\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(35\,a^5+\frac{585\,a^4\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{4032\,a^3\left(\sqrt{-ax+1}-1\right)^4}{x^2}+\frac{17640\,a^2\left(\sqrt{-ax+1}-1\right)^6}{x^3}+\frac{83790\,a\left(\sqrt{-ax+1}-1\right)^8}{x^4}\right)(ax)^{\frac{9}{2}}}{\left(\sqrt{-ax+1}-1\right)^9}}{80640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80640*(35*a^5*(sqrt(-a*x + 1) - 1)^9/(a*x)^(9/2) + 585*a^5*(sqrt(-a*x +
1) - 1)^7/(a*x)^(7/2) + 4032*a^5*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 17640
*a^5*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 83790*a^5*(sqrt(-a*x + 1) - 1)/sq
rt(a*x) - (35*a^5 + 585*a^4*(sqrt(-a*x + 1) - 1)^2/x + 4032*a^3*(sqrt(-a*x
+ 1) - 1)^4/x^2 + 17640*a^2*(sqrt(-a*x + 1) - 1)^6/x^3 + 83790*a*(sqrt(-a*x
 + 1) - 1)^8/x^4)*(a*x)^(9/2)/(sqrt(-a*x + 1) - 1)^9)/a
```

Mupad [**B**]
time = 2.83, size = 48, normalized size = 0.40

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{544\,a^4\,x^4}{315}+\frac{272\,a^3\,x^3}{315}+\frac{68\,a^2\,x^2}{105}+\frac{34\,a\,x}{63}+\frac{2}{9}\right)}{x^4\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^5*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((34*a*x)/63 + (68*a^2*x^2)/105 + (272*a^3*x^3)/315 + (54
4*a^4*x^4)/315 + 2/9))/(x^4*(a*x)^(1/2))
```

3.31 $\int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

```
[Out] 2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {156, 12, 94, 209}

$$2a\text{ArcTan}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\int \frac{2a}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\,\sqrt{1+x}\right)\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 0.95

$$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+4a\tan^{-1}\left(\sqrt{\frac{-1+x}{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 4*a*ArcTan[Sqrt[(-1 + x)/(1 + x)]]
```

Maple [A]
time = 0.12, size = 44, normalized size = 1.13

method	result	size
default	$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}}{x\sqrt{x^2-1}}$	44
risch	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}-\frac{2a\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{-1+x}\,\sqrt{1+x}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(
x^2-1)^(1/2)
```

Maxima [A]
time = 0.52, size = 21, normalized size = 0.54

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Fricas [A]
time = 0.80, size = 40, normalized size = 1.03

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\,\sqrt{x-1}-x\right)-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 26.12, size = 117, normalized size = 3.00

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\,\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\,\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\,\frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

Giac [A]
time = 0.78, size = 43, normalized size = 1.10

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

Mupad [B]
time = 4.08, size = 65, normalized size = 1.67

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}-a\left(\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x - 1)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] - a*(log(((x - 1)^(1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1) - log(((x - 1)^(
1/2) - 1i)/((x + 1)^(1/2) - 1)))*2i - ((x - 1)^(1/2)*(x + 1)^(1/2))/x
```

3.32 $\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

```
[Out] 2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {194, 156, 12, 94, 209}

$$2a\text{ArcTan}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 194

```
Int[(u_)^(m_.)*(v_)^(n_.)*(w_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[ExpandToS
um[u, x]^m*ExpandToSum[v, x]^n*ExpandToSum[w, x]^p*ExpandToSum[z, x]^q, x]
/; FreeQ[{m, n, p, q}, x] && LinearQ[{u, v, w, z}, x] &&  !LinearMatchQ[{u,
 v, w, z}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx &= \int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\int \frac{2a}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right) \\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 0.95

$$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+4a\tan^{-1}\left(\sqrt{\frac{-1+x}{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 4*a*ArcTan[Sqrt[(-1 + x)/(1 + x)]]
```

Maple [A]
time = 0.11, size = 44, normalized size = 1.13

method	result	size

default	$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}}{x\sqrt{x^2-1}}$	44
risch	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}-\frac{2a\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{-1+x}\,\sqrt{1+x}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(
x^2-1)^(1/2)
```

Maxima [A]
time = 0.50, size = 21, normalized size = 0.54

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Fricas [A]
time = 0.68, size = 40, normalized size = 1.03

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\,\sqrt{x-1}-x\right)-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 41.07, size = 117, normalized size = 3.00

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-(-a*x+1)**2)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

Giac [A]
time = 0.72, size = 43, normalized size = 1.10

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^{2}\right)-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^{4}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

Mupad [B]
time = 5.27, size = 444, normalized size = 11.38

$$a\ln\left(\frac{\sqrt{x-1}-i}{\sqrt{x+1}-1}\right)2i-a^2\,\mathrm{atan}\left(\frac{1024\,a^6}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}}+\frac{1024\,a^8}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}}-\frac{a^5\left(\sqrt{x-1}-i\right)1024i}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}\right)}-\frac{a^7\left(\sqrt{x-1}-i\right)1024i}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}\right)}\right)4i-a\ln\left(\frac{\left(\sqrt{x-1}-i\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)2i-\frac{\sqrt{x-1}-i}{4\left(\sqrt{x+1}-1\right)}+a^2\,\mathrm{acosh}\left(x\right)-\frac{\frac{5\left(\sqrt{x-1}-i\right)^2}{4\left(\sqrt{x+1}-1\right)^2}+\frac{1}{4}}{\frac{\left(\sqrt{x-1}-i\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{\sqrt{x-1}-i}{\sqrt{x+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x - 1)^2 - a^2*x^2)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] a*log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*2i - a^2*atan((1024*a^6)/(1
024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) +
 (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) + (1024*a^8)/(1024*a
^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8
*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) - (a^5*((x - 1)^(1/2) - 1
i)*1024i)/(((x + 1)^(1/2) - 1)*(1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) -
 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)
^(1/2) - 1))) - (a^7*((x - 1)^(1/2) - 1i)*1024i)/(((x + 1)^(1/2) - 1)*(1024
*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a
^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1))))*4i - a*log(((x - 1)^(
1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1)*2i - ((x - 1)^(1/2) - 1i)/(4*((x +
1)^(1/2) - 1)) + a^2*acosh(x) - ((5*((x - 1)^(1/2) - 1i)^2)/(4*((x + 1)^(1/
2) - 1)^2) + 1/4)/(((x - 1)^(1/2) - 1i)^3/((x + 1)^(1/2) - 1)^3 + ((x - 1)^
(1/2) - 1i)/((x + 1)^(1/2) - 1))
```

3.33 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}+\frac{2\sqrt{a}\,(aBe+A(b-be))F\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}$$

```
[Out] -2*a^(3/2)*B*EllipticE((1-c)^(1/2)*(b*x+a)^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2
))/b^2/(1-e)/(1-c)^(1/2)+2*(a*B*e+A*(-b*e+b))*EllipticF((1-c)^(1/2)*(b*x+a)
^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2))*a^(1/2)/b^2/(1-e)/(1-c)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {164, 114, 120}

$$\frac{2\sqrt{a}\,(aBe+A(b-be))F\left(\text{ArcSin}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}-\frac{2a^{3/2}BE\left(\text{ArcSin}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(-1 + e
)*x)/a]),x]
```

```
[Out] (-2*a^(3/2)*B*EllipticE[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e
)/(1 - c)])/(b^2*Sqrt[1 - c]*(1 - e)) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*El
lipticF[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e)/(1 - c)])/(b^2
*Sqrt[1 - c]*(1 - e))
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
```

```
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx}\,\sqrt{c+\frac{b(-1+c)x}{a}}\,\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\,\sqrt{c+\frac{b(-1+c)x}{a}}}\,dx}{b(1-e)} + \Big(A + \frac{}{b}$$

$$= -\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\,\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)} + \frac{2\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.14, size = 309, normalized size = 2.13

$$-\frac{2\sqrt{\frac{a}{-1+c}}\,(a+bx)^{3/2}\left(-B\sqrt{\frac{a}{-1+c}}\left(-1+c+\frac{a}{a+bx}\right)\left(-1+e+\frac{a}{a+bx}\right)-\frac{iaB(-1+e)\sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}}\,\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{-1+c}}}{\sqrt{a+bx}}\right)|\frac{-1+c}{-1+e}\right)}{\sqrt{a+bx}}+\frac{i(aBc+A(b-bc))(-1+e)\sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}}\,\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{-1+c}}}{\sqrt{a+bx}}\right)|\frac{-1+c}{-1+e}\right)}{\sqrt{a+bx}}\right)}{ab^2(-1+e)\sqrt{c+\frac{b(-1+c)x}{a}}\,\sqrt{e+\frac{b(-1+e)x}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(
-1 + e)*x)/a]),x]
```

```
[Out] (-2*Sqrt[a/(-1 + c)]*(a + b*x)^(3/2)*(-(B*Sqrt[a/(-1 + c)]*(-1 + c + a/(a +
 b*x))*(-1 + e + a/(a + b*x))) - (I*a*B*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x)
)/(-1 + c)]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[
```

```
a/(-1 + c)]/Sqrt[a + b*x]], (-1 + c)/(-1 + e)])/Sqrt[a + b*x] + (I*(a*B*c +
 A*(b - b*c))*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x))/(-1 + c)]*Sqrt[(-1 + e +
 a/(a + b*x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + c)]/Sqrt[a + b*x]]
, (-1 + c)/(-1 + e)])/Sqrt[a + b*x]))/(a*b^2*(-1 + e)*Sqrt[c + (b*(-1 + c)*
x)/a]*Sqrt[e + (b*(-1 + e)*x)/a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(127) = 254$.

time = 0.13, size = 604, normalized size = 4.17

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{(-1+c)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{-1+c}}\right)bce-A\,\mathrm{EllipticF}\left(\sqrt{\frac{(-1+c)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{-1+c}}\right)b\,e^{2}-B\,\mathrm{EllipticF}\left(\right.}{}$
elliptic	$\frac{\sqrt{\frac{(bx+a)(bcx+ac-bx)(bex+ae-bx)}{a^{2}}}\left(\frac{2A\left(-\frac{ae}{b(-1+e)}+\frac{ac}{b(-1+c)}\right)\sqrt{\frac{x+\frac{ac}{b(-1+c)}}{-\frac{ae}{b(-1+e)}+\frac{ac}{b(-1+c)}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{ac}{b(-1+c)}+\frac{a}{b}}}\sqrt{\frac{x+\frac{ae}{b(-1+e)}}{-\frac{ac}{b(-1+c)}+\cdots}}}{\sqrt{\frac{b^{3}ce\,x^{3}}{a^{2}}+\frac{3b^{2}ce\,x^{2}}{a}-\frac{b^{3}c\,x^{3}}{a^{2}}-\frac{b^{3}e\,x^{3}}{a^{2}}+3bcex-\frac{2b^{2}c\,x^{2}}{a}-\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2*(A*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(-1+c))^(1/2))
*b*c*e-A*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(-1+c))^(1
/2))*b*e^2-B*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(-1+c)
)^(1/2))*a*c*e+B*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(-
1+c))^(1/2))*a*e^2-A*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e
)/(-1+c))^(1/2))*b*c+A*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c
-e)/(-1+c))^(1/2))*b*e+B*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),(
(c-e)/(-1+c))^(1/2))*a*c-B*EllipticF(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2)
,((c-e)/(-1+c))^(1/2))*a*e-B*EllipticE(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/
2),((c-e)/(-1+c))^(1/2))*a*c+B*EllipticE(((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(
1/2),((c-e)/(-1+c))^(1/2))*a*e)*(-(-1+e)*(b*c*x+a*c-b*x)/a/(c-e))^(1/2)*(-(
b*x+a)*(-1+e)/a)^(1/2)*((-1+c)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2)*a/(b*x+a)^(1/
2)/((b*c*x+a*c-b*x)/a)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)/(-1+e)^2/(-1+c)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*x*(e - 1)
/a + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 1230, normalized size = 8.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="fricas")
```

```
[Out] -2/3*((B*a^3 - 3*A*a^2*b - (2*B*a^3 - 3*A*a^2*b)*c - (2*B*a^3 - 3*A*a^2*b -
 3*(B*a^3 - A*a^2*b)*c)*e)*sqrt(-(b^3*c - b^3 - (b^3*c - b^3)*e)/a^2)*weier
strassPInverse(4/3*(a^2*c^2 - a^2*c + a^2*e^2 + a^2 - (a^2*c + a^2)*e)/(b^2
*c^2 - 2*b^2*c + b^2 + (b^2*c^2 - 2*b^2*c + b^2)*e^2 - 2*(b^2*c^2 - 2*b^2*c
 + b^2)*e), 4/27*(2*a^3*c^3 - 3*a^3*c^2 - 3*a^3*c + 2*a^3*e^3 + 2*a^3 - 3*(
a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3*c + a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*
b^3*c - b^3 - (b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^3 + 3*(b^3*c^3 - 3*b^
3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e), 1/
3*(2*a*c + 3*(b*c - b)*x - (3*a*c + 3*(b*c - b)*x - 2*a)*e - a)/(b*c - (b*c
 - b)*e - b)) - 3*(B*a^2*b*c - B*a^2*b - (B*a^2*b*c - B*a^2*b)*e)*sqrt(-(b^
3*c - b^3 - (b^3*c - b^3)*e)/a^2)*weierstrassZeta(4/3*(a^2*c^2 - a^2*c + a^
2*e^2 + a^2 - (a^2*c + a^2)*e)/(b^2*c^2 - 2*b^2*c + b^2 + (b^2*c^2 - 2*b^2*
c + b^2)*e^2 - 2*(b^2*c^2 - 2*b^2*c + b^2)*e), 4/27*(2*a^3*c^3 - 3*a^3*c^2
- 3*a^3*c + 2*a^3*e^3 + 2*a^3 - 3*(a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3*c
+ a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3 - (b^3*c^3 - 3*b^3*c^2 + 3*b
^3*c - b^3)*e^3 + 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c^3
- 3*b^3*c^2 + 3*b^3*c - b^3)*e), weierstrassPInverse(4/3*(a^2*c^2 - a^2*c +
 a^2*e^2 + a^2 - (a^2*c + a^2)*e)/(b^2*c^2 - 2*b^2*c + b^2 + (b^2*c^2 - 2*b
^2*c + b^2)*e^2 - 2*(b^2*c^2 - 2*b^2*c + b^2)*e), 4/27*(2*a^3*c^3 - 3*a^3*c
^2 - 3*a^3*c + 2*a^3*e^3 + 2*a^3 - 3*(a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3
*c + a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3 - (b^3*c^3 - 3*b^3*c^2 +
3*b^3*c - b^3)*e^3 + 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c
^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e), 1/3*(2*a*c + 3*(b*c - b)*x - (3*a*c + 3
*(b*c - b)*x - 2*a)*e - a)/(b*c - (b*c - b)*e - b))))/(b^4*c^2 - 2*b^4*c +
b^4 + (b^4*c^2 - 2*b^4*c + b^4)*e^2 - 2*(b^4*c^2 - 2*b^4*c + b^4)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(c+b*(-1+c)*x/a)**(1/2)/(e+b*(-1+e)*x/a)**
(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,%%%{2,[3,1,0,0]%%%}+%%%{-2
,[3,0,0,0]%%%}+%%%{-2,[2,2,0,0]%%%}+%%%{2,[2,1,0,1]%%%}+%%%{2,[2,1,0,0]%%%}
+%%%{-2,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x}{\sqrt{c+\frac{b\,x\,\left(c-1\right)}{a}}\,\sqrt{e+\frac{b\,x\,\left(e-1\right)}{a}}\,\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a +
 b*x)^(1/2)),x)
```

```
[Out] int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a +
 b*x)^(1/2)), x)
```

3.34 $\int \frac{A+Bx}{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal. Leaf size=221

$$-\frac{2aB\sqrt{-bc+ad}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}\,(1-e)\sqrt{c+dx}}+\frac{2\sqrt{a}\,(aBe+A(b-be))\sqrt{\frac{b(c+}{bc-}}}{b^2}$$

```
[Out] 2*(a*B*e+A*(-b*e+b))*EllipticF((1-e)^(1/2)*(b*x+a)^(1/2)/a^(1/2),(-a*d/(-a*
d+b*c)/(1-e))^(1/2))*a^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)/b^2/(1-e)^(3/2)/(
d*x+c)^(1/2)-2*a*B*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),(-(-a*d+
b*c)*(1-e)/a/d)^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)/b^2/(1-
e)/d^(1/2)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {164, 115, 114, 122, 120}

$$\frac{2\sqrt{a}\,(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-e}\,\sqrt{a+bx}}{\sqrt{a}}\right)|-\frac{ad}{(bc-ad)(1-e)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}\,(1-e)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]),x]
```

```
[Out] (-2*a*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticE[ArcSin
[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], -(((b*c - a*d)*(1 - e))/(a*d)
)])/(b^2*Sqrt[d]*(1 - e)*Sqrt[c + d*x]) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticF[ArcSin[(Sqrt[1 - e]*Sqrt[a + b*x]
)/Sqrt[a]], -((a*d)/((b*c - a*d)*(1 - e)))])/(b^2*(1 - e)^(3/2)*Sqrt[c + d*
x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b(1-e)} + \left(A+\frac{aBe}{b-be}\right)\int \frac{}{\sqrt{a+b}}$$

$$= \frac{\left(\left(A+\frac{aBe}{b-be}\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}}{\sqrt{c+dx}}$$

$$= -\frac{2aB\sqrt{-bc+ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|-\right.}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.82, size = 312, normalized size = 1.41

$$-\frac{2\sqrt{\frac{a}{-1+e}}(a+bx)^{3/2}\left(-\frac{bB\sqrt{\frac{a}{-1+e}}(c+dx)(ae+b(-1+e)x)}{(a+bx)^2}-\frac{iaBd\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+bx}}\right)|\frac{(bc-ad)(-1+e)}{ad}\right)}{\sqrt{a+bx}}+\frac{id(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+bx}}\right)|\frac{(bc-ad)(-1+e)}{ad}\right)}{\sqrt{a+bx}}\right)}{ab^2d\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]
),x]
```

```
[Out] (-2*Sqrt[a/(-1 + e)]*(a + b*x)^(3/2)*(-((b*B*Sqrt[a/(-1 + e)]*(c + d*x)*(a*
e + b*(-1 + e)*x))/(a + b*x)^2) - (I*a*B*d*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[a/(-1 + e)
]/Sqrt[a + b*x]], ((b*c - a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x] + (I*d*(a*B*
e + A*(b - b*e))*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(-1 + e + a/(a + b*
x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + e)]/Sqrt[a + b*x]], ((b*c -
a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x]))/(a*b^2*d*Sqrt[c + d*x]*Sqrt[e + (b*(
-1 + e)*x)/a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(195) = 390.
time = 0.12, size = 940, normalized size = 4.25

method	result
elliptic	$\dfrac{\sqrt{\frac{(dx+c)(bx+a)(bex+ae-bx)}{a}}\left(\frac{2A\left(-\frac{a}{b}+\frac{ae}{b(-1+e)}\right)\sqrt{\frac{x+\frac{ae}{b(-1+e)}}{-\frac{a}{b}+\frac{ae}{b(-1+e)}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{ae}{b(-1+e)}+\frac{c}{d}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{ae}{b(-1+e)}+\frac{a}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{a}{b}}{-\frac{a}{b}+\cdots}}\cdots\right.}{\sqrt{\frac{b^2dex^3}{a}+2bdex^2+\frac{b^2cex^2}{a}-\frac{x^3db^2}{a}+adex+2bcex-bdx^2-\frac{b^2\cdots}{\cdots}}}\cdots\right.}{\cdots}$
default	$\dfrac{2\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d(bex+ae-bx)}{ade-bce+bc}}\sqrt{-\frac{(bx+a)(-1+e)}{a}}\sqrt{-\frac{(dx+c)b(-1+e)}{ade-bce+bc}}\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{d(bex+ae-bx)}{ade-bce+bc}},\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2)*(
-(b*x+a)*(-1+e)/a)^(1/2)*(-(d*x+c)*b*(-1+e)/(a*d*e-b*c*e+b*c))^(1/2)*(A*Ell
ipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^
(1/2))*a*b*d*e^2-A*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((
a*d*e-b*c*e+b*c)/d/a)^(1/2))*b^2*c*e^2-B*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*
e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a^2*d*e^2+B*EllipticF((d
*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a*
b*c*e^2-A*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c
*e+b*c)/d/a)^(1/2))*a*b*d*e+2*A*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b
*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*b^2*c*e+B*EllipticF((d*(b*e*x+a*e
-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a^2*d*e-2*B*E
llipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a
)^(1/2))*a*b*c*e-B*EllipticE((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((
a*d*e-b*c*e+b*c)/d/a)^(1/2))*a^2*d*e+B*EllipticE((d*(b*e*x+a*e-b*x)/(a*d*e-
b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*a*b*c*e-A*EllipticF((d*(b*
e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/d/a)^(1/2))*b^2*c+
B*EllipticF((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),((a*d*e-b*c*e+b*c)/
d/a)^(1/2))*a*b*c-B*EllipticE((d*(b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c))^(1/2),(
(a*d*e-b*c*e+b*c)/d/a)^(1/2))*a*b*c)/((b*e*x+a*e-b*x)/a)^(1/2)/(b*d*x^2+a*d
*x+b*c*x+a*c)/(-1+e)^2/b^2/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*x*(e - 1)/a + e)),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 1124, normalized size = 5.09

$$\frac{2\left(\left(Babc+(Ba^2-3Aab)d-(Babc+(2Ba^2-3Aab)d)e\right)\sqrt{\frac{b^2de-b^2d}{a}}\operatorname{weierstrassPInverse}\left(\frac{4\left(b^2c^2-abcd+a^2d^2+(b^2c^2-2abcd+a^2d^2)e^2-(2b^2c^2-3abcd+a^2d^2)e\right)}{3\left(b^2d^2e^2-2b^2d^2e+b^2d^2\right)},\frac{4\left(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3-2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)e^3+3(2b^3c^3-5ab^2c^2d+4a^2bcd^2-a^3d^3)e^2-3(2b^3c^3-4ab^2c^2d+a^2bcd^2+a^3d^3)e\right)}{27\left(b^3d^3e^3-3b^3d^3e^2+3b^3d^3e-b^3d^3\right)},-\frac{3bdx+bc+ad-(3bdx+bc+2ad)e}{3(bde-bd)}\right)-3(Babde-Babd)\sqrt{\frac{b^2de-b^2d}{a}}\operatorname{weierstrassZeta}\left(\frac{4\left(b^2c^2-abcd+a^2d^2+(b^2c^2-2abcd+a^2d^2)e^2-(2b^2c^2-3abcd+a^2d^2)e\right)}{3\left(b^2d^2e^2-2b^2d^2e+b^2d^2\right)},\frac{4\left(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3-2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)e^3+3(2b^3c^3-5ab^2c^2d+4a^2bcd^2-a^3d^3)e^2-3(2b^3c^3-4ab^2c^2d+a^2bcd^2+a^3d^3)e\right)}{27\left(b^3d^3e^3-3b^3d^3e^2+3b^3d^3e-b^3d^3\right)},\operatorname{weierstrassPInverse}\left(\frac{4\left(b^2c^2-abcd+a^2d^2+(b^2c^2-2abcd+a^2d^2)e^2-(2b^2c^2-3abcd+a^2d^2)e\right)}{3\left(b^2d^2e^2-2b^2d^2e+b^2d^2\right)},\frac{4\left(2b^3c^3-3ab^2c^2d-3a^2bcd^2+2a^3d^3-2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)e^3+3(2b^3c^3-5ab^2c^2d+4a^2bcd^2-a^3d^3)e^2-3(2b^3c^3-4ab^2c^2d+a^2bcd^2+a^3d^3)e\right)}{27\left(b^3d^3e^3-3b^3d^3e^2+3b^3d^3e-b^3d^3\right)},-\frac{3bdx+bc+ad-(3bdx+bc+2ad)e}{3(bde-bd)}\right)\right)\right)}{3\left(b^3d^2e^2-2b^3d^2e+b^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/3*((B*a*b*c + (B*a^2 - 3*A*a*b)*d - (B*a*b*c + (2*B*a^2 - 3*A*a*b)*d)*e)*
sqrt((b^2*d*e - b^2*d)/a)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*
d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^
2)*e)/(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^
2 - 3*(2*b^3*c^3 - 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 -
 3*b^3*d^3*e^2 + 3*b^3*d^3*e - b^3*d^3), -1/3*(3*b*d*x + b*c + a*d - (3*b*d
*x + b*c + 2*a*d)*e)/(b*d*e - b*d)) - 3*(B*a*b*d*e - B*a*b*d)*sqrt((b^2*d*e
 - b^2*d)/a)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2 + (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*e)/(b^2*d^2*e^
2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e^3 +
3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^2 - 3*(2*b^3*c^3
- 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 - 3*b^3*d^3*e^2 +
3*b^3*d^3*e - b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)
*e)/(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^2
- 3*(2*b^3*c^3 - 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 - 3
*b^3*d^3*e^2 + 3*b^3*d^3*e - b^3*d^3), -1/3*(3*b*d*x + b*c + a*d - (3*b*d*x
 + b*c + 2*a*d)*e)/(b*d*e - b*d))))/(b^3*d^2*e^2 - 2*b^3*d^2*e + b^3*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\sqrt{a + bx}\,\sqrt{c + dx}\,\sqrt{e + \frac{bex}{a} - \frac{bx}{a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(e+b*(-1+e)*x/a)**(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + b*e*x/a - b*x/a)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*x*(e - 1)/a + e)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x}{\sqrt{e + \frac{b\,x\,(e - 1)}{a}}\,\sqrt{a + b\,x}\,\sqrt{c + d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))
,x)
```

```
[Out] int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))
, x)
```

3.35 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3\,dx$

Optimal. Leaf size=281

$$-\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800}-\frac{12243139\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}{356400}-\frac{17561\sqrt{2-}}{}$$

```
[Out] 522167393/139968*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2
)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)-6489123157/699840*EllipticE(2/11*(2-3*x)^(1/
2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-1182926269
/1603800*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-12243139/356400*(7+5*x)
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-17561/8910*(7+5*x)^2*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/2970*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^
(1/2)*(1+4*x)^(1/2)+2/55*(7+5*x)^4*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/
2)
```

Rubi [A]
time = 0.26, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {167, 1614, 1629, 164, 115, 114, 122, 120}

$$\frac{522167393\sqrt{\frac{11}{6}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{23328\sqrt{2x-5}}-\frac{6489123157\sqrt{11}\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{699840\sqrt{5-2x}}+\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4-\frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}{2970}-\frac{17561\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}{8910}-\frac{12243139\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}{356400}-\frac{1182926269\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1603800}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (-1182926269*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1603800 - (1224313
9*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/356400 - (17561*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/8910 - (427*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/2970 + (2*Sqrt[2 - 3*x]*Sqrt[
-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^4)/55 - (6489123157*Sqrt[11]*Sqrt[-5 + 2*
x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(699840*Sqrt[5 - 2*
x]) + (522167393*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(23328*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
```

```
-1]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3\,dx = \frac{2}{55}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^4 + \frac{1}{55}\int \frac{(7+5}{\sqrt{2-}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-}$$

$$= -\frac{17561\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}{8910} - \frac{427\sqrt{2}}{}$$

$$= -\frac{12243139\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}{356400} - \frac{1756}{}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139}{}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139}{}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139}{}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139}{}$$

Mathematica [A]
time = 10.84, size = 135, normalized size = 0.48

$$\frac{24\sqrt{2-3x}\,\sqrt{1+4x}\,(3325071575-797747975x-670058262x^2-167736600x^3+67338000x^4+29160000x^5)-71380354727\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)\middle|\frac{1}{3}\right)+57438413230\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)\middle|\frac{1}{3}\right)}{15396480\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(3325071575 - 797747975*x - 670058262*x^2 -
 167736600*x^3 + 67338000*x^4 + 29160000*x^5) - 71380354727*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 57438413230*Sqrt
[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(15396
480*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.15, size = 154, normalized size = 0.55

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-8398080000x^{7}-15894144000x^{6}+57788380800x^{5}+29554530236\sqrt{1+4x}\,\sqrt{2-\right.}{}$
risch	$-\frac{(14580000x^{4}+70119000x^{3}+91429200x^{2}-106456131x-665014315)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+}}{641520\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{250x^{4}\sqrt{-24x^{3}+70x^{2}-21x-10}}{11}+\frac{64925x^{3}\sqrt{-24x^{3}+70x^{2}-2}}{594}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/15396480*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-8398080000*x^7-158
94144000*x^6+57788380800*x^5+29554530236*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/
2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-71380354727*(1+4*x
)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2)
,3^(1/2))+176080611456*x^4+141293068560*x^3-1085513167176*x^2+360716686200*
x+159603435600)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [A]
time = 0.20, size = 43, normalized size = 0.15

$$\frac{1}{641520}\left(14580000\,x^{4}+70119000\,x^{3}+91429200\,x^{2}-106456131\,x-665014315\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/641520*(14580000*x^4 + 70119000*x^3 + 91429200*x^2 - 106456131*x - 665014
315)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3,x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3, x)
```

3.36 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2\,dx$

Optimal. Leaf size=243

$$-\frac{5256763\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{97200}-\frac{8141\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}{2700}-\frac{61}{270}\sqrt{2-3x}\,\sqrt{-5$$

```
[Out] 5592499/23328*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)-17746949/29160*EllipticE(2/11*(2-3*x)^(1/2)*11^
(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-5256763/97200*(2
-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-8141/2700*(7+5*x)*(2-3*x)^(1/2)*(-
5+2*x)^(1/2)*(1+4*x)^(1/2)-61/270*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)+2/45*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$,
Rules used = {167, 1614, 1629, 164, 115, 114, 122, 120}

$$\frac{5592499\sqrt{\frac{11}{6}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{3888\sqrt{2x-5}}-\frac{17746949\sqrt{11}\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{29160\sqrt{5-2x}}+\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3-\frac{61}{270}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{8141\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}{2700}-\frac{5256763\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{97200}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (-5256763*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/97200 - (8141*Sqrt[2
- 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/2700 - (61*Sqrt[2 - 3*x]*Sqr
t[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/270 + (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/45 - (17746949*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipti
cE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(29160*Sqrt[5 - 2*x]) + (5592
499*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3])/(3888*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2\,dx = \frac{2}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3 + \frac{1}{45}\int \frac{(7+5}{\sqrt{2-}}$$

$$= -\frac{61}{270}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2 + \frac{2}{45}\sqrt{2-}$$

$$= -\frac{8141\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}{2700} - \frac{61}{270}\sqrt{2-}$$

$$= -\frac{5256763\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}}{}$$

$$= -\frac{5256763\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}}{}$$

$$= -\frac{5256763\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}}{}$$

$$= -\frac{5256763\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}}{}$$

Mathematica [A]
time = 8.85, size = 130, normalized size = 0.53

$$\frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,(6902575-2933650x-1649952x^2+147600x^3+216000x^4)-35493898\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)+27962495\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{116640\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(6902575 - 2933650*x - 1649952*x^2 + 147600*
x^3 + 216000*x^4) - 35493898*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3] + 27962495*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(116640*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.13, size = 149, normalized size = 0.61

method	result

default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-15552000x^6-4147200x^5+12899689\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\right.}{}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{50x^3\sqrt{-24x^3+70x^2-21x-10}}{9}+\frac{955x^2\sqrt{-24x^3+70x^2-21}}{54}\right.}{}$
risch	$-\frac{(108000x^3+343800x^2+34524x-1380515)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{19440\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/116640*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-15552000*x^6-4147200
*x^5+12899689*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(
1/11*(11+44*x)^(1/2),3^(1/2))-35493898*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)
*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+125816544*x^4+163495
440*x^3-604794324*x^2+171873450*x+82830900)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [A]
time = 0.20, size = 38, normalized size = 0.16

$$\frac{1}{19440}\left(108000\,x^3+343800\,x^2+34524\,x-1380515\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/19440*(108000*x^3 + 343800*x^2 + 34524*x - 1380515)*sqrt(4*x + 1)*sqrt(2*
x - 5)*sqrt(-3*x + 2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2,x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2, x)
```

3.37 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)\,dx$

Optimal. Leaf size=193

$$-\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780}+\frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2}+\frac{5}{28}\sqrt{2-3x}\,(-5+2x)^{3/2}(1+4x$$

```
[Out] 5/28*(-5+2*x)^(3/2)*(1+4*x)^(3/2)*(2-3*x)^(1/2)+72479/4536*EllipticF(1/11*3
3^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+13
6/105*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)-954811/22680*EllipticE(2/1
1*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/
2)-20911/3780*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {159, 164, 115, 114, 122, 120}

$$\frac{72479\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{756\sqrt{2x-5}}-\frac{954811\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{22680\sqrt{5-2x}}+\frac{5}{28}\sqrt{2-3x}\,(2x-5)^{3/2}(4x+1)^{3/2}+\frac{136}{105}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}-\frac{20911\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{3780}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (-20911*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3780 + (136*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/105 + (5*Sqrt[2 - 3*x]*(-5 + 2*x)^(3/2)*
(1 + 4*x)^(3/2))/28 - (954811*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*S
qrt[2 - 3*x])/Sqrt[11]], -1/2])/(22680*Sqrt[5 - 2*x]) + (72479*Sqrt[11/6]*S
qrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(756*Sqrt[-5
 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)\,dx = \frac{5}{28}\sqrt{2-3x}\,(-5+2x)^{3/2}(1+4x)^{3/2} + \frac{1}{28}\int \frac{\left(\frac{1249}{2}-1088x\right)}{\surd}$$

$$= \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{5}{28}\sqrt{2-3x}\,(-5+2x$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5}$$

Mathematica [A]
time = 1.54, size = 125, normalized size = 0.65

$$\frac{24\sqrt{2-3x}\,\sqrt{1+4x}\,(48475-37975x-6066x^2+5400x^3)-954811\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)+724790\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{45360\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(48475 - 37975*x - 6066*x^2 + 5400*x^3) - 9
54811*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3] + 724790*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(45360*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.14, size = 144, normalized size = 0.75

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-1555200x^5+264748\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\sqrt{1}\right.\right.}{}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{59x\sqrt{-24x^3+70x^2-21x-10}}{30}-\frac{277\sqrt{-24x^3+70x^2-21x}}{54}\right.}{}$
risch	$-\frac{(2700x^2+3717x-9695)(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{1890\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}-\frac{\left(-\frac{31\sqrt{22-33}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/45360*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-1555200*x^5+264748*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(
1/2),3^(1/2))-954811*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ell
ipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2395008*x^4+10468080*x^3-18808968*x^2+
3994200*x+2326800)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [A]
time = 0.24, size = 33, normalized size = 0.17

$$\frac{1}{1890}\left(2700\,x^2+3717\,x-9695\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/1890*(2700*x^2 + 3717*x - 9695)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\cdot(5x+7)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7), x)
```

3.38 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,dx$

Optimal. Leaf size=162

$$-\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2}-\frac{847\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{}}{}\right.\right.}{270\sqrt{5-2x}}$$

```
[Out] 121/108*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+1/10*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)-847/2
70*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(
1/2)/(5-2*x)^(1/2)-22/45*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {103, 159, 164, 115, 114, 122, 120}

$$\frac{121\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{18\sqrt{2x-5}}-\frac{847\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{270\sqrt{5-2x}}+\frac{1}{10}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}-\frac{22}{45}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (-22*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/45 + (Sqrt[2 - 3*x]*Sqrt[-
5 + 2*x]*(1 + 4*x)^(3/2))/10 - (847*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(270*Sqrt[5 - 2*x]) + (121*Sqrt[11/6]
*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(18*Sqrt[-
5 + 2*x])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
```

```
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
```

```
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,dx = \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} - \frac{1}{10}\int \frac{\left(\frac{99}{2}-44x\right)\sqrt{1+4x}}{\sqrt{2-3x}\,\sqrt{-5+2x}}\,dx$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

Mathematica [A]
time = 1.83, size = 120, normalized size = 0.74

$$\frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,(175-250x+72x^2) - 847\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right) + 605\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{540\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(175 - 250*x + 72*x^2) - 847*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 605*Sqrt[66]*Sqr
t[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(540*Sqrt[-5 +
 2*x])
```

Maple [A]
time = 0.13, size = 139, normalized size = 0.86

method	result

default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(121\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.\right.}{540(24x^3-}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{2x\sqrt{-24x^3+70x^2-21x-10}}{5}-\frac{7\sqrt{-24x^3+70x^2-21x-1}}{18}\right.}{}$
risch	$-\frac{(-35+36x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{90\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{\sqrt{22-33x}\,\sqrt{165-}}{36\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/540*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(121*(1+4*x)^(1/2)*(2-3*x
)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-847*
(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)
^(1/2),3^(1/2))-5184*x^4+20160*x^3-19236*x^2+2250*x+2100)/(24*x^3-70*x^2+21
*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [A]
time = 0.19, size = 28, normalized size = 0.17

$$\frac{1}{90}\,(36\,x-35)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")
[Out] 1/90*(36*x - 35)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2), x)
```

3.39 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{7+5x}\,dx$

Optimal. Leaf size=182

$$\frac{2}{15}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{427\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}-\frac{1253\sqrt{\frac{2}{33}}\,\sqrt{5-2x}}{3}$$

```
[Out] -1253/12375*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-
2*x)^(1/2)/(-5+2*x)^(1/2)-2691/1375*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),
55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-427/225*Ellipti
cE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*
x)^(1/2)+2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {167, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{1253\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{375\sqrt{2x-5}}-\frac{427\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}-\frac{2691\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{125\sqrt{11}\,\sqrt{2x-5}}+\frac{2}{15}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/15 - (427*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(225*Sqrt[5 - 2*
x]) - (1253*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(375*Sqrt[-5 + 2*x]) - (2691*Sqrt[5 - 2*x]*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(125*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{7+5x}\,dx = \frac{2}{15}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{15}\int \frac{-3-1190x+}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= \frac{2}{15}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{15}\int \frac{-\frac{11928}{25}+\frac{854x}{5}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= \frac{2}{15}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{1253}{375}\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}}$$

$$= \frac{2}{15}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{\left(1253\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right)\int \frac{}{\sqrt{2}}}{375\sqrt{-}}$$

$$= \frac{2}{15}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{427\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\right.}{225\sqrt{5-2x}}$$

Mathematica [A]
time = 5.85, size = 139, normalized size = 0.76

$$\frac{\sqrt{-5+2x}\left(1650\sqrt{2-3x}\,\sqrt{5-2x}\,\sqrt{1+4x}-23485\sqrt{11}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-3759\sqrt{11}\,F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+24219\sqrt{11}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{12375\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(1650*Sqrt[2 - 3*x]*Sqrt[5 - 2*x]*Sqrt[1 + 4*x] - 23485*Sqr
t[11]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 3759*Sqrt[11]*E
llipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 24219*Sqrt[11]*Ellipti
cPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(12375*Sqrt[5 - 2*x
])
```

Maple [A]
time = 0.14, size = 174, normalized size = 0.96

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(54488\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.\right.}{}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{-24x^3+70x^2-21x-10}}{15}-\frac{3976\sqrt{11+44x}\sqrt{22-33x}\sqrt{}}{15125\sqrt{-24x^3}}\right.}{}$
risch	$-\frac{2(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{15\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}-\frac{\left(\frac{854\sqrt{22-33x}\sqrt{165-66x}\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/24750*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(54488*(1+4*x)^(1/2)*(2-
3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+2
3485*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+
44*x)^(1/2),3^(1/2))-87048*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/
2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))+79200*x^3-231000*x^2+693
00*x+33000)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{5\,x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7), x)
```

3.40 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5(7+5x)}+\frac{6\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}}+\frac{152\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\mathrm{s}\right.}{125\sqrt{}}$$

```
[Out] 152/4125*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x
)^(1/2)/(-5+2*x)^(1/2)+26859/85250*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+6/25*EllipticE(2
/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(
1/2)-1/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]

time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {166, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\frac{152\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{125\sqrt{2x-5}}+\frac{6\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}}+\frac{26859\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{7750\sqrt{11}\,\sqrt{2x-5}}-\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{5(5x+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] -1/5*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (6*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt
[5 - 2*x]) + (152*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt
[1 + 4*x]], 1/3])/(125*Sqrt[-5 + 2*x]) + (26859*Sqrt[5 - 2*x]*EllipticPi[55
/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(7750*Sqrt[11]*Sqrt[-5 + 2
*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^2}\,dx = -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5(7+5x)} + \frac{1}{10}\int \frac{-21+140x-}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5(7+5x)} + \frac{1}{10}\int \frac{\frac{1204}{25}-\frac{72x}{5}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5(7+5x)} - \frac{18}{25}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx +$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5(7+5x)} + \frac{\left(152\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right)\int \frac{}{\sqrt{2-}}}{125\sqrt{-5}}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5(7+5x)} + \frac{6\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2}{}\right.\right.}{25\sqrt{5-2x}}$$

Mathematica [A]
time = 6.02, size = 130, normalized size = 0.69

$$\frac{\sqrt{-5+2x}\left(-\frac{51150\sqrt{2-3x}\,\sqrt{1+4x}}{7+5x}+\frac{3\sqrt{11}\left(20460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+9424F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-26859\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}}\right)}{255750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((-51150*Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (3*Sqrt[1
1]*(20460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 9424*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 26859*EllipticPi[55/124, Ar
cSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/255750
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(145) = 290$.
time = 0.14, size = 302, normalized size = 1.60

method	result

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{5(7+5x)}+\frac{602\sqrt{11+44x}\sqrt{22-33x}\sqrt{}}{15125\sqrt{-24x}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(18975\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}\,\mathrm{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.\right.}{}$
risch	$\frac{(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{5(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}-\frac{\left(-\frac{12\sqrt{22-33x}\sqrt{165-66x}\sqrt{33}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/31625*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(18975*(1+4*x)^(1/2)*(2
-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*
x-89530*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*
(11+44*x)^(1/2),-55/23,3^(1/2))*x+55430*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2
)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))*x+26565*(1+4*x)^(1/
2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1
/2))-125342*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1
/11*(11+44*x)^(1/2),-55/23,3^(1/2))+77602*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1
/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+151800*x^3-442750
*x^2+132825*x+63250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 + 70*x + 49), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2, x)
```

3.41 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2}+\frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{556140(7+5x)}-\frac{8953\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{}}{}\right.\right.}{1390350\sqrt{5-2x}}$$

```
[Out] 397/1960750*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-
2*x)^(1/2)/(-5+2*x)^(1/2)-14832503/3160729000*EllipticPi(2/11*(2-3*x)^(1/2)
*11^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-8953/
1390350*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2
*x)^(1/2)/(5-2*x)^(1/2)-1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+
5*x)^2+8953/556140*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {166, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\frac{397\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{89125\sqrt{2x-5}}-\frac{8953\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1390350\sqrt{5-2x}}-\frac{14832503\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{287339000\sqrt{11}\,\sqrt{2x-5}}+\frac{8953\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{556140(5x+7)}-\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{10(5x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] -1/10*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2 + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(556140*(7 + 5*x)) - (8953*Sqrt[11]
*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(13903
50*Sqrt[5 - 2*x]) + (397*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/1
1]*Sqrt[1 + 4*x]], 1/3])/(89125*Sqrt[-5 + 2*x]) - (14832503*Sqrt[5 - 2*x]*E
llipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(287339000*Sqr
t[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^3}\,dx = -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2} + \frac{1}{20}\int \frac{-21+140x-}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{556140(7+5x)}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{556140(7+5x)}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{556140(7+5x)}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{556140(7+5x)}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{556140(7+5x)}$$

Mathematica [A]
time = 6.21, size = 134, normalized size = 0.59

$$\frac{\sqrt{-5+2x}\left(\frac{17050\sqrt{2-3x}\,\sqrt{1+4x}\,(7057+44765x)}{(7+5x)^2}+\frac{\sqrt{11}\left(-61059460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+5759676F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+44497509\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}}\right)}{9482187000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7057 + 44765*x))/(7 +
5*x)^2 + (Sqrt[11]*(-61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] + 5759676*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 4449
7509*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5
- 2*x]))/9482187000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(175) = 350$.
time = 0.14, size = 434, normalized size = 1.91

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{10(7+5x)^2}+\frac{8953\sqrt{-24x^3+70x^2-21x-}}{556140(7+5x)}\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(7057+44765x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{556140(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{8953\sqrt{22-33x}\,\sqrt{16}}{}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(512860900\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3517585500*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(512860900*(1+4*x)^
(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3
^(1/2))*x^2+283138625*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*El
lipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x^2-741625150*(1+4*x)^(1/2)*(2-3*x)^(
1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))
*x^2+1436010520*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Elliptic
F(1/11*(11+44*x)^(1/2),3^(1/2))*x+792788150*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^
(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x-2076550420*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^
(1/2),-55/23,3^(1/2))*x+1005207364*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-
2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+554951705*(1+4*x)^(1/2)*
(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2)
)-1453585294*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(
1/11*(11+44*x)^(1/2),-55/23,3^(1/2))+6795327000*x^4-18748451150*x^3+2821424
375*x^2+3768732275*x+446355250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3 + 525*x^2 + 73
5*x + 343), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{(5x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3,x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3, x)
```

3.42 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^4}\,dx$

Optimal. Leaf size=263

$$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3}+\frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1668420(7+5x)^2}+\frac{16830401\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{30929169960(7+5x)}$$

```
[Out] 15664616449/175780782606000*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1
/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+24957247/327135451500*E
llipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-
5+2*x)^(1/2)-16830401/77322924900*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2
*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-1/15*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3+8953/1668420*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)/(7+5*x)^2+16830401/30929169960*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(
1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.25, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {166, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$\frac{24957247\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{4956597750\sqrt{66}\,\sqrt{2x-5}}-\frac{16830401\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{77322924900\sqrt{5-2x}}+\frac{15664616449\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{15980071146000\sqrt{11}\,\sqrt{2x-5}}+\frac{16830401\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{30929169960(5x+7)}+\frac{8953\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1668420(5x+7)^2}-\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{15(5x+7)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

```
[Out] -1/15*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3 + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1668420*(7 + 5*x)^2) + (16830401*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(30929169960*(7 + 5*x)) - (16830
401*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(77322924900*Sqrt[5 - 2*x]) + (24957247*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(4956597750*Sqrt[66]*Sqrt[-5 + 2*x]) + (
15664616449*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[
11]], -1/2])/(15980071146000*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^4}\,dx = -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{1}{30}\int \frac{-21+140x-7}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{1668420(7+5x)^2}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{1668420(7+5x)^2}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{1668420(7+5x)^2}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{1668420(7+5x)^2}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{1668420(7+5x)^2}$$

$$= -\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{1668420(7+5x)^2}$$

Mathematica [A]
time = 6.48, size = 139, normalized size = 0.53

$$\frac{\sqrt{-5+2x}\left(\frac{17050\sqrt{2-3x}\,\sqrt{1+4x}\,(-75460017+2007981640x+420760025x^2)}{(7+5x)^3}+\frac{\sqrt{11}\left(-114783334820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+120693246492F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-46993849347\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}}\right)}{527342347818000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-75460017 + 2007981640
*x + 420760025*x^2))/(7 + 5*x)^3 + (Sqrt[11]*(-114783334820*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 120693246492*EllipticF[ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2] - 46993849347*EllipticPi[55/124, ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/527342347818000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(205) = 410$.

time = 0.14, size = 566, normalized size = 2.15

method	result
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(-\frac{\sqrt{-24x^{3}+70x^{2}-21x-10}}{15(7+5x)^{3}}+\frac{8953\sqrt{-24x^{3}+70x^{2}-21x}}{1668420(7+5x)^{2}}\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(420760025x^{2}+2007981640x-75460017)\sqrt{(2-3x)\left(-5+2x\right)\left(1+4x\right)}}{30929169960(7+5x)^{3}\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{2-3x}}-$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(274048323500\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+}}{11}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/195626999997000*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(274048323500
*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x
)^(1/2),3^(1/2))*x^3-2661307158125*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-
2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x^3-3916154112250*(1+4*x
)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2
),-55/23,3^(1/2))*x^3+1151002958700*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5
-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))*x^2-11177490064125*(1+4
*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/
2),3^(1/2))*x^2-16447847271450*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)
^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*x^2+1611404142180*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(
1/2),3^(1/2))*x-15648486089775*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)
^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x-23026986180030*(1+4*x)^(1/
2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55
/23,3^(1/2))*x+751988599684*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1
```

```
/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-7302626841895*(1+4*x)^(1/2)*(2-
3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))-1
0745926884014*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi
(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))-63871371795000*x^5-118520111883250*x^
4+844601251369975*x^3-326733155441000*x^2-116981861971975*x+4772846075250)/
(24*x^3-70*x^2+21*x+10)/(7+5*x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4 + 3500*x^3 + 7
350*x^2 + 6860*x + 2401), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{(5x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**4,x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4, x)
```

3.43 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$

Optimal. Leaf size=570

$$\frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b}-\frac{2\sqrt{-de+cf}\,(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{g+hx}\,E\Big(\sin}{3b^2d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

```
[Out] 2/3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b-2/3*(3*a*d*f*h-b*(c*f*h+d*e
*h+d*f*g))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/
(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/
2)/b^2/d/h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(3*a^2*d*
f*h^2-3*a*b*(c*f+d*e)*h^2-b^2*(d*g*(-e*h+f*g)-c*h*(2*e*h+f*g)))*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^3/
d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*f+b*e)*(-a*h+b*g)*EllipticPi(
f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e
)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h
*x+g)/(-c*h+d*g))^(1/2)/b^3/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]

time = 0.87, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {167, 1621, 175, 552, 551, 164, 115, 114, 122, 121}

$$\frac{2\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,(3a^2dfh^2-3abh^2(cf+de)-(b^2(dg(fg-eh)-ch(2eh+fg))))\,F\left(\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^3d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{g+hx}}-\frac{2(be-af)(bg-ah)\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^3\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}-\frac{2\sqrt{g+hx}\,\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,(3adfh-b(cfh+deh+dfg))E\left(\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^2d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{\frac{d(g+hx)}{dg-ch}}}+\frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], (
(d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^2*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(
g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a^2*d*f*h^2 - 3*a*b*(d*e
 + c*f)*h^2 - b^2*(d*g*(f*g - e*h) - c*h*(f*g + 2*e*h)))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqr
t[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^3*d
*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]
*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d
```

```
*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^3*Sqrt[f]*Sq
rt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{a+bx}\,dx = \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} + \frac{\int \frac{3bceg-a(deg+cfg+ceh)+2(b(deg+cfg+ce}{(a+bx)\sqrt{c+d}}}{}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} + \frac{\int \frac{2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+}{\sqrt{c+dx}\,\sqrt{e+f}}}{3b}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{(2(bc-ad)(be-af)(bg-ah))\mathrm{Su}}{}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{\left(2(bc-ad)(be-af)(bg-ah)\sqrt{}\right.}{}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}\,(3adfh-b(dfg+d}{}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}\,(3adfh-b(dfg+d}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.38, size = 1254, normalized size = 2.20

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(3*b^2*e*g - 3*a*b*f*g + (b^2*f*g^2)/h - 3*a*b*e*h + (b^2*
e^2*h)/f - (b^2*c^2*f*h)/d^2 + (3*a*b*c*f*h)/d + 2*b^2*f*g*x + 2*b^2*e*h*x
- 3*a*b*f*h*x + (b^2*c*f*h*x)/d + b^2*f*h*x^2 - (b^2*c*e*g)/(c + d*x) - (3*
a*b*d*e*g)/(c + d*x) + (3*a*b*c*f*g)/(c + d*x) + (3*a*b*c*e*h)/(c + d*x) +
(b^2*c^3*f*h)/(d^2*(c + d*x)) - (3*a*b*c^2*f*h)/(d*(c + d*x)) + (b^2*d*e^2*
g)/(c*f + d*f*x) - (b^2*c*e^2*h)/(c*f + d*f*x) + (b^2*d*e*g^2)/(c*h + d*h*x
) - (b^2*c*f*g^2)/(c*h + d*h*x) - (I*b*Sqrt[-c + (d*g)/h]*(3*a*d*f*h - b*(d
*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt
[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c
 + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/d^2 + (I*b*Sqrt[-c + (d*g)/h]*(
-(b*f*g) - 2*b*e*h + 3*a*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*g)/h]
/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/d + ((3*I)*b^2*e*g*Sqrt[
-c + (d*g)/h]*h*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g
+ h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*Arc
Sinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/(
d*g - c*h) + ((3*I)*a^2*f*Sqrt[-c + (d*g)/h]*h^2*Sqrt[c + d*x]*Sqrt[(d*(e +
 f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*h
 - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d
*e*h - c*f*h)/(d*f*g - c*f*h)])/(d*g - c*h) + ((3*I)*a*b*f*g*Sqrt[-c + (d*g
)/h]*h*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(
h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt
[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/(-(d*g) +
c*h) + ((3*I)*a*b*e*Sqrt[-c + (d*g)/h]*h^2*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))
/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d
*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h -
 c*f*h)/(d*f*g - c*f*h)])/(-(d*g) + c*h)))/(3*b^3*Sqrt[e + f*x]*Sqrt[g + h*
x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3669 vs. $2(507) = 1014$.
time = 0.16, size = 3670, normalized size = 6.44

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ce}}{3b}\right.}{}$

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x,method=_RETURNVERBO
SE)
```

```
[Out] 2/3*(-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(
e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-
d*g))^(1/2))*b^2*d^2*f^2*g^3+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h
-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^
(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a^2*d^2*e*f*
h^3-3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(
e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-
b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a^2*d^2*f^2*g*h^2-3*(-(h*x+g)*f/(e*h-
f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*Ellipti
cPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*
h-d*g))^(1/2))*a*b*d^2*e^2*h^3+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c
*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g)
)^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c*d*e^
2*h^3+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h
/(e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*
h-d*g))^(1/2))*a^2*d^2*f^2*g*h^2+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/
(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h-f*g
))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*d^2*e^2*h^3+(-(h*x+g)*f/(e*h-
f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*Ellipti
cF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c^2*e*
f*h^3-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(
e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-
d*g))^(1/2))*b^2*c^2*f^2*g*h^2-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h
-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h-f*g))^(
1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c*d*e^2*h^3-2*(-(h*x+g)*f/(e*h-f*
g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticF
((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*d^2*e^2*
g*h^2-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(
e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-
d*g))^(1/2))*b^2*c^2*e*f*h^3+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d
*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/
2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c^2*f^2*g*h^2-(-(h*x+g)*f/(e*h-f*g)
)^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((
-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c*d*e^2*h^
3+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-
f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g)
)^(1/2))*b^2*d^2*e^2*g*h^2-3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d
*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/
```

```
2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a^2*d^2*e*f*h^3+b^2*c*d*e*f*h^3*x+b^2*c
*d*f^2*g*h^2*x+b^2*d^2*e*f*g*h^2*x+b^2*c*d*e*f*g*h^2+b^2*c*d*f^2*h^3*x^2+b^
2*d^2*e*f*h^3*x^2+b^2*d^2*f^2*g*h^2*x^2+b^2*d^2*f^2*h^3*x^3-3*(-(h*x+g)*f/(
e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*Ell
ipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*d^
2*e*f*g*h^2-3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*
x+e)*h/(e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d
/f/(c*h-d*g))^(1/2))*a*b*d^2*f^2*g^2*h+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c
)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticF((-(h*x+g)*f/(e*h
-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c*d*f^2*g^2*h+2*(-(h*x+g)
*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)
*EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^
2*d^2*e*f*g^2*h+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*
((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*
g)*d/f/(c*h-d*g))^(1/2))*a*b*d^2*f^2*g^2*h+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*(
(d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*
f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*e*f*h^3+(-(h*x+
g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/
2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*
b^2*c*d*e*f*g*h^2-3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2
)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-
f*g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*f^2*g*h^2-3*(-(h*x+g)*f/(e*h-f*g))^(1/2)
*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+
g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/
2))*a*b*c*d*e*f*h^3+3*(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1
/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*
h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*f^2*g*h^2+3*(
-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g
))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(h*x+g)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)/(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x),x)
```

```
[Out] int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x), x)
```

3.44 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^3}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=243

$$\frac{46134551\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{38880}+\frac{26291}{540}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)+\frac{1679}{756}\sqrt{2-3x}\,\sqrt{-5}$$

```
[Out] -2161804579/326592*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1
/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+2629157597/163296*EllipticE(2/11*(2-3*x)^(
1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+46134551
/38880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+26291/540*(7+5*x)*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1679/756*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)+1/9*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {168, 1614, 1629, 164, 115, 114, 122, 120}

$$-\frac{2161804579\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{54432\sqrt{2x-5}}+\frac{2629157597\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{163296\sqrt{5-2x}}+\frac{1}{9}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^3+\frac{1679}{756}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2+\frac{26291}{540}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{46134551\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{38880}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (46134551*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/38880 + (26291*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/540 + (1679*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/756 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/9 + (2629157597*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipt
icE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(163296*Sqrt[5 - 2*x]) - (21
61804579*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]
], 1/3])/(54432*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^3}{\sqrt{-5+2x}}\,dx = \frac{1}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3 - \frac{1}{18}\int \frac{(7+5x)^2\,(-699 -}{\sqrt{2-3x}\,\sqrt{-5 -}}$$

$$= \frac{1679}{756}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= \frac{26291}{540}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) + \frac{1679}{756}\sqrt{2-3x}\,\sqrt{-5}$$

$$= \frac{46134551\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= \frac{46134551\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= \frac{46134551\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= \frac{46134551\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\,\sqrt{-5+2x}$$

Mathematica [A]
time = 11.35, size = 130, normalized size = 0.53

$$\frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,(-455686385+51484034x+21329208x^2+8614800x^3+1512000x^4)+2629157597\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)\middle|\frac{1}{3}\right)-2161804579\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)\middle|\frac{1}{3}\right)}{326592\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-455686385 + 51484034*x + 21329208*x^2 + 86
14800*x^3 + 1512000*x^4) + 2629157597*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcS
in[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 2161804579*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(326592*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.39, size = 149, normalized size = 0.61

method	result

default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(108864000x^6+574905600x^5+1227098543\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\right.}{}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{125x^3\sqrt{-24x^3+70x^2-21x-10}}{9}+\frac{86075x^2\sqrt{-24x^3+70x^2-}}{756}\right.}{}$
risch	$-\frac{(756000x^3+6197400x^2+26158104x+91137277)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{54432\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/326592*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(108864000*x^6+57490560
0*x^5+1227098543*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ellipti
cF(1/11*(11+44*x)^(1/2),3^(1/2))-2629157597*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^
(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+1259114976*x^4+
2963596608*x^3-34609891236*x^2+13052783142*x+5468236620)/(24*x^3-70*x^2+21*
x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [A]
time = 0.35, size = 38, normalized size = 0.16

$$\frac{1}{54432}\left(756000\,x^3+6197400\,x^2+26158104\,x+91137277\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/54432*(756000*x^3 + 6197400*x^2 + 26158104*x + 91137277)*sqrt(4*x + 1)*sq
rt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2-3x}\,\sqrt{4x+1}\,(5x+7)^3}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**3/sqrt(2*x - 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{\left(5\,x+7\right)}^3}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2), x)
```

3.45 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^2}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=205

$$\frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080}+\frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)+\frac{1}{7}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1$$

```
[Out] -1679161/4536*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)+8198333/9072*EllipticE(2/11*(2-3*x)^(1/2)*11^(1
/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+73207/1080*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+173/60*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(
1/2)*(1+4*x)^(1/2)+1/7*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {168, 1614, 1629, 164, 115, 114, 122, 120}

$$-\frac{1679161\sqrt{\frac{11}{6}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{756\sqrt{2x-5}}+\frac{8198333\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}}+\frac{1}{7}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2+\frac{173}{60}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{73207\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1080}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (73207*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1080 + (173*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/60 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/7 + (8198333*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE
[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (1679161
*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(756*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^2}{\sqrt{-5+2x}}\,dx = \frac{1}{7}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2 - \frac{1}{14}\int \frac{(7+5x)\,(-543-}{\sqrt{2-3x}\,\sqrt{-5+}}$$

$$= \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) + \frac{1}{7}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

Mathematica [A]
time = 8.85, size = 125, normalized size = 0.61

$$\frac{12\sqrt{2-3x}\,\sqrt{1+4x}\,(-717955+102592x+46836x^2+10800x^3)+8198333\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)-6716644\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{18144\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-717955 + 102592*x + 46836*x^2 + 10800*x^3
) + 8198333*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3] - 6716644*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(18144*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.14, size = 144, normalized size = 0.70

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(1555200x^5+3753266\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\sqrt{11}\right.\right.}{}$

elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(\frac{25x^2\sqrt{-24x^3+70x^2-21x-10}}{7}+\frac{293x\sqrt{-24x^3+70x^2-21x}}{12}\right.}{}$
risch	$-\frac{\left(5400x^2+36918x+143591\right)\left(-2+3x\right)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{\left(2-3x\right)\left(-5+2x\right)\left(1+4x\right)}}{1512\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{2-3x}}-\frac{\left(\frac{17533\sqrt{22-}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/18144*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(1555200*x^5+3753266*(1+
4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1
/2),3^(1/2))-8198333*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ell
ipticE(1/11*(11+44*x)^(1/2),3^(1/2))+6096384*x^4+11703888*x^3-110665104*x^2
+40615092*x+17230920)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [A]
time = 0.28, size = 33, normalized size = 0.16

$$\frac{1}{1512}\left(5400\,x^2+36918\,x+143591\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/1512*(5400*x^2 + 36918*x + 143591)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x
+ 2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,(5x+7)^2}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**2/sqrt(2*x - 5), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{\left(5\,x+7\right)}^2}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2), x)
```

3.46 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=162

$$\frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2}+\frac{1397\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.}{27\sqrt{5-2x}}$$

```
[Out] -4543/216*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*
x)^(1/2)/(-5+2*x)^(1/2)+1/4*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)+1397
/27*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^
(1/2)/(5-2*x)^(1/2)+95/18*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {159, 164, 115, 114, 122, 120}

$$-\frac{4543\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{36\sqrt{2x-5}}+\frac{1397\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27\sqrt{5-2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}+\frac{95}{18}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (95*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 + (Sqrt[2 - 3*x]*Sqrt[-5
 + 2*x]*(1 + 4*x)^(3/2))/4 + (1397*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin
[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(27*Sqrt[5 - 2*x]) - (4543*Sqrt[11/6]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5
 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
```

```
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) && !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) && !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{1}{20}\int \frac{\left(\frac{1065}{2}-950x\right)\sqrt{1+4x}}{\sqrt{2-3x}\,\sqrt{-5+2x}}\,dx$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} -$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} -$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} -$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} -$$

Mathematica [**A**]
time = 2.18, size = 120, normalized size = 0.74

$$\frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,(-995+218x+72x^2)+5588\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)-4543\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{216\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-995 + 218*x + 72*x^2) + 5588*Sqrt[66]*Sqrt
[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 4543*Sqrt[66]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(216*Sqrt[-
5 + 2*x])
```

Maple [**A**]
time = 0.17, size = 139, normalized size = 0.86

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(2453\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right.}{5184x^3-1512}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(x\sqrt{-24x^3+70x^2-21x-10}+\frac{199\sqrt{-24x^3+70x^2-21x-}}{36}\right.}{}$
risch	$-\frac{(199+36x)(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}-\frac{\left(\frac{179\sqrt{22-33x}\sqrt{165-}}{792}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/216*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2453*(1+4*x)^(1/2)*(2-3*x
)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-5588
*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x
)^(1/2),3^(1/2))+5184*x^4+13536*x^3-79044*x^2+27234*x+11940)/(24*x^3-70*x^2
+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [A]
time = 0.33, size = 28, normalized size = 0.17

$$\frac{1}{36}\left(36\,x+199\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/36*(36*x + 199)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\cdot(5x+7)}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)/sqrt(2*x - 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\left(5\,x+7\right)}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2), x)
```

3.47 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=131

$$\frac{1}{3}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{55\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}}-\frac{11\sqrt{\frac{22}{3}}\,\sqrt{5-2x}\,F\left(\right.}{3\sqrt{}}$$

```
[Out] -11/9*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)+55/18*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/
2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+1/3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {103, 164, 115, 114, 122, 120}

$$-\frac{11\sqrt{\frac{22}{3}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{3\sqrt{2x-5}}+\frac{55\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}}+\frac{1}{3}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3 + (55*Sqrt[11]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(18*Sqrt[5 - 2*x]) -
 (11*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(3*Sqrt[-5 + 2*x])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
```

```
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{1}{3}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{1}{3}\int \frac{-\frac{33}{2}+55x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$$

$$= \frac{1}{3}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{55}{6}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx - \frac{121}{3}\int \frac{}{\sqrt{2}}$$

$$= \frac{1}{3}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{\left(11\sqrt{22}\,\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}}}{3\sqrt{-5+2x}}$$

$$= \frac{1}{3}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{55\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right.}{18\sqrt{5-2x}}$$

Mathematica [A]
time = 1.25, size = 115, normalized size = 0.88

$$\frac{12\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x} + 55\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right) - 44\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{36\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 55*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 44*Sqrt[66]*Sqrt[5 - 2*x]*El
lipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.13, size = 134, normalized size = 1.02

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(22\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right.}{864x^3-2520x^2+756}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-24x^3+70x^2-21x-10}}{3}+\frac{\sqrt{11+44x}\sqrt{22-33x}\sqrt{11\cdots}}{22\sqrt{-24x^3+\cdots}}\right.}{\cdots}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{3\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}-\frac{\left(\frac{\sqrt{22-33x}\sqrt{165-66x}\sqrt{33\cdots}}{66\sqrt{-24x^3+\cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(22*(1+4*x)^(1/2)*(2-3*x)^(
1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55*(1+4
*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/
2),3^(1/2))+288*x^3-840*x^2+252*x+120)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [A]
time = 0.31, size = 23, normalized size = 0.18

$$\frac{1}{3}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/sqrt(2*x - 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2), x)
```

3.48 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)}\,dx$

Optimal. Leaf size=151

$$\frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}}-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{25\sqrt{-5+2x}}+\frac{69\sqrt{}}{}$$

```
[Out] -41/825*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+69/275*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,
1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+2/5*EllipticE(2/11*(2-
3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {169, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{2\sqrt{11}\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}}+\frac{69\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{11}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (2*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(5*Sqrt[5 - 2*x]) - (41*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt
[3/11]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (69*Sqrt[5 - 2*x]*Ellipt
icPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]*Sqrt[-
5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 169

```
Int[(Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)])/(((a_.) + (b_.)*(x_
))*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[(b*e - a*f)*((b*g - a*h)/b^
2), Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + D
ist[1/b^2, Int[Simp[b*f*g + b*e*h - a*f*h + b*f*h*x, x]/(Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)}\,dx = \frac{1}{25}\int \frac{109-60x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx - \frac{713}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}$$

$$= -\left(\frac{6}{5}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\right) - \frac{41}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx +$$

$$= -\frac{\left(41\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{25\sqrt{-5+2x}} + \frac{\left(1426\sqrt{\frac{3}{11}}\sqrt{\right.}{}$$

$$= \frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}} - \frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\sin^{-1}\right.}{25\sqrt{-5+}}$$

Mathematica [A]
time = 2.50, size = 95, normalized size = 0.63

$$\frac{\sqrt{5-2x}\left(-110E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+41F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+69\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{25\sqrt{-55+22x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (Sqrt[5 - 2*x]*(-110*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
41*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 69*EllipticPi[55/1
24, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(25*Sqrt[-55 + 22*x])
```

Maple [A]
time = 0.13, size = 67, normalized size = 0.44

method	result
default	$\frac{\left(69\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+55\,\mathrm{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-124\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\right)\sqrt{}}{275\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(\frac{109\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.}{3025\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/275*(69*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+55*EllipticE(1/11*(11+44*
x)^(1/2),3^(1/2))-124*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))*(5-2
*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 - 11*x - 35), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\cdot(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)), x)
```

3.49 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)}-\frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}}-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\sin\right.}{25\sqrt{-\right.}$$

```
[Out] -2/275*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^
(1/2)/(-5+2*x)^(1/2)-6101/221650*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/
124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-2/195*EllipticE(2/
11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1
/2)+1/39*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {170, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{25\sqrt{2x-5}}-\frac{2\sqrt{11}\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}}-\frac{6101\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{20150\sqrt{11}\sqrt{2x-5}}+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*(7 + 5*x)) - (2*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(195*Sqr
t[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticPi[55/12
4, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(20150*Sqrt[11]*Sqrt[-5 + 2*x
])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^2}\,dx = \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{-29+120x-24x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{\frac{768}{25}-\frac{24x}{5}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx + -$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39(7+5x)} + \frac{2}{65}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx - \frac{6}{25}\int \frac{}{\sqrt{2-}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}}}{25\sqrt{-5+2x}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39(7+5x)} - \frac{2\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\right.}{195\sqrt{5-2x}}$$

Mathematica [A]
time = 5.97, size = 130, normalized size = 0.69

$$\frac{\frac{51150\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x}}{7+5x} + 3\sqrt{55-22x}\left(6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right) + 14508F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right) - 18303\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)\right)}{1994850\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] ((51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22*
x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 18303*EllipticPi[55/124, Ar
cSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1994850*Sqrt[-5 + 2*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(145) = 290.
time = 0.14, size = 302, normalized size = 1.60

method	result

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{128\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\text{EllipticF}\left(\frac{\sqrt{11+44}}{11}\right.}{39325\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(39560\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{39(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{4\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{3}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/246675*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(39560*(1+4*x)^(1/2)*(2
-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))*
x+6325*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1
1+44*x)^(1/2),3^(1/2))*x-61010*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)
^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*x+55384*(1+4*x)^(1/2
)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/
2))+8855*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*
(11+44*x)^(1/2),3^(1/2))-85414*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)
^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))+151800*x^3-442750*x^
2+132825*x+63250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 + 15*x^2 - 252*
x - 245), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

3.50 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2}-\frac{361\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{481988(7+5x)}+\frac{361\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.}{1204970\sqrt{5-2x}}$$

```
[Out] -6655867/8217895400*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2^(
1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-6101/15293850*EllipticF(1/11*33
^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+361
/1204970*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+
2*x)^(1/2)/(5-2*x)^(1/2)+1/78*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7
+5*x)^2-361/481988*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {170, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{6101\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{231725\sqrt{66}\,\sqrt{2x-5}}+\frac{361\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1204970\sqrt{5-2x}}-\frac{6655867\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{747081400\sqrt{11}\,\sqrt{2x-5}}-\frac{361\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{481988(5x+7)}+\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{78(5x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(78*(7 + 5*x)^2) - (361*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(481988*(7 + 5*x)) + (361*Sqrt[11]*Sq
rt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1204970*
Sqrt[5 - 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(231725*Sqrt[66]*Sqrt[-5 + 2*x]) - (6655867*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(747081400*Sqrt[11
]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^3}\,dx = \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2} - \frac{1}{156}\int \frac{-37+100x+24x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{481988(7+5x)} - \frac{\int}{\sqrt{2}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{481988(7+5x)} - \frac{\int}{\sqrt{2}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{481988(7+5x)} - \frac{1083\int}{}$$

$$(6101$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{481988(7+5x)} -$$

$$= \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{481988(7+5x)} + \frac{361\sqrt{1}}{}$$

Mathematica [A]
time = 5.83, size = 135, normalized size = 0.60

$$\frac{-\frac{17050\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x}\,(-10957+5415x)}{(7+5x)^2} - 3\sqrt{55-22x}\left(2462020E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 9834812F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) + 6655867\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{24653686200\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] ((-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(-10957 + 5415*x))/(7 + 5*x
)^2 - 3*Sqrt[55 - 22*x]*(2462020*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11
]], -1/2] - 9834812*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 6
655867*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(2465
3686200*Sqrt[-5 + 2*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(175) = 350.
time = 0.14, size = 434, normalized size = 1.93

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-24x^3+70x^2-21x-10}}{78(7+5x)^2}-\frac{361\sqrt{-24x^3+70x^2-21x-}}{481988(7+5x)}\right.}{}$
risch	$\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(-10957+5415x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{1445964(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{361\sqrt{22-33x}\,\sqrt{1}}{}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(205130100\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/9145722300*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(205130100*(1+4*x)^
(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3
^(1/2))*x^2-34249875*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ell
ipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x^2-332793350*(1+4*x)^(1/2)*(2-3*x)^(1
/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*
x^2+574364280*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(
1/11*(11+44*x)^(1/2),3^(1/2))*x-95899650*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/
2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x-931821380*(1+4*x
)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2
),-55/23,3^(1/2))*x+402054996*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^
(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-67129755*(1+4*x)^(1/2)*(2-3*x
)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))-6522
74966*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1
1+44*x)^(1/2),-55/23,3^(1/2))-821997000*x^4+4060763850*x^3-5570459125*x^2+1
112864775*x+693030250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4 + 425*x^3 - 11
55*x^2 - 2989*x - 1715), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2 - 3x}\,\sqrt{4x + 1}}{\sqrt{2x - 5}\,(5x + 7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2 - 3\,x}\,\sqrt{4\,x + 1}}{\sqrt{2\,x - 5}\,{(5\,x + 7)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

3.51 $\int \frac{\sqrt{2-3x}\,(7+5x)^3}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=205

$$\frac{110743}{864}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{121}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)+\frac{5}{28}\sqrt{2-3x}\,\sqrt{-5+2x}$$

```
[Out] -25260049/36288*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)
*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+15629623/9072*EllipticE(2/11*(2-3*x)^(1/2)*11
^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+110743/864*(2-3
*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+121/24*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x
)^(1/2)*(1+4*x)^(1/2)+5/28*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {180, 1614, 1629, 164, 115, 114, 122, 120}

$$-\frac{25260049\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{6048\sqrt{2x-5}}+\frac{15629623\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{9072\sqrt{5-2x}}+\frac{5}{28}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2+\frac{121}{24}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{110743}{864}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (110743*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/864 + (121*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 + (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2
*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/28 + (15629623*Sqrt[11]*Sqrt[-5 + 2*x]*Ellip
ticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (252
60049*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/(6048*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^3}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{28}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2 - \frac{1}{56}\int \frac{(7+5x)\,(-7223+2667x}{\sqrt{2-3x}\,\sqrt{-5+2x}}$$

$$= \frac{121}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) + \frac{5}{28}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4}$$

$$= \frac{110743}{864}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,($$

$$= \frac{110743}{864}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,($$

$$= \frac{110743}{864}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,($$

$$= \frac{110743}{864}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,($$

Mathematica [A]
time = 18.71, size = 125, normalized size = 0.61

$$\frac{30\sqrt{2-3x}\,\sqrt{1+4x}\,(-1041565+188566x+64224x^2+10800x^3)+31259246\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)-25260049\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{36288\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-1041565 + 188566*x + 64224*x^2 + 10800*x^
3) + 31259246*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3] - 25260049*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sq
rt[1 + 4*x]], 1/3])/(36288*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.13, size = 144, normalized size = 0.70

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(13261655\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\right.\right.}{}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{125x^2\sqrt{-24x^3+70x^2-21x-10}}{28}+\frac{905x\sqrt{-24x^3+70x^2-21x}}{24}\right.}{}$
risch	$-\frac{5(5400x^2+45612x+208313)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{6048\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{1653901\sqrt{22}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/36288*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(13261655*(1+4*x)^(1/2)*
(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)
)-31259246*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/1
1*(11+44*x)^(1/2),3^(1/2))+3888000*x^5+21500640*x^4+57602160*x^3-407101740*
x^2+144920790*x+62493900)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [A]
time = 0.25, size = 33, normalized size = 0.16

$$\frac{5}{6048}\left(5400\,x^2+45612\,x+208313\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 5/6048*(5400*x^2 + 45612*x + 208313)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x
+ 2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)^3}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**3/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,{\left(5\,x+7\right)}^3}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.52 $\int \frac{\sqrt{2-3x}\,(7+5x)^2}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=167

$$\frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)+\frac{44569\sqrt{11}\,\sqrt{-5+2x}\,E\Big(\sin^{-1}$$
$$\overline{432\sqrt{5-2x}}$$

```
[Out] -17533/432*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2
*x)^(1/2)/(-5+2*x)^(1/2)+44569/432*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/
2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+68/9*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)+1/4*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {180, 1629, 164, 115, 114, 122, 120}

$$-\frac{17533\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{72\sqrt{2x-5}}+\frac{44569\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{432\sqrt{5-2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{68}{9}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (68*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (Sqrt[2 - 3*x]*Sqrt[-5
 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/4 + (44569*Sqrt[11]*Sqrt[-5 + 2*x]*Elliptic
E[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(432*Sqrt[5 - 2*x]) - (17533*S
qrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
72*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^2}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)-\frac{1}{40}\int \frac{-5155+3605x+10880x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4}}$$

$$=\frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$=\frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$=\frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$=\frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

Mathematica [A]
time = 16.19, size = 120, normalized size = 0.72

$$\frac{120\sqrt{2-3x}\,\sqrt{1+4x}\,(-335+89x+18x^2)+44569\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)-35066\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{864\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-335 + 89*x + 18*x^2) + 44569*Sqrt[66]*Sq
rt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 35066*Sqrt[6
```

```
6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(864*Sqr
t[-5 + 2*x])
```

Maple [A]
time = 0.13, size = 139, normalized size = 0.83

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(16060\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\ldots\right.\right.}{20736x^3\ldots}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{5x\sqrt{-24x^3+70x^2-21x-10}}{4}+\frac{335\sqrt{-24x^3+70x^2-21x\ldots}}{36}\ldots\right.}{\ldots}$
risch	$-\frac{5(67+9x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{4997\sqrt{22-33x}\,\sqrt{165-\ldots}}{8712\ldots}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/864*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(16060*(1+4*x)^(1/2)*(2-3*
x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-445
69*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2))+25920*x^4+117360*x^3-540120*x^2+179640*x+80400)/(24*x^3-
70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [A]
time = 0.30, size = 28, normalized size = 0.17

$$\frac{5}{36}\,(9\,x+67)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] 5/36*(9*x + 67)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2-3x}\,(5x+7)^2}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**2/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{2-3\,x}\,{\left(5\,x+7\right)}^2}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.53 $\int \frac{\sqrt{2-3x}\,(7+5x)}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=131

$$\frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{241\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}}-\frac{179\sqrt{\frac{11}{6}}\,\sqrt{5-2x}}{}$$

```
[Out] -179/72*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+241/36*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^
(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+5/12*(2-3*x)^(1/2)*(-5+2*x)^(1
/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {159, 164, 115, 114, 122, 120}

$$-\frac{179\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{12\sqrt{2x-5}}+\frac{241\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{36\sqrt{5-2x}}+\frac{5}{12}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/12 + (241*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(36*Sqrt[5 - 2*x
]) - (179*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(12*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
```

```
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{12}\int \frac{\frac{441}{2}-482x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$$

$$= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{241}{12}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx - \frac{1969}{24}\int$$

$$= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{\left(179\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\right)\int \frac{}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}}}}{12\sqrt{-5+2x}}$$

$$= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{241\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3}}{\sqrt{11}}\right.\right.}{36\sqrt{5-2x}}$$

Mathematica [A]
time = 2.16, size = 115, normalized size = 0.88

$$\frac{30\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x} + 241\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right) - 179\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{72\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 241*Sqrt[66]*Sqrt[5 - 2*x]*Ell
ipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 179*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(72*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.13, size = 134, normalized size = 1.02

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(55\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right.}{1728x^3-5040x^2+15}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{5\sqrt{-24x^3+70x^2-21x-10}}{12}+\frac{147\sqrt{11+44x}\sqrt{22-33x}\sqrt{}}{968\sqrt{-24x^3}}\right.}{}$
risch	$-\frac{5(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{12\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}-\frac{\left(\frac{49\sqrt{22-33x}\sqrt{165-66x}\sqrt{3}}{968\sqrt{-24x^3}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/72*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(55*(1+4*x)^(1/2)*(2-3*x)^(
1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-241*(1+
4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1
/2),3^(1/2))+720*x^3-2100*x^2+630*x+300)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [A]
time = 0.22, size = 23, normalized size = 0.18

$$\frac{5}{12}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] 5/12*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\left(5\,x+7\right)}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.54 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}$$

```
[Out] 1/4*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))*22^(1/2)*(5-2*x)^(1/2)/(
-5+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {115, 114}

$$\frac{\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\,E\left(\text{ArcSin}\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[11/2]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[1 + 4*x]/Sqrt[11]], 3])/(2*
Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{\sqrt{5-2x}\int \frac{\sqrt{\frac{8}{11}-\frac{12x}{11}}}{\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x}}\,dx}{\sqrt{2}\,\sqrt{-5+2x}}$$
$$= \frac{\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 111 vs. 2(47) = 94.
time = 1.52, size = 111, normalized size = 2.36

$$-\frac{\frac{2(-5+2x)(-2+3x)}{\sqrt{\frac{1}{2}+2x}}+\sqrt{11}\sqrt{\frac{-5+2x}{1+4x}}\sqrt{\frac{-2+3x}{1+4x}}\,(1+4x)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right)\middle|3\right)}{2\sqrt{2-3x}\,\sqrt{-10+4x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/2*((2*(-5 + 2*x)*(-2 + 3*x))/Sqrt[1/2 + 2*x] + Sqrt[11]*Sqrt[(-5 + 2*x)/
(1 + 4*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*EllipticE[ArcSin[Sqrt[11/3]
/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x])
```

Maple [**A**]
time = 0.13, size = 33, normalized size = 0.70

method	result
default	$\frac{\mathrm{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{4\sqrt{-5+2x}}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{\cdots}\right.}{121\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.55 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}}-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{-5+2x}}$$

```
[Out] -1/55*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)-3/55*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*
I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {181, 122, 120, 174, 552, 551}

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] -1/5*(Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/Sqrt[-5 + 2*x] - (3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2
 - 3*x])/Sqrt[11]], -1/2])/(5*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
```

```
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 181

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}\,dx = -\left(\frac{3}{5}\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx\right) + \frac{31}{5}\int \frac{}{\sqrt{2-3x}\,\sqrt{-5}}$$

$$= -\left(\frac{62}{5}\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-}\right.\right.$$

$$= -\frac{\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}} - \frac{\left(62\sqrt{\frac{3}{11}}\,\sqrt{5-\right.}{}$$

$$= -\frac{\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}} - \frac{3\sqrt{5-2x}\,\Pi\left(\right.}{5}$$

Mathematica [A]
time = 2.50, size = 70, normalized size = 0.68

$$\frac{3\sqrt{5-2x}\left(F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{5\sqrt{-55+22x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
[Out] (3*Sqrt[5 - 2*x]*(EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - Ell
ipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(5*Sqrt[-55 + 2
2*x])
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.50

method	result
default	$-\frac{\left(69\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-124\,\text{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{1265\sqrt{-5+2x}}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{3\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.}{605\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/1265*(69*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-124*EllipticPi(1/11*(11
+44*x)^(1/2),-55/23,3^(1/2)))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 - 34*x^2 - 151*
x - 35), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\cdot(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

3.56 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)}+\frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}}-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\Big(}{115\ldots}$$

```
[Out] -2/1265*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)-3571/1019590*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+2/897*EllipticE(
2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^
(1/2)-5/897*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {183, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{115\sqrt{2x-5}}+\frac{2\sqrt{11}\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{897\sqrt{5-2x}}-\frac{3571\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{92690\sqrt{11}\sqrt{2x-5}}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*(7 + 5*x)) + (2*Sqrt[1
1]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(897
*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*S
qrt[1 + 4*x]], 1/3])/(115*Sqrt[-5 + 2*x]) - (3571*Sqrt[5 - 2*x]*EllipticPi[
55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(92690*Sqrt[11]*Sqrt[-5
+ 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 183

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h)))
```

```
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}\,dx = -\frac{5\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{-479+336x+120x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}}{1794}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{\frac{168}{5}+24x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}}{1794}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{897(7+5x)} - \frac{2}{299}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx -$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{897(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right)\int \frac{}{\sqrt{2-3}}}{115\sqrt{-5-}}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{897(7+5x)} + \frac{2\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{}}{}\right.\right.}{897\sqrt{5-2x}}$$

Mathematica [A]
time = 6.17, size = 130, normalized size = 0.69

$$\frac{-\frac{51150\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x}}{7+5x} - 3\sqrt{55-22x}\left(6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right) - 14508F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right) + 10713\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)\right)}{9176310\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ellip
ticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 10713*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(9176310*Sqrt[-5 + 2*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(145) = 290$.
time = 0.16, size = 302, normalized size = 1.60

method	result

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{28\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.}{180895\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(35710\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},\right.\right.}{}$
risch	$\frac{5(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{897(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{4\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{3}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/1134705*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(35710*(1+4*x)^(1/2)*
(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23
,3^(1/2))*x-14260*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ellipt
icF(1/11*(11+44*x)^(1/2),3^(1/2))*x+6325*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/
2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x+49994*(1+4*x)^(1
/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-5
5/23,3^(1/2))-19964*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Elli
pticF(1/11*(11+44*x)^(1/2),3^(1/2))+8855*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/
2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+151800*x^3-442750*
x^2+132825*x+63250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4 + 110*x^3 - 99
3*x^2 - 1232*x - 245), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

3.57 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{5\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1794(7+5x)^2}-\frac{26825\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{33257172(7+5x)}+\frac{5365\sqrt{11}\,\sqrt{-5+2x}\,E\Big(\sin^{-1}\Big(}{16628586\sqrt{5-}}$$

```
[Out] -16369941/37802318840*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2
^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-13243/70351710*EllipticF(1/11
*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+
5365/16628586*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)
*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-5/1794*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)/(7+5*x)^2-26825/33257172*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7
+5*x)
```

Rubi [A]

time = 0.20, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {183, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{13243\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{1065935\sqrt{66}\,\sqrt{2x-5}}+\frac{5365\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{16628586\sqrt{5-2x}}-\frac{16369941\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{3436574440\sqrt{11}\,\sqrt{2x-5}}-\frac{26825\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{33257172(5x+7)}-\frac{5\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1794(5x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1794*(7 + 5*x)^2) - (26825
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(33257172*(7 + 5*x)) + (5365*S
qrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])
/(16628586*Sqrt[5 - 2*x]) - (13243*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(1065935*Sqrt[66]*Sqrt[-5 + 2*x]) - (16369941*Sqrt[
5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(343
6574440*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 183

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*Sq
```

```
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h)))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)^3}\,dx = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{\int \frac{-1063+1372x-120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}}{3588}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{33257172(7+5x)}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{33257172(7+5x)}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{33257172(7+5x)}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{33257172(7+5x)}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{33257172(7+5x)}$$

Mathematica [**A**]
time = 5.94, size = 142, normalized size = 0.63

$$\frac{-17050\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x}\,(56093+26825x)-\sqrt{55-22x}\,(7+5x)^2\left(36589300E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-64043148F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+49109823\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{113406956520\sqrt{-5+2x}\,(7+5x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(56093 + 26825*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(36589300*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 64043148*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 491
09823*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(11340
6956520*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(175) = 350$.
time = 0.18, size = 434, normalized size = 1.93

method	result
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\left(-\frac{5\sqrt{-24x^{3}+70x^{2}-21x-10}}{1794\left(7+5x\right)^{2}}-\frac{26825\sqrt{-24x^{3}+70x^{2}-21}}{33257172\left(7+5x\right)}\right.}{}$
risch	$\frac{5\left(-2+3x\right)\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(56093+26825x\right)\sqrt{\left(2-3x\right)\left(-5+2x\right)\left(1+4x\right)}}{33257172\left(7+5x\right)^{2}\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{2-3x}}-\frac{\left(-\frac{5365\sqrt{22-33x}\,\sqrt{}}{}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(254612300\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/42070322580*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(254612300*(1+4*x)
^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),
3^(1/2))*x^2-169668125*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*E
llipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x^2-818497050*(1+4*x)^(1/2)*(2-3*x)^
(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)
)*x^2+712914440*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Elliptic
F(1/11*(11+44*x)^(1/2),3^(1/2))*x-475070750*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^
(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x-2291791740*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^
(1/2),-55/23,3^(1/2))*x+499040108*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2
*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-332549525*(1+4*x)^(1/2)*(
2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))
-1604254218*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1
/11*(11+44*x)^(1/2),-55/23,3^(1/2))-4072035000*x^4+3361851350*x^3+212721451
25*x^2-9147233975*x-3547882250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5 + 1950*x^4 -
4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

3.58 $\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}-\frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}}{}$$

```
[Out] 2*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g
))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g
))^(1/2)/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*EllipticPi(f^(1/2)*(d*x+c)
^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g
))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g
))^(1/2)/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {181, 122, 121, 175, 552, 551}

$$\frac{2\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}-\frac{2\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d
*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((
d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) - (
2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*
g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt
[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 181

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \frac{d\int \frac{1}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e}}}{b}$$

$$= -\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\,\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,d\right.}{b}$$

$$= -\frac{\left(2(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\,\sqrt{}}\right.}{b\sqrt{e+fx}}$$

$$= \frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\right.}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

$$= \frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\right.}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.18, size = 202, normalized size = 0.69

$$-\frac{2i\sqrt{c+dx}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\left(F\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)-\Pi\left(\frac{b(-de+cf)}{(bc-ad)f};i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)\right)}{b\sqrt{\frac{f(c+dx)}{d(e+fx)}}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*Sqrt[c + d*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*(EllipticF[I*ArcSinh[
Sqrt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - Ellipt
icPi[(b*(-(d*e) + c*f))/((b*c - a*d)*f), I*ArcSinh[Sqrt[(f*(c + d*x))/(d*e
- c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(b*Sqrt[(f*(c + d*x))/(d*(e +
f*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(259) = 518$.

time = 0.12, size = 668, normalized size = 2.28

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2d\left(-\frac{e}{f}+\frac{g}{h}\right)\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\ldots\right)}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+c\ldots}}\right)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{\ldots}}$
default	$\frac{2\left(\mathrm{EllipticPi}\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-bg)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ade\,h^2-\mathrm{EllipticPi}\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-bg)},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)adfgh-\ldots\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2*(EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*
g)*d/f/(c*h-d*g))^(1/2))*a*d*e*h^2-EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),
(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*d*f*g*h-Elliptic
Pi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h
-d*g))^(1/2))*b*c*e*h^2+EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b
/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*c*f*g*h-EllipticF((-(h*x+g)
*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*d*e*h^2+EllipticF((-
(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*d*f*g*h+Ellip
ticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d*e*g*
h-EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b
*d*f*g^2)*((f*x+e)*h/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*(-(h*x+g)
*f/(e*h-f*g))^(1/2)/h/f/b*(h*x+g)^(1/2)*(f*x+e)^(1/2)*(d*x+c)^(1/2)/(a*h-b*
g)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
```

```
[Out] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

3.59 $\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=449

$$\frac{2d\sqrt{-fg+eh}\,\sqrt{c+dx}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\,\sqrt{-\frac{f(c+dx)}{de-cf}}\,\sqrt{g+hx}}+\frac{2(bc-ad)\sqrt{-de+cf}\,\sqrt{\;}}{}$$

```
[Out] 2*(-a*d+b*c)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/
f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g
)/(-c*h+d*g))^(1/2)/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*d+b*c)*El
lipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,(
(-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1
/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)+2*
d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)
/h)^(1/2))*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/b/f/h
^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {185, 122, 121, 175, 552, 551, 115, 114}

$$\frac{2(bc-ad)\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}+\frac{2d\sqrt{c+dx}\,\sqrt{eh-fg}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\,\sqrt{g+hx}\,\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(b*f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g +
 h*x]) + (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*
Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e +
f*x]*Sqrt[g + h*x]) - (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/(
(b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
```

```
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 185

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
```

```
1/2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \left(\frac{d(bc-ad)}{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} + \frac{(bc-ad)^2}{b^2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\right.$$

$$= \frac{d\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(d(bc-ad))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}}{b^2}$$

$$= -\frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\right.}{b^2}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\right.}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\right.}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\right.}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.17, size = 1198, normalized size = 2.67

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b^2*d^2*e^2*f*g*Sqrt[-e + (f*g)/h]) + b^2*c*d*e*f^2*g*Sqrt[-e + (f*g
)/h] + a*b*d^2*e*f^2*g*Sqrt[-e + (f*g)/h] - a*b*c*d*f^3*g*Sqrt[-e + (f*g)/h
] + b^2*d^2*e^3*Sqrt[-e + (f*g)/h]*h - b^2*c*d*e^2*f*Sqrt[-e + (f*g)/h]*h -
 a*b*d^2*e^2*f*Sqrt[-e + (f*g)/h]*h + a*b*c*d*e*f^2*Sqrt[-e + (f*g)/h]*h +
b^2*d^2*e*f*g*Sqrt[-e + (f*g)/h]*(e + f*x) - a*b*d^2*f^2*g*Sqrt[-e + (f*g)/
h]*(e + f*x) - 2*b^2*d^2*e^2*Sqrt[-e + (f*g)/h]*h*(e + f*x) + b^2*c*d*e*f*S
qrt[-e + (f*g)/h]*h*(e + f*x) + 2*a*b*d^2*e*f*Sqrt[-e + (f*g)/h]*h*(e + f*x
) - a*b*c*d*f^2*Sqrt[-e + (f*g)/h]*h*(e + f*x) + b^2*d^2*e*Sqrt[-e + (f*g)/
h]*h*(e + f*x)^2 - a*b*d^2*f*Sqrt[-e + (f*g)/h]*h*(e + f*x)^2 + I*b*d^2*(b*
e - a*f)*(f*g - e*h)*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt
[(f*(g + h*x))/(h*(e + f*x))]*EllipticE[I*ArcSinh[Sqrt[-e + (f*g)/h]/Sqrt[e
 + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] - I*b*f*(a*d^2*(-(f*g) + e*h)
 + b*(d^2*e*g - 2*c*d*e*h + c^2*f*h))*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e
+ f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticF[I*ArcSinh[Sqrt[-e
+ (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] + I*b^2*c^2*
f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/
(h*(e + f*x))]*EllipticPi[-((b*e*h - a*f*h)/(b*f*g - b*e*h)), I*ArcSinh[Sqr
t[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] - (2*I)
*a*b*c*d*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g
 + h*x))/(h*(e + f*x))]*EllipticPi[-((b*e*h - a*f*h)/(b*f*g - b*e*h)), I*Ar
cSinh[Sqrt[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))
] + I*a^2*d^2*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[
(f*(g + h*x))/(h*(e + f*x))]*EllipticPi[-((b*e*h - a*f*h)/(b*f*g - b*e*h)),
 I*ArcSinh[Sqrt[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) +
e*h))]))/(b^2*f^2*(-(b*e) + a*f)*Sqrt[-e + (f*g)/h]*h*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1550 vs. $2(398) = 796$.
time = 0.12, size = 1551, normalized size = 3.45

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2d(ad-2bc)\left(-\frac{e}{f}+\frac{g}{h}\right)\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}},\sqrt{\cdots}\right.}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+\cdots}}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2*(EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*
a^2*d^2*e*h^3-EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*
g))^(1/2))*a^2*d^2*f*g*h^2-EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g
)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*e*h^3+EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2
),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*f*g*h^2-EllipticF((-(h*x+g)*f/(e
*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*d^2*e*g*h^2+EllipticF((
-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*d^2*f*g^2*
h+EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b
^2*c*d*e*g*h^2-EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d
*g))^(1/2))*b^2*c*d*f*g^2*h-EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*
g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*e*h^3+EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/
2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*f*g*h^2+EllipticE((-(h*x+g)*f/(
e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*d^2*e*g*h^2-EllipticE(
(-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*d^2*f*g^2
*h+EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*
b^2*c*d*e*g*h^2-EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-
d*g))^(1/2))*b^2*c*d*f*g^2*h-EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f
*g)*d/f/(c*h-d*g))^(1/2))*b^2*d^2*e*g^2*h+EllipticE((-(h*x+g)*f/(e*h-f*g))^
(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*d^2*f*g^3-EllipticPi((-(h*x+g)*f
/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*
a^2*d^2*e*h^3+EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*
g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a^2*d^2*f*g*h^2+2*EllipticPi((-(h*x+g)*
f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))
*a*b*c*d*e*h^3-2*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h
-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*b*c*d*f*g*h^2-EllipticPi((-(h*x+g)
*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2)
)*b^2*c^2*e*h^3+EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-
b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b^2*c^2*f*g*h^2)*((f*x+e)*h/(e*h-f*g)
)^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*(-(h*x+g)*f/(e*h-f*g))^(1/2)/h^2/f/b^2*
(h*x+g)^(1/2)*(f*x+e)^(1/2)*(d*x+c)^(1/2)/(a*h-b*g)/(d*f*h*x^3+c*f*h*x^2+d*
e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
```

```
[Out] int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

3.60 $\int \frac{(7+5x)^4}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=203

$$-\frac{120355}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{305}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)-\frac{25}{84}\sqrt{2-3x}\,\sqrt{-5+2x}$$

```
[Out] 392989907/133056*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2
)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)-5109835/756*EllipticE(2/11*(2-3*x)^(1/2)*11^
(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-120355/288*(2-3*
x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-305/24*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)
^(1/2)*(1+4*x)^(1/2)-25/84*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {173, 1614, 1629, 164, 115, 114, 122, 120}

$$\frac{392989907\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2016\sqrt{66}\,\sqrt{2x-5}}-\frac{5109835\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{756\sqrt{5-2x}}-\frac{25}{84}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2-\frac{305}{24}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)-\frac{120355}{288}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-120355*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/288 - (305*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 - (25*Sqrt[2 - 3*x]*Sqrt[-5 +
 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/84 - (5109835*Sqrt[11]*Sqrt[-5 + 2*x]*Elli
pticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(756*Sqrt[5 - 2*x]) + (392
989907*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(201
6*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = -\frac{25}{84}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2 + \frac{1}{168}\int \frac{(7+5x)\,(48}{\sqrt{2-3}}$$

$$= -\frac{305}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) - \frac{25}{84}\sqrt{2-3x}\,\sqrt{-5+}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= -\frac{120355}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\,\sqrt{-5+2x}$$

Mathematica [A]
time = 31.37, size = 125, normalized size = 0.62

$$\frac{-1650\sqrt{2-3x}\,\sqrt{1+4x}\,(-210245+50078x+10608x^2+1200x^3) - 449665480\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right) + 392989907\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{133056\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-1650*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-210245 + 50078*x + 10608*x^2 + 1200*x^
3) - 449665480*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 +
4*x]], 1/3] + 392989907*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*
Sqrt[1 + 4*x]], 1/3])/(133056*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.15, size = 144, normalized size = 0.71

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(279638761\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44}}{11}\right.\right.}{}$

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{625x^2\sqrt{-24x^3+70x^2-21x-10}}{84}-\frac{675x\sqrt{-24x^3+70x^2-2}}{8}\right.}{}$
risch	$\frac{25(600x^2+6804x+42049)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{2016\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{752233\sqrt{22-33}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/133056*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(279638761*(1+4*x)^(1/
2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1
/2))-449665480*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE
(1/11*(11+44*x)^(1/2),3^(1/2))+23760000*x^5+200138400*x^4+900068400*x^3-461
1000900*x^2+1569263850*x+693808500)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [A]
time = 0.25, size = 33, normalized size = 0.16

$$-\frac{25}{2016}\left(600\,x^2+6804\,x+42049\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] -25/2016*(600*x^2 + 6804*x + 42049)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x +
 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^4}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**4/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**4/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5\,x+7)^4}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^4/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^4/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.61 $\int \frac{(7+5x)^3}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=165

$$-\frac{2135}{108}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)-\frac{487585\sqrt{11}\,\sqrt{-5+2x}\,E}{1296\sqrt{}}$$

```
[Out] 2474201/14256*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)-487585/1296*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/
2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-2135/108*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-5/12*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {173, 1629, 164, 115, 114, 122, 120}

$$\frac{2474201\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{216\sqrt{66}\,\sqrt{2x-5}}-\frac{487585\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{1296\sqrt{5-2x}}-\frac{5}{12}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)-\frac{2135}{108}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/108 - (5*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/12 - (487585*Sqrt[11]*Sqrt[-5 + 2*x]
*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1296*Sqrt[5 - 2*x])
+ (2474201*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/
(216*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = -\frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)+\frac{1}{120}\int \frac{34985+1048}{\sqrt{2-3x}\,\sqrt{-5}}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

Mathematica [A]
time = 28.48, size = 120, normalized size = 0.73

$$\frac{-6600\sqrt{2-3x}\,\sqrt{1+4x}\,(-490+151x+18x^2)-5363435\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)+4948402\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{28512\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-6600*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-490 + 151*x + 18*x^2) - 5363435*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 4948402
```

```
*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
28512*Sqrt[-5 + 2*x])
```

Maple [A]
time = 0.14, size = 139, normalized size = 0.84

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(4118336\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.\right.}{}$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\left(-\frac{25x\sqrt{-24x^{3}+70x^{2}-21x-10}}{12}-\frac{1225\sqrt{-24x^{3}+70x^{2}-2}}{54}\right.}{}$
risch	$\frac{25(98+9x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{\left(2-3x\right)\left(-5+2x\right)\left(1+4x\right)}}{108\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{2-3x}}+\frac{\left(-\frac{3023\sqrt{22-33x}\,\sqrt{165-}}{13068}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/28512*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(4118336*(1+4*x)^(1/2)*
(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)
)-5363435*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11
*(11+44*x)^(1/2),3^(1/2))+1425600*x^4+11365200*x^3-44028600*x^2+14176800*x+
6468000)/(24*x^3-70*x^2+21*x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [A]
time = 0.24, size = 28, normalized size = 0.17

$$-\frac{25}{108}\,(9\,x+98)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] -25/108*(9*x + 98)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^3}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**3/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x+7)^3}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.62 $\int \frac{(7+5x)^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=129

$$-\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{2135\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{108\sqrt{5-2x}}+\frac{24353\sqrt{5-2x}\,F\ldots}{36\ldots}$$

```
[Out] 24353/2376*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2
*x)^(1/2)/(-5+2*x)^(1/2)-2135/108*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2
*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-25/36*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {173, 24, 164, 115, 114, 122, 120}

$$\frac{24353\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{36\sqrt{66}\,\sqrt{2x-5}}-\frac{2135\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{108\sqrt{5-2x}}-\frac{25}{36}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 - (2135*Sqrt[11]*Sqrt[-
5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(108*Sqrt[5 -
 2*x]) + (24353*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(36*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
```

```
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rubi steps

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{72}\int \frac{21021+74795x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{}}$$

$$= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{\int \frac{75075+213500x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}}{1800}$$

$$= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{2135}{36}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}$$

$$= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{\left(24353\sqrt{5-2x}\right)\int \frac{}{\sqrt{2-3x}}}{36\sqrt{22}\,\sqrt{-}}$$

$$= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} - \frac{2135\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin\right.}{108\sqrt{5-}}$$

Mathematica [A]
time = 26.24, size = 115, normalized size = 0.89

$$\frac{1650\sqrt{2-3x}\,(5-2x)\sqrt{1+4x} - 23485\sqrt{66}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right) + 24353\sqrt{66}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{2376\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (1650*Sqrt[2 - 3*x]*(5 - 2*x)*Sqrt[1 + 4*x] - 23485*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 24353*Sqrt[66]*Sqrt[5 -
2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2376*Sqrt[-5 + 2*x]
)
```

Maple [A]
time = 0.14, size = 134, normalized size = 1.04

method	result

default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(26089\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.\right.}{2376(24x^{3}-}$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\left(-\frac{25\sqrt{-24x^{3}+70x^{2}-21x-10}}{36}+\frac{91\sqrt{11+44x}\,\sqrt{22-33x}}{264\sqrt{-24x}}\right.}{}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{91\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{3}}{792\sqrt{-24x^{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2376*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(26089*(1+4*x)^(1/2)*(2-
3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-2
3485*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+
44*x)^(1/2),3^(1/2))+39600*x^3-115500*x^2+34650*x+16500)/(24*x^3-70*x^2+21*
x+10)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [A]
time = 0.21, size = 23, normalized size = 0.18

$$-\frac{25}{36}\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] -25/36*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^2}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**2/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x+7)^2}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.63 $\int \frac{7+5x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=98

$$-\frac{5\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{6\sqrt{5-2x}}+\frac{13\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)\middle|\frac{1}{3}\right)}{\sqrt{-5+2x}}$$

```
[Out] 13/22*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)-5/6*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2)
)*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {164, 115, 114, 122, 120}

$$\frac{13\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)\middle|\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{6\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(6*Sqrt[5 - 2*x]) + (13*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqr
t[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{7+5x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{2}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx + \frac{39}{2}\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$$

$$= \frac{\left(39\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x}}\,dx}{\sqrt{22}\,\sqrt{-5+2x}} + \frac{\left(5\sqrt{-5+2x}\right.}{}$$

$$= -\frac{5\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{6\sqrt{5-2x}} + \frac{13\sqrt{\frac{3}{22}}\,\sqrt{5-}}{}$$

Mathematica [A]

time = 8.26, size = 187, normalized size = 1.91

$$\frac{220\sqrt{1+4x}\,(10-19x+6x^2)+55\sqrt{22}\,\sqrt{\frac{-5+2x}{1+4x}}\,\sqrt{\frac{-2+3x}{1+4x}}\,(1+4x)^2E\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right)\middle|3\right)-124\sqrt{22}\,\sqrt{\frac{-5+2x}{1+4x}}\,\sqrt{\frac{-2+3x}{1+4x}}\,(1+4x)^2F\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right)\middle|3\right)}{132\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (220*Sqrt[1 + 4*x]*(10 - 19*x + 6*x^2) + 55*Sqrt[22]*Sqrt[(-5 + 2*x)/(1 + 4
*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*EllipticE[ArcSin[Sqrt[11/3]/Sqr
t[1 + 4*x]], 3] - 124*Sqrt[22]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*Sqrt[(-2 + 3*x)/(
1 + 4*x)]*(1 + 4*x)^2*EllipticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(132*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x))
```

Maple [A]
time = 0.16, size = 51, normalized size = 0.52

method	result
default	$\frac{\left(124\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-55\,\mathrm{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{132\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(\frac{7\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{\right.}}{121\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/132*(124*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2)))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x+7}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x+7}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.64 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{\sqrt{-5+2x}}$$

```
[Out] 1/33*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1
/2)/(-5+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {122, 120}

$$\frac{\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/Sqrt[-5 + 2*x]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
```

```
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{\left(\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x}}\,dx}{\sqrt{-5+2x}}$$
$$= \frac{\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{\sqrt{-5+2x}}$$

Mathematica [A]
time = 1.16, size = 79, normalized size = 1.65

$$-\frac{\sqrt{\frac{-2+3x}{1+4x}}\,(1+4x)\sqrt{\frac{-10+4x}{11+44x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right)\middle|3\right)}{\sqrt{2-3x}\,\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
[Out] -((Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*Sqrt[(-10 + 4*x)/(11 + 44*x)]*Ellip
ticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]))
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.69

method	result
default	$\frac{\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{11\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\right.}{121\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{-24x^3+70x^2-21x-10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.65 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}\,dx$

Optimal. Leaf size=51

$$-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\,\sqrt{-5+2x}}$$

```
[Out] -3/341*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)
^(1/2)*11^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {174, 552, 551}

$$-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\,\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(31*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\cdots}}\right.}{\sqrt{-5+2x}}$$

$$= -\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A]
time = 2.48, size = 99, normalized size = 1.94

$$-\frac{3(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\left(F\left(\sin^{-1}\left(\frac{\sqrt{11}}{2\sqrt{2-3x}}\right)\middle|-2\right)-\Pi\left(\frac{124}{55};\sin^{-1}\left(\frac{\sqrt{11}}{2\sqrt{2-3x}}\right)\middle|-2\right)\right)}{31\sqrt{1+4x}\sqrt{-\frac{55}{2}+11x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(EllipticF[ArcSin[Sqrt
[11]/(2*Sqrt[2 - 3*x])], -2] - EllipticPi[124/55, ArcSin[Sqrt[11]/(2*Sqrt[2
 - 3*x])], -2]))/(31*Sqrt[1 + 4*x]*Sqrt[-55/2 + 11*x])
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.67

method	result
default	$\frac{4\,\text{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{253\sqrt{-5+2x}}$

elliptic	$\frac{4\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\right.}{2783\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{-24x^{3}+70x^{2}-21x-10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 4/253*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*(5-2*x)^(1/2)*22^(1/2
)/(-5+2*x)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4 - 182*x^3 - 3
85*x^2 + 197*x + 70), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\cdot(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

3.66 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27807(7+5x)}+\frac{10\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27807\sqrt{5-2x}}-\frac{2\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,F\Big(}{71}$$

```
[Out] -2/7843*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)-8953/6321458*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+10/27807*Ellipti
cE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*
x)^(1/2)-25/27807*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.13, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {178, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{2\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{713\sqrt{2x-5}}+\frac{10\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{27807\sqrt{5-2x}}-\frac{8953\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{574678\sqrt{11}\,\sqrt{2x-5}}-\frac{25\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{27807(5x+7)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + (10*Sq
rt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(27807*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3])/(713*Sqrt[-5 + 2*x]) - (8953*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(574678*Sqrt[11]*S
qrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 178

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b^2*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(
```

```
b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}\,dx = -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27807(7+5x)} + \frac{\int \frac{7777-16}{\sqrt{2-3x}\,\sqrt{-5+}}}{55}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27807(7+5x)} + \frac{\int \frac{-168-1}{\sqrt{2-3x}\,\sqrt{-5+}}}{5561}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27807(7+5x)} - \frac{10\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1-}}}{9269}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27807(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right.}{}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27807(7+5x)} + \frac{10\sqrt{11}\,\sqrt{-5+2x}}{27}$$

Mathematica [A]
time = 5.02, size = 130, normalized size = 0.69

$$\frac{-\frac{51150\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x}}{7+5x} - 3\sqrt{55-22x}\left(6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right) - 14508F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right) + 26859\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)\right)}{56893122\sqrt{-5+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ellip
ticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 26859*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(56893122*Sqrt[-5 + 2*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(145) = 290$.
time = 0.14, size = 302, normalized size = 1.60

method	result

elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{28\sqrt{11+44x}\sqrt{22-33x}\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11}\right.}{1121549\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(14260\sqrt{1+4x}\sqrt{2-3x}\sqrt{22}\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right.\right.}{}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\sqrt{1+4x}\sqrt{(2-3x)(-5+2x)(1+4x)}}{27807(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\sqrt{2-3x}}+\frac{\left(\frac{20\sqrt{22-33x}\sqrt{165-66x}\sqrt{33}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/7035171*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(14260*(1+4*x)^(1/2)*(
2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))
*x-6325*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(
11+44*x)^(1/2),3^(1/2))*x-89530*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x
)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*x+19964*(1+4*x)^(1/
2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1
/2))-8855*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11
*(11+44*x)^(1/2),3^(1/2))-125342*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*
x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))-151800*x^3+442750*
x^2-132825*x-63250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="fricas")
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5 - 70*x^4 - 31
99*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

3.67 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2}-\frac{223825\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1030972332(7+5x)}+\frac{44765\sqrt{11}\,\sqrt{-5+2x}\,E\Big(\sin^{-1}}{515486166\sqrt{5-}}$$

```
[Out] -48493305/234374376808*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*
2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-24007/436180602*EllipticF(1/
11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2
)+44765/515486166*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(
1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-25/55614*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)/(7+5*x)^2-223825/1030972332*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)
^(1/2)/(7+5*x)
```

Rubi [A]
time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {178, 1618, 1621, 174, 552, 551, 164, 115, 114, 122, 120}

$$-\frac{24007\sqrt{5-2x}\,F\left(\text{ArcSin}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{6608797\sqrt{66}\,\sqrt{2x-5}}+\frac{44765\sqrt{11}\,\sqrt{2x-5}\,E\left(\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{515486166\sqrt{5-2x}}-\frac{48493305\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{21306761528\sqrt{11}\,\sqrt{2x-5}}-\frac{223825\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1030972332(5x+7)}-\frac{25\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{55614(5x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) - (223
825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1030972332*(7 + 5*x)) + (4
4765*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
-1/2])/(515486166*Sqrt[5 - 2*x]) - (24007*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sq
rt[3/11]*Sqrt[1 + 4*x]], 1/3])/(6608797*Sqrt[66]*Sqrt[-5 + 2*x]) - (4849330
5*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2
])/(21306761528*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0]
&& GtQ[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[(d*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] &&
GtQ[((-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x]
&& GtQ[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || P
osQ[-f/b]))
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 178

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b^2*(a + b*x)^(m + 1)*Sq
```

```
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(
b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
```

```
q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3}\,dx = -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2} + \frac{\int \frac{16079-6}{\sqrt{2-3x}\,\sqrt{-5+}}}{11}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\,\sqrt{}}{103097233}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\,\sqrt{}}{103097233}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\,\sqrt{}}{103097233}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\,\sqrt{}}{103097233}$$

$$= -\frac{25\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\,\sqrt{}}{103097233}$$

Mathematica [A]
time = 6.84, size = 142, normalized size = 0.63

$$\frac{-17050\sqrt{2-3x}\,(-5+2x)\sqrt{1+4x}\,(81209+44765x)-\sqrt{55-22x}\,(7+5x)^2\left(61059460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-116097852F\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+145479915\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{703123130424\sqrt{-5+2x}\,(7+5x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(81209 + 44765*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 116097852*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14
5479915*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(703
123130424*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(175) = 350$.

time = 0.14, size = 434, normalized size = 1.93

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25\sqrt{-24x^3+70x^2-21x-10}}{55614(7+5x)^2}-\frac{223825\sqrt{-24x^3+70x^2-21\ldots}}{1030972332(7+5x)}\right.}{\ldots}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(81209+44765x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{1030972332(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{44765\sqrt{22-33x}\,\sqrt{1\ldots}}{\ldots}\right.}{\ldots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(510436700\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\ldots\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/260835999996*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(510436700*(1+4*x
)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2)
,3^(1/2))*x^2-283138625*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*
EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x^2-2424665250*(1+4*x)^(1/2)*(2-3*x
)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/
2))*x^2+1429222760*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ellip
ticF(1/11*(11+44*x)^(1/2),3^(1/2))*x-792788150*(1+4*x)^(1/2)*(2-3*x)^(1/2)*
22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*x-6789062700
*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(11+44*
x)^(1/2),-55/23,3^(1/2))*x+1000455932*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*
(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-554951705*(1+4*x)^(1/
2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1
/2))-4752343890*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Elliptic
Pi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))-6795327000*x^4+7492177550*x^3+30009
373625*x^2-13617971675*x-5136469250)/(24*x^3-70*x^2+21*x+10)/(7+5*x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^6 + 3850*x^5 -
 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

3.68 $\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=137

$$\frac{2\sqrt{-fg+eh}\,i\sqrt{c+dx}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\,\sqrt{-\frac{f(c+dx)}{de-cf}}\,\sqrt{g+hx}}$$

```
[Out] 2*i*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*
e)/h)^(1/2))*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/f/h
^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 115, 114}

$$\frac{2i\sqrt{c+dx}\,\sqrt{eh-fg}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\,\sqrt{g+hx}\,\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(f*g) + e*h]*i*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h
*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
```

```
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rubi steps

$$\int \frac{68c+68dx}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = 68\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

$$= \frac{\left(68\sqrt{c+dx}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\,\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{\sqrt{\frac{f(c+dx)}{-de+cf}}\,\sqrt{g+hx}}$$

$$= \frac{136\sqrt{-fg+eh}\,\sqrt{c+dx}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|\right.}{f\sqrt{h}\,\sqrt{-\frac{f(c+dx)}{de-cf}}\,\sqrt{g+hx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.42, size = 180, normalized size = 1.31

$$-\frac{2ii\sqrt{c+dx}\,\sqrt{g+hx}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)\right)}{h\sqrt{\frac{f(c+dx)}{d(e+fx)}}\,\sqrt{e+fx}\,\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*i*Sqrt[c + d*x]*Sqrt[g + h*x]*(EllipticE[I*ArcSinh[Sqrt[(f*(c + d*x
))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - EllipticF[I*ArcSinh[Sq
rt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(h*Sqrt[
(f*(c + d*x))/(d*(e + f*x))]*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
```

Maple [A]
time = 0.11, size = 210, normalized size = 1.53

method	result
default	$-\frac{2i\left(ce\,h^{2}-cfgh-degh+df\,g^{2}\right)\sqrt{\frac{(fx+e)h}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\ \mathrm{EllipticE}\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)\sqrt{dx+\cdots}}{h^{2}f\left(dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg\right)}$
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2ci\left(-\frac{e}{f}+\frac{g}{h}\right)\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}}\,\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\cdots\right.}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+c\cdots}}\cdots\right.}{\sqrt{dx+c\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2*i*(c*e*h^2-c*f*g*h-d*e*g*h+d*f*g^2)*((f*x+e)*h/(e*h-f*g))^(1/2)*((d*x+c)
*h/(c*h-d*g))^(1/2)*(-(h*x+g)*f/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h
-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))/h^2/f*(d*x+c)^(1/2)*(f*x+e)^(
1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x
+d*e*g*x+c*e*g)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((I*d*x + I*c)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 718, normalized size = 5.24

$$-\frac{2\left(3i\sqrt{dfh}\,dfh\,\mathrm{weierstrassZeta}\left(\frac{4\left(d^{2}f^{2}g^{2}-cdf^{2}gh+c^{2}f^{2}h^{2}+d^{2}h^{2}e^{2}-\left(d^{2}fgh+cdfh^{2}\right)e\right)}{3d^{2}f^{2}h^{2}},-\frac{4\left(2d^{3}f^{3}g^{3}-3cd^{2}f^{3}g^{2}h-3c^{2}df^{3}gh^{2}+2c^{3}f^{3}h^{3}+2d^{3}h^{3}e^{3}-3\left(d^{3}fgh^{2}+cd^{2}fh^{3}\right)e^{2}-3\left(d^{3}f^{2}g^{2}h-4cd^{2}f^{2}gh^{2}+c^{2}df^{2}h^{3}\right)e\right)}{27d^{3}f^{3}h^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(d^{2}f^{2}g^{2}-cdf^{2}gh+c^{2}f^{2}h^{2}+d^{2}h^{2}e^{2}-\left(d^{2}fgh+cdfh^{2}\right)e\right)}{3d^{2}f^{2}h^{2}},-\frac{4\left(2d^{3}f^{3}g^{3}-3cd^{2}f^{3}g^{2}h-3c^{2}df^{3}gh^{2}+2c^{3}f^{3}h^{3}+2d^{3}h^{3}e^{3}-3\left(d^{3}fgh^{2}+cd^{2}fh^{3}\right)e^{2}-3\left(d^{3}f^{2}g^{2}h-4cd^{2}f^{2}gh^{2}+c^{2}df^{2}h^{3}\right)e\right)}{27d^{3}f^{3}h^{3}},\frac{3dfhx+dfg+cfh+dhe}{3dfh}\right)\right)+\left(idfg-2icfh+idhe\right)\sqrt{dfh}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(d^{2}f^{2}g^{2}-cdf^{2}gh+c^{2}f^{2}h^{2}+d^{2}h^{2}e^{2}-\left(d^{2}fgh+cdfh^{2}\right)e\right)}{3d^{2}f^{2}h^{2}},-\frac{4\left(2d^{3}f^{3}g^{3}-3cd^{2}f^{3}g^{2}h-3c^{2}df^{3}gh^{2}+2c^{3}f^{3}h^{3}+2d^{3}h^{3}e^{3}-3\left(d^{3}fgh^{2}+cd^{2}fh^{3}\right)e^{2}-3\left(d^{3}f^{2}g^{2}h-4cd^{2}f^{2}gh^{2}+c^{2}df^{2}h^{3}\right)e\right)}{27d^{3}f^{3}h^{3}},\frac{3dfhx+dfg+cfh+dhe}{3dfh}\right)\right)}{3df^{2}h^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="fricas")
```

```
[Out] -2/3*(3*I*sqrt(d*f*h)*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - c*d*f^2*g*h
+ c^2*f^2*h^2 + d^2*h^2*e^2 - (d^2*f*g*h + c*d*f*h^2)*e)/(d^2*f^2*h^2), -4/
27*(2*d^3*f^3*g^3 - 3*c*d^2*f^3*g^2*h - 3*c^2*d*f^3*g*h^2 + 2*c^3*f^3*h^3 +
 2*d^3*h^3*e^3 - 3*(d^3*f*g*h^2 + c*d^2*f*h^3)*e^2 - 3*(d^3*f^2*g^2*h - 4*c
*d^2*f^2*g*h^2 + c^2*d*f^2*h^3)*e)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(
d^2*f^2*g^2 - c*d*f^2*g*h + c^2*f^2*h^2 + d^2*h^2*e^2 - (d^2*f*g*h + c*d*f*
h^2)*e)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*c*d^2*f^3*g^2*h - 3*c^2*d*f
^3*g*h^2 + 2*c^3*f^3*h^3 + 2*d^3*h^3*e^3 - 3*(d^3*f*g*h^2 + c*d^2*f*h^3)*e^
2 - 3*(d^3*f^2*g^2*h - 4*c*d^2*f^2*g*h^2 + c^2*d*f^2*h^3)*e)/(d^3*f^3*h^3),
 1/3*(3*d*f*h*x + d*f*g + c*f*h + d*h*e)/(d*f*h))) + (I*d*f*g - 2*I*c*f*h +
 I*d*h*e)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - c*d*f^2*g*h +
c^2*f^2*h^2 + d^2*h^2*e^2 - (d^2*f*g*h + c*d*f*h^2)*e)/(d^2*f^2*h^2), -4/27
*(2*d^3*f^3*g^3 - 3*c*d^2*f^3*g^2*h - 3*c^2*d*f^3*g*h^2 + 2*c^3*f^3*h^3 + 2
*d^3*h^3*e^3 - 3*(d^3*f*g*h^2 + c*d^2*f*h^3)*e^2 - 3*(d^3*f^2*g^2*h - 4*c*d
^2*f^2*g*h^2 + c^2*d*f^2*h^3)*e)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + c*
f*h + d*h*e)/(d*f*h)))/(d*f^2*h^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$i\int\frac{\sqrt{c+dx}}{\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] i*Integral(sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((I*d*x + I*c)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{c\,i+d\,i\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.69 $\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=284

$$\frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2\sqrt{-de+cf}\,(bg-ah)\sqrt{\frac{d(e}{de}}}{}$$

```
[Out] 2*b*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-a*h+b*g)*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {164, 115, 114, 122, 121}

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\text{ArcSin}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\text{ArcSin}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
 - (2*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx}{h} + \frac{(-bg+ah)\int \frac{1}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+}}}{h}$$

$$= \frac{\left((-bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\,\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\,\sqrt{g+}}}{h\sqrt{e+fx}}$$

$$= \frac{2b\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(d}{f(}\right.}{d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$= \frac{2b\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(d}{f(}\right.}{d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.06, size = 319, normalized size = 1.12

$$-\frac{2\left(-bd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)-ib(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)|\frac{dfg-cfh}{deh-cfh}\right)+id(be-af)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)|\frac{dfg-cfh}{deh-cfh}\right)\right)}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*b*(d*e - c*f)*h*(c
 + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c +
d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*
h)/(d*e*h - c*f*h)] + I*d*(b*e - a*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/
(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*Sqrt[-
c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(250) = 500$.

time = 0.11, size = 566, normalized size = 1.99

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(\frac{2a\left(-\frac{e}{f}+\frac{g}{h}\right)\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}},\sqrt{\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{c}{d}}}\right)}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+c}}\right.}{\sqrt{dx+c}}$
default	$-\frac{2\left(\mathrm{EllipticF}\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)ade\,h^{2}-\mathrm{EllipticF}\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\sqrt{\frac{(eh-fg)d}{f(ch-dg)}}\right)adfgh-\mathrm{EllipticF}\left(\sqrt{-\frac{(h}{e}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -2*(EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))
*a*d*e*h^2-EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))
^(1/2))*a*d*f*g*h-EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*
h-d*g))^(1/2))*b*c*e*h^2+EllipticF((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*
d/f/(c*h-d*g))^(1/2))*b*c*f*g*h+EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*
h-f*g)*d/f/(c*h-d*g))^(1/2))*b*c*e*h^2-EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/
2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*c*f*g*h-EllipticE((-(h*x+g)*f/(e*h-f*
g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d*e*g*h+EllipticE((-(h*x+g)*f/
(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d*f*g^2)*((f*x+e)*h/(e*
h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*(-(h*x+g)*f/(e*h-f*g))^(1/2)/h^2/
f/d*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^
2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 725, normalized size = 2.55

$$-\frac{2\left(3\sqrt{dfh}\,bdfh\,\mathrm{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-cdf^2gh+c^2f^2h^2+d^2h^2e^2-\left(d^2fgh+cdfh^2\right)e\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\,cd^2f^3g^2h-3\,c^2df^3gh^2+2\,c^3f^3h^3+2\,d^3h^3e^3-3\left(d^3fgh^2+cd^2fh^3\right)e^2-3\left(d^3f^2g^2h-4\,cd^2f^2gh^2+c^2df^2h^3\right)e\right)}{27\,d^3f^3h^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(d^2f^2g^2-cdf^2gh+c^2f^2h^2+d^2h^2e^2-\left(d^2fgh+cdfh^2\right)e\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\,cd^2f^3g^2h-3\,c^2df^3gh^2+2\,c^3f^3h^3+2\,d^3h^3e^3-3\left(d^3fgh^2+cd^2fh^3\right)e^2-3\left(d^3f^2g^2h-4\,cd^2f^2gh^2+c^2df^2h^3\right)e\right)}{27\,d^3f^3h^3},\frac{3\,dfhx+dfg+cfh+dhe}{3\,dfh}\right)\right)+\left(bdfg+bdhe+\left(bc-3\,ad\right)fh\right)\sqrt{dfh}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(d^2f^2g^2-cdf^2gh+c^2f^2h^2+d^2h^2e^2-\left(d^2fgh+cdfh^2\right)e\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\,cd^2f^3g^2h-3\,c^2df^3gh^2+2\,c^3f^3h^3+2\,d^3h^3e^3-3\left(d^3fgh^2+cd^2fh^3\right)e^2-3\left(d^3f^2g^2h-4\,cd^2f^2gh^2+c^2df^2h^3\right)e\right)}{27\,d^3f^3h^3},\frac{3\,dfhx+dfg+cfh+dhe}{3\,dfh}\right)\right)}{3\,d^2f^2h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(3*sqrt(d*f*h)*b*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - c*d*f^2*g*h
+ c^2*f^2*h^2 + d^2*h^2*e^2 - (d^2*f*g*h + c*d*f*h^2)*e)/(d^2*f^2*h^2), -4/
27*(2*d^3*f^3*g^3 - 3*c*d^2*f^3*g^2*h - 3*c^2*d*f^3*g*h^2 + 2*c^3*f^3*h^3 +
 2*d^3*h^3*e^3 - 3*(d^3*f*g*h^2 + c*d^2*f*h^3)*e^2 - 3*(d^3*f^2*g^2*h - 4*c
*d^2*f^2*g*h^2 + c^2*d*f^2*h^3)*e)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(
d^2*f^2*g^2 - c*d*f^2*g*h + c^2*f^2*h^2 + d^2*h^2*e^2 - (d^2*f*g*h + c*d*f*
h^2)*e)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*c*d^2*f^3*g^2*h - 3*c^2*d*f
^3*g*h^2 + 2*c^3*f^3*h^3 + 2*d^3*h^3*e^3 - 3*(d^3*f*g*h^2 + c*d^2*f*h^3)*e^
2 - 3*(d^3*f^2*g^2*h - 4*c*d^2*f^2*g*h^2 + c^2*d*f^2*h^3)*e)/(d^3*f^3*h^3),
 1/3*(3*d*f*h*x + d*f*g + c*f*h + d*h*e)/(d*f*h))) + (b*d*f*g + b*d*h*e + (
b*c - 3*a*d)*f*h)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - c*d*f^
2*g*h + c^2*f^2*h^2 + d^2*h^2*e^2 - (d^2*f*g*h + c*d*f*h^2)*e)/(d^2*f^2*h^2
), -4/27*(2*d^3*f^3*g^3 - 3*c*d^2*f^3*g^2*h - 3*c^2*d*f^3*g*h^2 + 2*c^3*f^3
*h^3 + 2*d^3*h^3*e^3 - 3*(d^3*f*g*h^2 + c*d^2*f*h^3)*e^2 - 3*(d^3*f^2*g^2*h
 - 4*c*d^2*f^2*g*h^2 + c^2*d*f^2*h^3)*e)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*
f*g + c*f*h + d*h*e)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx}{\sqrt{c + dx}\,\sqrt{e + fx}\,\sqrt{g + hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.70 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

```
[Out] -2*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c
)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e
))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {175, 552, 551}

$$-\frac{2\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c
 - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\,\sqrt{g-\frac{ch}{d}}\cdots}\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{(bc-ad-bx^2)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\cdots}\right.}{\sqrt{e+fx}\cdots}$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{}{(bc-ad-bx^2)\sqrt{1\cdots}}\right.}{\sqrt{e+fx}\,\sqrt{g\cdots}}$$

$$= -\frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin\cdots\right.}{(bc-ad)\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.54, size = 226, normalized size = 1.37

$$\frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\,\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)\middle|\frac{dfg-cfh}{deh-cfh}\right)-\Pi\left(-\frac{bcf-adf}{bde-bcf};i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right)\middle|\frac{dfg-cfh}{deh-cfh}\right)\right)}{(-bc+ad)\sqrt{-c+\frac{de}{f}}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g -
c*f*h)/(d*e*h - c*f*h)] - EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*
ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]
))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Maple [A]
time = 0.12, size = 222, normalized size = 1.35

method	result
default	$-\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{-\frac{(hx+g)f}{eh-fg}}\,\sqrt{\frac{(dx+c)h}{ch-dg}}\,\sqrt{\frac{(fx+e)h}{eh-fg}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(hx+g)f}{eh-fg}},\frac{(eh-fg)b}{f(ah-bg)},\ldots\right.}{f(ah-bg)(dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg)}$
elliptic	$\frac{2\sqrt{(dx+c)\,(fx+e)\,(hx+g)}\,\left(-\frac{e}{f}+\frac{g}{h}\right)\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}}\,\sqrt{\frac{x+\frac{c}{d}}{-\frac{g}{h}+\frac{c}{d}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{g}{h}+\frac{e}{f}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{g}{h}}{-\frac{e}{f}+\frac{g}{h}}},\frac{-\frac{g}{h}+\frac{e}{f}}{-\frac{g}{h}+\frac{a}{b}},\ldots\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f*(-(h*x+g)*f/(e*h-f*g))^(1/2)
*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+
g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/
2))*(e*h-f*g)/(a*h-b*g)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.71 $\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=393

$$\frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}}-\frac{2d\sqrt{h}\,\sqrt{-fg+eh}\,\sqrt{c+dx}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\Bigg(\sin^{-1}\Bigg(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{-fg+eh}}}{(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\,\sqrt{g+hx}}$$

```
[Out] 2*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^
(1/2)-2*b*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-
a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-
c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^2/f^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2)-2*d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*
(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))*h^(1/2)*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h
*x+g)/(-e*h+f*g))^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(-f*(d*x+c)/(-c*f+
d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {185, 106, 21, 115, 114, 175, 552, 551}

$$-\frac{2d\sqrt{h}\,\sqrt{c+dx}\,\sqrt{eh-fg}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}\,(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}}-\frac{2b\sqrt{cf-de}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\mathrm{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}\,(bc-ad)^2}+\frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{\sqrt{c+dx}\,(bc-ad)(de-cf)(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*Sq
rt[c + d*x]) - (2*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h
*x))/(f*g - e*h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*
h]], -((d*(f*g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)*(d*e - c*f)*(d*g - c
*h)*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*b*Sqrt[-(d*e) +
c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*Sqrt[f]*
Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 185

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
```

```
1/2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \int \left(-\frac{d}{(bc-ad)(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}} + \frac{}{(bc-a}\right.$$

$$= \frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx}{bc-ad} - \frac{d\int \frac{}{(c+dx)^{3/2}\sqrt{}}}{}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(2b)\text{Subst}\left(\int \frac{}{(bc-}\right.}{}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(dfh)\int \frac{\sqrt{c}}{\sqrt{e+fx}}}{(bc-ad)(de-}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{\left(2b\sqrt{\frac{d(e+fx)}{de-cf}}\right.}{}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2d\sqrt{h}\,\sqrt{-fg+}}{(b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.84, size = 322, normalized size = 0.82

$$\frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left((bc-ad)fE\left(i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{dg}{h}}}{\sqrt{c+dx}}\right)\middle|\frac{deh-cfh}{dfg-cfh}\right)+(bde-2bcf+adf)F\left(i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{dg}{h}}}{\sqrt{c+dx}}\right)\middle|\frac{deh-cfh}{dfg-cfh}\right)+b(-de+cf)\Pi\left(-\frac{bch-adh}{bdg-bch};i\sinh^{-1}\left(\frac{\sqrt{-c+\frac{dg}{h}}}{\sqrt{c+dx}}\right)\middle|\frac{deh-cfh}{dfg-cfh}\right)\right)}{(bc-ad)^2(-de+cf)\sqrt{-c+\frac{dg}{h}}\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*((b*c - a*d)*f*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] + (b*d*e - 2*b*c*f + a*d*f)*EllipticF[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] + b*(-(d*e) + c*f)*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/((b*c - a*d)^2*(-(d*e) + c*f)*Sqrt[-c + (d*g)/h]*Sqrt[e + f*x]*Sqrt[g + h*x])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1977 vs. $2(353) = 706$.
time = 0.12, size = 1978, normalized size = 5.03

method	result
elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2\left(dfh\,x^2+dehx+dfgx+deg\right)d}{\left(c^2fh-cdeh-cdfg+d^2eg\right)(ad-bc)\sqrt{\left(x+\frac{c}{d}\right)\left(dfh\,x^2+dehx+dfgx+deg\right)}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)*((-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*c*d*e*f*h^3-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*c*d*f^2*g*h^2-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*d^2*e*f*g*h^2+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*a*d^2*f^2*g^2*h-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*c*d*e*f*g*h^2+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c

```
*h-d*g))^(1/2))*b*c*d*f^2*g^2*h+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*
h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE((-(h*x+g)*f/(e*h-f*g))^
(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d^2*e*f*g^2*h-(-(h*x+g)*f/(e*h-f*g
))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticE(
(-(h*x+g)*f/(e*h-f*g))^(1/2),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d^2*f^2*g^3
-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f
*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),
((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*c^2*e*f*h^3+(-(h*x+g)*f/(e*h-f*g))^(1/2)
*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+
g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/
2))*b*c^2*f^2*g*h^2+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2
)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-
f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*c*d*e^2*h^3-(-(h*x+g)
*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)
*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)
*d/f/(c*h-d*g))^(1/2))*b*c*d*f^2*g^2*h-(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c
)*h/(c*h-d*g))^(1/2)*((f*x+e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*
h-f*g))^(1/2),(e*h-f*g)*b/f/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d^
2*e^2*g*h^2+(-(h*x+g)*f/(e*h-f*g))^(1/2)*((d*x+c)*h/(c*h-d*g))^(1/2)*((f*x+
e)*h/(e*h-f*g))^(1/2)*EllipticPi((-(h*x+g)*f/(e*h-f*g))^(1/2),(e*h-f*g)*b/f
/(a*h-b*g),((e*h-f*g)*d/f/(c*h-d*g))^(1/2))*b*d^2*e*f*g^2*h+a*d^2*f^2*h^3*x
^2-b*d^2*f^2*g*h^2*x^2+a*d^2*e*f*h^3*x+a*d^2*f^2*g*h^2*x-b*d^2*e*f*g*h^2*x-
b*d^2*f^2*g^2*h*x+a*d^2*e*f*g*h^2-b*d^2*e*f*g^2*h)/f/h/(c*h-d*g)/(c*f-d*e)/
(a*h-b*g)/(a*d-b*c)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*
x+d*e*g*x+c*e*g)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\,(c+dx)^{\frac{3}{2}}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(1/((a + b*x)*(c + d*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)), x)
```

3.72 $\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=875

$$\frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}}+\frac{2bd^2\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}}-\frac{4d^2(dfg+deh-2cfh)}{3(bc-ad)(de-cf)^2(}$$

```
[Out] 2/3*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c
)^(3/2)+2*b*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d
*g)/(d*x+c)^(1/2)-4/3*d^2*(-2*c*f*h+d*e*h+d*f*g)*(f*x+e)^(1/2)*(h*x+g)^(1/2
)/(-a*d+b*c)/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)+4/3*d*(-2*c*f*h+d*e*h+
d*f*g)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*
h+d*g))^(1/2))*f^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c
)/(c*f-d*e)^(3/2)/(-c*h+d*g)^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2
/3*(-3*c*f*h+d*e*h+2*d*f*g)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2)
,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d
*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/(c*f-d*e)^(3/2)/(-c*h+d*g)/(f*x+e)^(1
/2)/(h*x+g)^(1/2)-2*b^2*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b
*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)
*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^3/f^(
1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*b*d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h
-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))*h^(1/2)*(e*h-f*g)^(1/2)*(d*
x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d*g)/
(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]

time = 0.94, antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {185, 106, 157, 164, 115, 114, 122, 121, 21, 175, 552, 551}

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\text{ArcSin}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)b}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}+\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}\,b}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}}+\frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(cf-de)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2\sqrt{f}(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(cf-de)^{3/2}(dg-ch)\sqrt{e+fx}\sqrt{g+hx}}-\frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}}+\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)*(d*g - c*h)*
(c + d*x)^(3/2)) + (2*b*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(d*
e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) - (4*d^2*(d*f*g + d*e*h - 2*c*f*h)*Sqrt
[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)^2*(d*g - c*h)^2*Sqrt[c
+ d*x]) + (4*d*Sqrt[f]*(d*f*g + d*e*h - 2*c*f*h)*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
```

```
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2
)*(d*g - c*h)^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*b*d*Sqr
t[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*Sqrt[-((f*(c + d*
x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*Sqrt[f]*(2*d*f*g + d*e*h - 3*c*f*h)*S
qrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[Ar
cSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*Sqrt[e + f*x]*Sqrt
[g + h*x]) - (2*b^2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)),
ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g
 - c*h))])/((b*c - a*d)^3*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
```

```
*x]
```

Rule 185

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \int \left(-\frac{d}{(bc-ad)(c+dx)^{5/2}\sqrt{e+fx}\,\sqrt{g+hx}} - \frac{}{(bc-a}\right.$$

$$= \frac{b^2 \int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{(bd)\int \frac{}{(c+dx)}}{}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{}}{(bc-ad)^2(de}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{}}{(bc-ad)^2(de}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{}}{(bc-ad)^2(de}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{}}{(bc-ad)^2(de}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{}}{(bc-ad)^2(de}$$

$$= \frac{2d^2\sqrt{e+fx}\,\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{}}{(bc-ad)^2(de}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.32, size = 4180, normalized size = 4.78

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*((2*d^2)/(3*(b*c - a*d)*(-(d*e) +
 c*f)*(-(d*g) + c*h)*(c + d*x)^2) + (2*d^2*(3*b*d^2*e*g - 5*b*c*d*f*g + 2*a
*d^2*f*g - 5*b*c*d*e*h + 2*a*d^2*e*h + 7*b*c^2*f*h - 4*a*c*d*f*h))/(3*(b*c
- a*d)^2*(-(d*e) + c*f)^2*(-(d*g) + c*h)^2*(c + d*x))) + (2*(c + d*x)^(3/2)
*(-3*b^2*c*d^2*e*f*g*Sqrt[-c + (d*g)/h]*h + 3*a*b*d^3*e*f*g*Sqrt[-c + (d*g)
/h]*h + 5*b^2*c^2*d*f^2*g*Sqrt[-c + (d*g)/h]*h - 7*a*b*c*d^2*f^2*g*Sqrt[-c
+ (d*g)/h]*h + 2*a^2*d^3*f^2*g*Sqrt[-c + (d*g)/h]*h + 5*b^2*c^2*d*e*f*Sqrt[
-c + (d*g)/h]*h^2 - 7*a*b*c*d^2*e*f*Sqrt[-c + (d*g)/h]*h^2 + 2*a^2*d^3*e*f*
Sqrt[-c + (d*g)/h]*h^2 - 7*b^2*c^3*f^2*Sqrt[-c + (d*g)/h]*h^2 + 11*a*b*c^2*
d*f^2*Sqrt[-c + (d*g)/h]*h^2 - 4*a^2*c*d^2*f^2*Sqrt[-c + (d*g)/h]*h^2 - (3*
b^2*c*d^4*e^2*g^2*Sqrt[-c + (d*g)/h])/(c + d*x)^2 + (3*a*b*d^5*e^2*g^2*Sqrt
[-c + (d*g)/h])/(c + d*x)^2 + (8*b^2*c^2*d^3*e*f*g^2*Sqrt[-c + (d*g)/h])/(c
 + d*x)^2 - (10*a*b*c*d^4*e*f*g^2*Sqrt[-c + (d*g)/h])/(c + d*x)^2 + (2*a^2*
d^5*e*f*g^2*Sqrt[-c + (d*g)/h])/(c + d*x)^2 - (5*b^2*c^3*d^2*f^2*g^2*Sqrt[-
c + (d*g)/h])/(c + d*x)^2 + (7*a*b*c^2*d^3*f^2*g^2*Sqrt[-c + (d*g)/h])/(c +
 d*x)^2 - (2*a^2*c*d^4*f^2*g^2*Sqrt[-c + (d*g)/h])/(c + d*x)^2 + (8*b^2*c^2
*d^3*e^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 - (10*a*b*c*d^4*e^2*g*Sqrt[-c
+ (d*g)/h]*h)/(c + d*x)^2 + (2*a^2*d^5*e^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x
)^2 - (20*b^2*c^3*d^2*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 + (28*a*b*c^2
*d^3*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 - (8*a^2*c*d^4*e*f*g*Sqrt[-c +
 (d*g)/h]*h)/(c + d*x)^2 + (12*b^2*c^4*d*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d
*x)^2 - (18*a*b*c^3*d^2*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 + (6*a^2*c^
2*d^3*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 - (5*b^2*c^3*d^2*e^2*Sqrt[-c
+ (d*g)/h]*h^2)/(c + d*x)^2 + (7*a*b*c^2*d^3*e^2*Sqrt[-c + (d*g)/h]*h^2)/(c
 + d*x)^2 - (2*a^2*c*d^4*e^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 + (12*b^2*
c^4*d*e*f*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 - (18*a*b*c^3*d^2*e*f*Sqrt[-c
 + (d*g)/h]*h^2)/(c + d*x)^2 + (6*a^2*c^2*d^3*e*f*Sqrt[-c + (d*g)/h]*h^2)/(
c + d*x)^2 - (7*b^2*c^5*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 + (11*a*b*c
^4*d*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 - (4*a^2*c^3*d^2*f^2*Sqrt[-c +
 (d*g)/h]*h^2)/(c + d*x)^2 - (3*b^2*c*d^3*e*f*g^2*Sqrt[-c + (d*g)/h])/(c +
d*x) + (3*a*b*d^4*e*f*g^2*Sqrt[-c + (d*g)/h])/(c + d*x) + (5*b^2*c^2*d^2*f^
2*g^2*Sqrt[-c + (d*g)/h])/(c + d*x) - (7*a*b*c*d^3*f^2*g^2*Sqrt[-c + (d*g)/
h])/(c + d*x) + (2*a^2*d^4*f^2*g^2*Sqrt[-c + (d*g)/h])/(c + d*x) - (3*b^2*c
*d^3*e^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) + (3*a*b*d^4*e^2*g*Sqrt[-c + (d*
g)/h]*h)/(c + d*x) + (16*b^2*c^2*d^2*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)
- (20*a*b*c*d^3*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) + (4*a^2*d^4*e*f*g*Sq
rt[-c + (d*g)/h]*h)/(c + d*x) - (17*b^2*c^3*d*f^2*g*Sqrt[-c + (d*g)/h]*h)/(
```

```
c + d*x) + (25*a*b*c^2*d^2*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) - (8*a^2*c
*d^3*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) + (5*b^2*c^2*d^2*e^2*Sqrt[-c + (
d*g)/h]*h^2)/(c + d*x) - (7*a*b*c*d^3*e^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)
 + (2*a^2*d^4*e^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) - (17*b^2*c^3*d*e*f*Sqr
t[-c + (d*g)/h]*h^2)/(c + d*x) + (25*a*b*c^2*d^2*e*f*Sqrt[-c + (d*g)/h]*h^2
)/(c + d*x) - (8*a^2*c*d^3*e*f*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) + (14*b^2*
c^4*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) - (22*a*b*c^3*d*f^2*Sqrt[-c + (d*
g)/h]*h^2)/(c + d*x) + (8*a^2*c^2*d^2*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)
 + (I*(b*c - a*d)*f*(-(d*g) + c*h)*(2*a*d*(d*f*g + d*e*h - 2*c*f*h) + b*(3*
d^2*e*g + 7*c^2*f*h - 5*c*d*(f*g + e*h)))*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(
c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticE[I*ArcSinh[
Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c
 + d*x] - (I*(d*g - c*h)*(a^2*d^2*f*(2*d*f*g + d*e*h - 3*c*f*h) + b^2*(3*d^
3*e^2*g - 9*c^3*f^2*h - 3*c*d^2*e*(3*f*g + e*h) + 2*c^2*d*f*(4*f*g + 5*e*h)
) + a*b*d*f*(3*d^2*e*g + 9*c^2*f*h - c*d*(7*f*g + 5*e*h)))*Sqrt[1 - c/(c +
d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*Ell
ipticF[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g
- c*f*h)])/Sqrt[c + d*x] + ((3*I)*b^2*d^4*e^2*g^2*Sqrt[1 - c/(c + d*x) + (d
*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-
((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*
x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] - ((6*I)*b^2*c*d^3*e*f
*g^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*
g)/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[
Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c
 + d*x] + ((3*I)*b^2*c^2*d^2*f^2*g^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d
*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d
*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h -
 c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] - ((6*I)*b^2*c*d^3*e^2*g*h*Sqrt[1 -
 c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*
x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16646 vs. $2(781) = 1562$.

time = 0.20, size = 16647, normalized size = 19.03

method	result

elliptic	$\frac{\sqrt{(dx+c)(fx+e)(hx+g)}\left(-\frac{2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+c}}{3\left(c^2fh-cdeh-cdfg+d^2eg\right)(ad-bc)\left(x+\frac{c}{d}\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)), x)
```

3.73 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*f+b),2^(1/2)*(d/(c*f+d))^(1
/2))*(f*(d*x+c)/(c*f+d))^(1/2)/(a*f+b)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {174, 552, 551}

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\text{ArcSin}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-fx}\,\sqrt{1+fx}}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af-bx^2)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af-bx^2)\sqrt{1-\frac{\cdots}{(c}}}\right.}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.69, size = 203, normalized size = 2.74

$$\frac{2i(c+dx)\sqrt{\frac{d(-1+fx)}{f(c+dx)}}\,\sqrt{\frac{d+dfx}{cf+dfx}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{-d+cf}{d+cf}\right)-\Pi\left(\frac{bcf-adf}{bd+bcf};i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{-d+cf}{d+cf}\right)\right)}{(-bc+ad)\sqrt{-\frac{d+cf}{f}}\,\sqrt{1-f^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(71) = 142$.
time = 0.13, size = 184, normalized size = 2.49

method	result
default	$-\frac{2(cf-d)\,\mathrm{EllipticPi}\left(\sqrt{\frac{(dx+c)f}{cf-d}},-\frac{(cf-d)b}{f(ad-bc)},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{(fx+1)d}{cf-d}}\,\sqrt{-\frac{(fx-1)d}{cf+d}}\,\sqrt{\frac{(dx+c)f}{cf-d}}\,\sqrt{fx+1}\,\sqrt{-fx+}}{f(ad-bc)(d\,f^2x^3+c\,f^2x^2-dx-c)}$
elliptic	$\frac{2\sqrt{-\left(f^2x^2-1\right)(dx+c)}\,\left(\frac{c}{d}-\frac{1}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x-\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x+\frac{1}{f}}{-\frac{c}{d}+\frac{1}{f}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}},\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\right)}{\sqrt{dx+c}\,\sqrt{-fx+1}\,\sqrt{fx+1}\,b\sqrt{-d\,f^2x^3-c\,f^2x^2+dx+c}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(f*x+1)^(1/2)*(-f*x+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d
-b*c)/(d*f^2*x^3+c*f^2*x^2-d*x-c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{-fx+1}\,\sqrt{fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f*x+1)**(1/2)/(f*x+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f*x + 1)*sqrt(f*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{1-f\,x}\,\sqrt{f\,x+1}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.74 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^2x^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*f+b),2^(1/2)*(d/(c*f+d))^(1
/2))*(f*(d*x+c)/(c*f+d))^(1/2)/(a*f+b)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {946, 174, 552, 551}

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\text{ArcSin}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-fx}\,\sqrt{1+fx}}\,dx$$

$$= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af-bx^2)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx, x, \sqrt{1$$

$$= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af-bx^2)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}$$

$$\sqrt{c+dx}$$

$$= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 203, normalized size = 2.74

$$\frac{2i(c+dx)\sqrt{\frac{d(-1+fx)}{f(c+dx)}}\sqrt{\frac{d+dfx}{cf+dfx}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{-d+cf}{d+cf}\right)-\Pi\left(\frac{bcf-adf}{bd+bcf};i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{-d+cf}{d+cf}\right)\right)}{(-bc+ad)\sqrt{-\frac{d+cf}{f}}\sqrt{1-f^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(71) = 142$.
time = 0.13, size = 181, normalized size = 2.45

method	result
default	$-\frac{2(cf-d)\,\mathrm{EllipticPi}\left(\sqrt{\frac{(dx+c)f}{cf-d}},-\frac{(cf-d)b}{f(ad-bc)},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{(fx+1)d}{cf-d}}\,\sqrt{-\frac{(fx-1)d}{cf+d}}\,\sqrt{\frac{(dx+c)f}{cf-d}}\,\sqrt{-f^2x^2+1}\,\sqrt{d\cdots}}{f(ad-bc)(d\,f^2x^3+c\,f^2x^2-dx-c)}$
elliptic	$\frac{2\sqrt{-\left(f^2x^2-1\right)(dx+c)}\,\left(\frac{c}{d}-\frac{1}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x-\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x+\frac{1}{f}}{-\frac{c}{d}+\frac{1}{f}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}},\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\right)}{\sqrt{-f^2x^2+1}\,\sqrt{dx+c}\,b\sqrt{-d\,f^2x^3-c\,f^2x^2+dx+c}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(-f^2*x^2+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d-b*c)/(d*f
^2*x^3+c*f^2*x^2-d*x-c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(fx-1)(fx+1)}\,(a+bx)\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(f*x - 1)*(f*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{1-f^2\,x^2}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.75 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^2x}\,\sqrt{1+f^2x}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+
d))^(1/2))*(f^2*(d*x+c)/(c*f^2+d))^(1/2)/(a*f^2+b)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {174, 552, 551}

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\text{ArcSin}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{(af^2+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^2x}\,\sqrt{1+f^2x}}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af^2-bx^2)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af^2-bx^2)\sqrt{1-}}\right.}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.67, size = 218, normalized size = 2.53

$$\frac{2i(c+dx)\sqrt{\frac{d(-1+f^2x)}{f^2(c+dx)}}\sqrt{\frac{d(1+f^2x)}{f^2(c+dx)}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{-d+cf^2}{d+cf^2}\right)-\Pi\left(\frac{(bc-ad)f^2}{b(d+cf^2)};i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{-d+cf^2}{d+cf^2}\right)\right)}{(-bc+ad)\sqrt{-c-\frac{d}{f^2}}\sqrt{1-f^4x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(83) = 166$.
time = 0.13, size = 212, normalized size = 2.47

method	result
default	$-\frac{2(c f^2-d)\,\mathrm{EllipticPi}\left(\sqrt{\frac{(dx+c)f^2}{c f^2-d}}, -\frac{(c f^2-d)b}{f^2(ad-bc)}, \sqrt{\frac{c f^2-d}{c f^2+d}}\right)\sqrt{-\frac{(f^2x+1)d}{c f^2-d}}\,\sqrt{-\frac{(f^2x-1)d}{c f^2+d}}\,\sqrt{\frac{(dx+c)f^2}{c f^2-d}}\,\sqrt{f^2x+1}}{f^2(ad-bc)(d f^4x^3+c f^4x^2-dx-c)}$
elliptic	$\frac{2\sqrt{-(f^4x^2-1)(dx+c)}\,\left(\frac{c}{d}-\frac{1}{f^2}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}}\,\sqrt{\frac{x-\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}\,\sqrt{\frac{x+\frac{1}{f^2}}{-\frac{c}{d}+\frac{1}{f^2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}}, \frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}+\frac{a}{b}}, \sqrt{\frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}\right)}{\sqrt{dx+c}\,\sqrt{-f^2x+1}\,\sqrt{f^2x+1}\,b\sqrt{-d f^4x^3-c f^4x^2+dx+c}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d
-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-
1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(f^2*x+1)^(1/2)*(-f^2*x
+1)^(1/2)*(d*x+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(sqrt(f^2*x + 1)*sqrt(-f^2*x + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{-f^2x+1}\,\sqrt{f^2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x+1)**(1/2)/(f**2*x+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f**2*x + 1)*sqrt(f**2*x + 1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)\,\sqrt{1-f^2\,x}\,\sqrt{x\,f^2+1}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)), x)
```

3.76 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^4x^2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+
d))^(1/2))*(f^2*(d*x+c)/(c*f^2+d))^(1/2)/(a*f^2+b)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {946, 174, 552, 551}

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\text{ArcSin}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{(af^2+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^2x}\,\sqrt{1+f^2x}}\,dx$$

$$= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af^2-bx^2)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx, x, \sqrt{\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,(b+af^2-bx^2)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)}}}\right.}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 218, normalized size = 2.53

$$\frac{2i(c+dx)\sqrt{\frac{d(-1+f^2x)}{f^2(c+dx)}}\,\sqrt{\frac{d(1+f^2x)}{f^2(c+dx)}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{-d+cf^2}{d+cf^2}\right)-\Pi\left(\frac{(bc-ad)f^2}{b(d+cf^2)};i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{-d+cf^2}{d+cf^2}\right)\right)}{(-bc+ad)\sqrt{-c-\frac{d}{f^2}}\,\sqrt{1-f^4x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.
time = 0.12, size = 205, normalized size = 2.38

method	result
default	$-\frac{2(c f^2-d) \operatorname{EllipticPi}(\sqrt{\frac{(dx+c)f^2}{c f^2-d}}, -\frac{(c f^2-d) b}{f^2(ad-bc)}, \sqrt{\frac{c f^2-d}{c f^2+d}}) \sqrt{-\frac{(f^2x+1)d}{c f^2-d}} \sqrt{-\frac{(f^2x-1)d}{c f^2+d}} \sqrt{\frac{(dx+c)f^2}{c f^2-d}} \sqrt{-f^4x^2+ \cdots}}{f^2(ad-bc)(d f^4x^3+c f^4x^2-dx-c)}$
elliptic	$\frac{2\sqrt{-(f^4x^2-1)(dx+c)} (\frac{c}{d}-\frac{1}{f^2}) \sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}} \sqrt{\frac{x-\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}} \sqrt{\frac{x+\frac{1}{f^2}}{-\frac{c}{d}+\frac{1}{f^2}}} \operatorname{EllipticPi}(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^2}}}, \frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}+\frac{a}{b}}, \sqrt{\frac{-\frac{c}{d}+\frac{1}{f^2}}{-\frac{c}{d}-\frac{1}{f^2}}}) \cdots}{\sqrt{-f^4x^2+1} \sqrt{dx+c}\, b\sqrt{-d f^4x^3-c f^4x^2+dx+c} (-\frac{c}{d}+\frac{a}{b})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d
-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-
1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(-f^4*x^2+1)^(1/2)*(d*x
+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(f^{2}x-1\right)\left(f^{2}x+1\right)}\left(a+bx\right)\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**4*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(f**2*x - 1)*(f**2*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{1-f^{4}\,x^{2}}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

3.77 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}\,dx$

Optimal. Leaf size=471

$$-\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}}-\frac{70489981\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1658880}-83363\sqrt{2-}$$

```
[Out] -83363/34560*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/2
400*(7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/25*(7+5*x)^(
7/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-57691792727443/213497856000
*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*
I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5
+2*x)^(1/2)/(1+4*x)^(1/2)-1450582567/3686400*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7
+5*x)^(1/2)/(-5+2*x)^(1/2)-70489981/1658880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)*(7+5*x)^(1/2)-245264762213/2289254400*(1/(4+2*(1+4*x)/(2-3*x)))
^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^
(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)
/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+1450582567/7372800*EllipticE(1/23*8
97^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(
1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {167, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\frac{1450582567\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{2457600\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{57691792727443(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{497664000\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}-\frac{245264762213\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{99532800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{25}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}-\frac{427\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}{2400}-\frac{83363\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}{34560}-\frac{70489981\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{1658880}-\frac{1450582567\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{3686400\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] (-1450582567*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3686400*Sqrt[-5 +
2*x]) - (70489981*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
/1658880 - (83363*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2
))/34560 - (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))
/2400 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(7/2))/25 + (
1450582567*Sqrt[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[Ar
cSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(2457600*Sqrt[(2
 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (245264762213*Sqrt[11/23]*Sqrt[7 + 5*x]
*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(9953280
0*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (57691792727443*(2 - 3*x)*Sqr
```

```
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcS
in[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(497664000*Sqrt[429
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
```

```
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}\,dx = \frac{1}{25}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{7/2} + \frac{1}{50}\int \frac{(7}{\sqrt{}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}{2400} + \frac{1}{25}\sqrt{2}$$

$$= -\frac{83363\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{34560} - \frac{427}{}$$

$$= -\frac{70489981\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1658880} - \frac{8}{}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{}}{}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{}}{}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{}}{}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{}}{}$$

Mathematica [A]
time = 45.86, size = 350, normalized size = 0.74

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(78331458618\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-62507925572\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(6(1118234665415+5225923788019x+2861488598626x^2-795166559320x^3-849459145920x^4-288728294400x^5+71414784000x^6+39813120000x^7)-60033082963\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)\right)}{398131200\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] -1/398131200*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(78331458618*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 62507925572*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(6*(
1118234665415 + 5225923788019*x + 2861488598626*x^2 - 795166559320*x^3 - 84
9459145920*x^4 - 288728294400*x^5 + 71414784000*x^6 + 39813120000*x^7) - 60
033082963*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x
 + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2
+ 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [A]
time = 0.18, size = 841, normalized size = 1.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/9820901376000*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(2
42812114590870*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2
+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+
5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+1038452269093974*(-253*(7+5*x)/(-2+3
*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2
+3*x))^(1/2)*x^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*
I*897^(1/2))-756094404310245*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)
*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(
1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-323749486121160*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/
39*I*897^(1/2))-1384603025458632*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(
1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Elliptic
Pi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+100812587241
3660*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/
2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))+107916495373720*(-253*(7+5*x)/(-2+3*x))^(1/2)*13
^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*
EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+461534341819
```

```
544*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2
)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))
^(1/2),-69/55,1/39*I*897^(1/2))-336041957471220*(-253*(7+5*x)/(-2+3*x))^(1/
2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(
1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+5892540
825600000*x^7+10569745105920000*x^6-42733231212672000*x^5-12572420089188960
0*x^4-117688626612156600*x^3+423514620039641130*x^2+773462850245752095*x+16
5504321654747075)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2), x)
```

3.78 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}\,dx$

Optimal. Leaf size=429

$$-\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{69120}-\frac{427\sqrt{2-3x}\,\sqrt{-}$$

```
[Out] -427/1440*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/20*(7+
5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-145131624827/88957440
00*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/3
9*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(
-5+2*x)^(1/2)/(1+4*x)^(1/2)-1471781/51200*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*
x)^(1/2)/(-5+2*x)^(1/2)-267029/69120*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)*(7+5*x)^(1/2)-982275517/95385600*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*
(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(
1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1
/2)/((7+5*x)/(5-2*x))^(1/2)+1471781/102400*EllipticE(1/23*897^(1/2)*(1+4*x)
^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5
-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {167, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\frac{1471781\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{102400\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-\frac{145131624827(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{20736000\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}-\frac{982275517\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{4147200\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{20}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{5/2}-\frac{427\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}}{1440}-\frac{267029\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}{69120}-\frac{1471781\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{51200\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] (-1471781*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(51200*Sqrt[-5 + 2*x])
 - (267029*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/69120
- (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/1440 + (
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/20 + (1471781*S
qrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(102400*Sqrt[(2 - 3*x)/(5 - 2*
x)]*Sqrt[7 + 5*x]) - (982275517*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[
Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(4147200*Sqrt[-5 + 2*x]*Sq
rt[(7 + 5*x)/(5 - 2*x)]) - (145131624827*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)
]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[
```

```
7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(20736000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[
1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
```

```
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}\,dx = \frac{1}{20}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2} + \frac{1}{40}\int \frac{(7+\cdots}{\sqrt{\cdots}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{1440} + \frac{1}{20}\sqrt{2-\cdots}$$

$$= -\frac{267029\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{69120} - \frac{427\sqrt{\cdots}}{\cdots}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x\cdots}}{\cdots}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x\cdots}}{\cdots}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x\cdots}}{\cdots}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}} - \frac{267029\sqrt{2-3x\cdots}}{\cdots}$$

Mathematica [A]
time = 44.12, size = 345, normalized size = 0.80

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(7391284182\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-5426733148\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(186(3497259535+16491468251x+9107809874x^2-4479491480x^3-3503236800x^4+40320000x^5+414720000x^6)-4681665317\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)\right)}{514252800\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] -1/514252800*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7391284182*Sqrt[682]*Sqrt[(-5 -
 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/
39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 5426733148*Sqrt[682]*Sqrt[(-5 -
18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/3
9]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(
3497259535 + 16491468251*x + 9107809874*x^2 - 4479491480*x^3 - 3503236800*x
^4 + 40320000*x^5 + 414720000*x^6) - 4681665317*Sqrt[682]*(2 - 3*x)^2*Sqrt[
(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[11
7/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 -
3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(392) = 784$.

time = 0.15, size = 836, normalized size = 1.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/409204224000*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(97
2452761830*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x
))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)
/(-2+3*x))^(1/2),1/39*I*897^(1/2))+2612369246886*(-253*(7+5*x)/(-2+3*x))^(1
/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^
(1/2)*x^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(
1/2))-2301431308605*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x
)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-1296603682440*(-253*(7+5*x)/(-2
+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(
-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1
/2))-3483158995848*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)
/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*
(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+3068575078140*(-253*(7+5*x
)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4
*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*8
97^(1/2))+432201227480*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253
*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+1161052998616*(-253*(7+5*x)/(-2+
3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-
2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*8
97^(1/2))-1022858359380*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5
+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+61380633600000*x^6+596756160000
```

```
0*x^5-518496562584000*x^4-662987136497400*x^3+1348001400401370*x^2+24408197
58489255*x+517611897477675)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2), x)
```

3.79 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,dx$

Optimal. Leaf size=391

$$-\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}}+\frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}-\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}$$

```
[Out] -65750101/92664000*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^
(1/2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(
1/2)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-13027/4800*(2-3*x)^(1/2)*(1+4*x
)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-1/9*(2-3*x)^(3/2)*(-5+2*x)^(1/2)*(1+4*
x)^(1/2)*(7+5*x)^(1/2)+23/240*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7
+5*x)^(1/2)-1368371/993600*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-
3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3
*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x
)/(5-2*x))^(1/2)+13027/9600*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {167, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\frac{13027\sqrt{\frac{143}{3}}\sqrt{\frac{5x+7}{5-2x}}\sqrt{2-3x}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{3200\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{65750101\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}(2-3x)\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{216000\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}-\frac{1368371\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{43200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}(2-3x)^{3/2}+\frac{23}{240}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}-\frac{13027\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}}{4800\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

```
[Out] (-13027*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4800*Sqrt[-5 + 2*x]) +
(23*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/240 - ((2 - 3
*x)^(3/2)*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/9 + (13027*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(3200*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1368371*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(43200*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) - (65750101*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(216000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,dx = -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{1}{18}\int \frac{\sqrt{2}}{\sqrt{}}$$

$$= \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{1}{9}(2-3x)^{3}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5}$$

Mathematica [A]
time = 39.88, size = 340, normalized size = 0.87

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(7269066\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-4532324\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(186(3848705+17658613x+7278862x^2-7723240x^3-2184000x^4+1152000x^5)-2120971\sqrt{682}(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)\right)}{5356800\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

```
[Out] -1/5356800*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7269066*Sqrt[682]*Sqrt[(-5 - 18*x
 + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*S
qrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 4532324*Sqrt[682]*Sqrt[(-5 - 18*x + 8
*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[
```

```
(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(3848705
+ 17658613*x + 7278862*x^2 - 7723240*x^3 - 2184000*x^4 + 1152000*x^5) - 212
0971*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10
*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-
2 + 3*x)]], 39/62]))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x
)]*(-5 - 18*x + 8*x^2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(362) = 724$.
time = 0.14, size = 831, normalized size = 2.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4262544000*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)*(1354
687290*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2
+3*x))^(1/2),1/39*I*897^(1/2))+1183501818*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^
(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x
^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-2
263376115*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x)
)^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/
(-2+3*x))^(1/2),1/39*I*897^(1/2))-1806249720*(-253*(7+5*x)/(-2+3*x))^(1/2)*
13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2
)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-15780024
24*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x)
)^(1/2),-69/55,1/39*I*897^(1/2))+3017834820*(-253*(7+5*x)/(-2+3*x))^(1/2)*1
3^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)
*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+602083240
*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*2
3^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/
2),1/39*I*897^(1/2))+526000808*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/
2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1
/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-1005944940*(-253
*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2
)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/3
9*I*897^(1/2))+170501760000*x^5-323242920000*x^4-1143078136200*x^3+10773079
70310*x^2+2613563017065*x+569627583525)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2), x)
```

3.80 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{\sqrt{7+5x}}\,dx$

Optimal. Leaf size=351

$$-\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}}+\frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}+\frac{427\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{7+}{5-}}}{400\sqrt{}}$$

```
[Out] 1008833/3861000*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-427/600*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)*(7+5*x)^(1/2)-20057/41400*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)+427/1200*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x
)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/(
(2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {167, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\frac{427\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{400\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{1008833(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{9000\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}-\frac{20057\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1800\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{10}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}-\frac{427\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{600\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (-427*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(600*Sqrt[-5 + 2*x]) + (Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + (427*Sqrt[143/
3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(400*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (20057*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]
/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1800*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (1008833*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(9000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

<u>Rule 171</u>

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

<u>Rule 176</u>

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

<u>Rule 182</u>

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

<u>Rule 429</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{1}{20}\int \frac{-3-}{\sqrt{2-3x}\,\sqrt{-5}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

Mathematica [**A**]
time = 28.77, size = 347, normalized size = 0.99

$$\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(66960(2-3x)+\frac{-238266\sqrt{682}(-2+3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+117924\sqrt{682}(-2+3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-7\sqrt{\frac{7+5x}{-2+3x}}\left(-102114(-35-151x-34x^2+40x^3)+13947\sqrt{682}(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{(2-3x)\left(\frac{7+5x}{-2+3x}\right)^{3/2}(5+18x-8x^2)}\right)}{669600\sqrt{2-3x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) + (-238266*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 117924*Sqrt[682]
*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSi
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 7*Sqrt[(7 + 5*x)/(-2 +
 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) + 13947*Sqrt[682]*(2 - 3*x)
^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Ellip
```

```
ticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/((2
 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(669600*Sqrt[2 -
 3*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(332) = 664.
time = 0.16, size = 826, normalized size = 2.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/177606000*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)*(19856
430*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2
)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*
x))^(1/2),1/39*I*897^(1/2))-18158994*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)
*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*El
lipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-247297
05*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))-26475240*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Ellip
ticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+24211992*(-253*(7
+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*(
(1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69
/55,1/39*I*897^(1/2))+32972940*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/
2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(
1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+8825080*(-253*(7+5*x)/
(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x
)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(
1/2))-8070664*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+
3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)
/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-10990980*(-253*(7+5*x)/(-2+3*x))^
(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x)
)^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+2131
272000*x^4-10816205400*x^3-391621230*x^2+32127593355*x+7879046175)/(120*x^4
-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/sqrt(5*x + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2), x)
```

3.81 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5\sqrt{7+5x}}+\frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{25\sqrt{-5+2x}}-\frac{3\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\right.\right.}{25\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+}}$$

```
[Out] -26474/160875*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2)
,-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*
429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-2/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)/(7+5*x)^(1/2)+6/25*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5
+2*x)^(1/2)+296/1725*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^
(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(
1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2
*x))^(1/2)-3/25*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*
I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*
x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {166, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$-\frac{3\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{25\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{375\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}+\frac{296\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{75\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{6\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{25\sqrt{2x-5}}-\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{5\sqrt{5x+7}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) + (6*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(25*Sqrt[-5 + 2*x]) - (3*Sqrt[429]*S
qrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1
 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(25*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5
*x]) + (296*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(75*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
- (26474*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*E
llipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39
])/(375*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{1}{5}\int \frac{-21+140x-}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{25\sqrt{-5+2x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{25\sqrt{-5+2x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{25\sqrt{-5+2x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{25\sqrt{-5+2x}}$$

Mathematica [A]
time = 37.47, size = 330, normalized size = 0.95

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(-558\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+262\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(186(-415-1569x+394x^2+120x^3)-427\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)\right)}{4650\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] -1/4650*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-558*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x
^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62] + 262*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)
/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-415 - 1569*x + 39
4*x^2 + 120*x^3) - 427*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqr
t[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*
```

```
Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(
7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(332) = 664$.
time = 0.13, size = 821, normalized size = 2.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3700125*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)*(146520
*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*2
3^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))
^(1/2),1/39*I*897^(1/2))-238266*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1
/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*Ellipti
cPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-173745*(-25
3*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/
2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2
),1/39*I*897^(1/2))-195360*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*(
(-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23
*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+317688*(-253*(7+5*x)/(-2+3
*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2
+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*
897^(1/2))+231660*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/
(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7
+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+65120*(-253*(7+5*x)/(-2+3*x))^(1/2)
*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/
2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-105896*(-
253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(
1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2)
,-69/55,1/39*I*897^(1/2))-77220*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1
/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1
/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-17760600*x^3-58313970*x
^2+232219845*x+61422075)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 +
 70*x + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2), x)
```

3.82 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}-\frac{35812\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{2085525\sqrt{-5+2x}}$$

```
[Out] -2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+496/53625*(2
-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I*8
97^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5+2*
x)^(1/2)/(1+4*x)^(1/2)+17906/417105*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)/(7+5*x)^(1/2)-35812/2085525*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(
-5+2*x)^(1/2)-496/39675*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x
))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x)
)^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(
5-2*x))^(1/2)+17906/2085525*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = $\{166, 1618, 1616, 1612, 176, 429, 171, 551, 182, 435\}$

$$\frac{17906\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{53475\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{496(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{125\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}-\frac{496\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1725\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{35812\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2085525\sqrt{2x-5}}+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{417105\sqrt{5x+7}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^(3/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(417105*Sqrt[7 + 5*x]) - (35
812*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2085525*Sqrt[-5 + 2*x]) + (
17906*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[
(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(53475*Sqrt[(2 - 3*x)
/(5 - 2*x)]*Sqrt[7 + 5*x]) - (496*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTa
n[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1725*Sqrt[-5 + 2*x]*Sqr
t[(7 + 5*x)/(5 - 2*x)]) + (496*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((
1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/
Sqrt[2 - 3*x]], -23/39])/(125*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
```

```
+ c*e*h))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{1}{15}\int \frac{-21+140x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{417105\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{417105\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{417105\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{417105\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{15(7+5x)^{3/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{417105\sqrt{7+5x}}$$

Mathematica [A]
time = 34.42, size = 366, normalized size = 0.94

$$\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(\frac{30\sqrt{6-9x}\,(34864+44765x)}{(7+5x)^2}-\frac{2\left(80577\sqrt{682}\,(-2+3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)-37053\sqrt{682}\,(-2+3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(-241731(-35-151x-34x^2+40x^3)+48438\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)\right)}{\sqrt{3}\,(2-3x)^{3/2}\left(\frac{7+5x}{-2+3x}\right)^{3/2}(5+18x-8x^2)}\right)}{6256575\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*((30*Sqrt[6 - 9*x]*(34864 + 447 65*x))/(7 + 5*x)^2 - (2*(80577*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18 *x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 37053*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8* x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]] , 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-241731*(-35 - 151*x - 34*x^2 + 40*x ^3) + 48438*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11 *x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[3]*(2 - 3*x)^(3/2)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(6256575*Sqrt[3])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(362) = 724$.
time = 0.14, size = 1077, normalized size = 2.75

method	result
elliptic	$\dfrac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{375\left(x+\frac{7}{5}\right)^2}+\frac{\cdots}{\sqrt{(x\cdots}}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x,method=_RETU RNVERBOSE)

[Out] -2/2638189125*(23871240*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5 +2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-2 53*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))*x^3-19027800*(-253*(7+5 *x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1 +4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*8

```
97^(1/2))*x^3+22158675*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253
*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3+1591416*(-253*(7+5*x)/(-2+3*
x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+
3*x))^(1/2)*x^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I
*897^(1/2))-1268520*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x
)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+1477245*(-253*(7+5*x)/(-2+3*x))
^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x
))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))
-33950208*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x)
)^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(
-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+27061760*(-253*(7+5*x)/(-2+3*x))^(1
/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^
(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-3151
4560*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/
2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))+14853216*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ellipti
cPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-11839520*(-
253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(
1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),
1/39*I*897^(1/2))+13787620*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*(
(-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(
-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+18264778400*x^3-24248673680*
x^2-49321411370*x-10529423575)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(1/2)/(
120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3
+ 525*x^2 + 735*x + 343), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{(5x+7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2), x)
```

3.83 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}}+\frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}}+\frac{1426348\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2319687747\sqrt{7+5x}}$$

```
[Out] -2/25*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+17906/208552
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+1426348/23196877
47*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)-2852696/1159843
8735*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-48884/2206485
45*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1
+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1
/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+1426348
/11598438735*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*8
97^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))
^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {166, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$\frac{1426348\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{297395865\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-\frac{48884\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{9593415\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}-\frac{2852696\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{11598438735\sqrt{2x-5}}+\frac{1426348\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{2319687747\sqrt{5x+7}}+\frac{17906\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{2085525(5x+7)^{3/2}}-\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(25*(7 + 5*x)^(5/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2085525*(7 + 5*x)^(3/2)) +
(1426348*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5
*x]) - (2852696*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(11598438735*Sqr
t[-5 + 2*x]) + (1426348*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]
*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(29
7395865*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (48884*Sqrt[11/23]*Sqrt[
7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/
(9593415*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{1}{25}\int \frac{-21+140x -}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2085525(7+5x)^{3/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2085525(7+5x)^{3/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2085525(7+5x)^{3/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2085525(7+5x)^{3/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2085525(7+5x)^{3/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{2085525(7+5x)^{3/2}}$$

<u>Mathematica</u> [A]
time = 39.74, size = 251, normalized size = 0.76

$$-\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}\,(137502130+880765228x+1137407943x^2-729949210x^3+50105384x^4)+713174\sqrt{682}\,(-2+3x)(7+5x)^3\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-236555\sqrt{682}\,(-2+3x)(7+5x)^3\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{11598438735\sqrt{2-3x}\,(7+5x)^{5/2}\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(137502130
+ 880765228*x + 1137407943*x^2 - 729949210*x^3 + 50105384*x^4) + 713174*Sqr
```

```
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
cE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 236555*Sqrt[68
2]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(11598438735*Sqrt[
2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(311) = 622$.
time = 0.13, size = 913, normalized size = 2.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/266764090905*(170482950*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((
-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-
253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^4-160464150*(-253*(7+5*x)/(
-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)
/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1
/2))*x^4+250041660*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)
/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+
5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-235347420*(-253*(7+5*x)/(-2+3*x)
)^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*
x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^
3-226552898*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*
x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x
)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+213239026*(-253*(7+5*x)/(-2+3*x))^(1/2)
*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/
2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-23337
2216*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/
2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))+219657592*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)
*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Elli
pticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+148509592*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39
*I*897^(1/2))-139782104*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5
+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+35725138792*x^4-520453786730*x^
3+810971863359*x^2+627985607564*x+98039018690)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)
*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4
+ 3500*x^3 + 7350*x^2 + 6860*x + 2401), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2), x)
```

3.84 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}}+\frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}}+\frac{23758016\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}{57992193675(7+5x)^{3/2}}$$

```
[Out] -2/35*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+2558/695175*
(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+23758016/579921936
75*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+32843987836/451
524900265803*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)-65687
975672/2257624501329015*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^
(1/2)-1212290288/42949018635705*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x
)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)
/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((
7+5*x)/(5-2*x))^(1/2)+32843987836/2257624501329015*EllipticE(1/23*897^(1/2)
*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7
+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {166, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$\frac{32843987836\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{57887807726385\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{1212290288\sqrt{\frac{11}{23}}\sqrt{5x+7}F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1867348636335\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{65687975672\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2257624501329015\sqrt{2x-5}}+\frac{32843987836\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{451524900265803\sqrt{5x+7}}+\frac{23758016\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{57992193675(5x+7)^{3/2}}+\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{695175(5x+7)^{5/2}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(35*(7 + 5*x)^(7/2)) + (255
8*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(695175*(7 + 5*x)^(5/2)) + (2
3758016*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(57992193675*(7 + 5*x)^
(3/2)) + (32843987836*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(45152490
0265803*Sqrt[7 + 5*x]) - (65687975672*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 +
5*x])/(2257624501329015*Sqrt[-5 + 2*x]) + (32843987836*Sqrt[11/39]*Sqrt[2 -
 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x]
)/Sqrt[-5 + 2*x]], -23/39])/(57887807726385*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1212290288*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1867348636335*Sqrt[-5 + 2*x]*Sqrt
[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d
*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{1}{35}\int \frac{-21+140x-}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{695175(7+5x)^{5/2}}$$

Mathematica [A]
time = 34.95, size = 259, normalized size = 0.70

$$\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(-\frac{(-2+3x)(15395515423270+113490310442229x+54668919175710x^2+10263746198750x^3)}{(7+5x)^4}+\frac{242\left(203578437\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)-67859479\sqrt{682}(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+19017205\sqrt{682}(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)}{\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)}\right)}{2257624501329015\sqrt{2-3x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-(((-2 + 3*x)*(1539551542327
0 + 113490310442229*x + 54668919175710*x^2 + 10263746198750*x^3))/(7 + 5*x)
^4) + (242*(203578437*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 6785
9479*Sqrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 19017205*Sqrt[682]
*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/
39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5
- 18*x + 8*x^2))))/(2257624501329015*Sqrt[2 - 3*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(341) = 682$.
time = 0.14, size = 1088, normalized size = 2.94

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{21875\left(x+\frac{7}{5}\right)^4}+\frac{2558\sqrt{}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/51925363530567345*(21139311897000*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ellipti
cF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^5-18474743157750*
(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23
^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2
```

```
),1/39*I*897^(1/2))*x^5+60599360771400*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/
2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Elli
pticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^4-529609303855
50*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(
1/2),1/39*I*897^(1/2))*x^4+15314257063160*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^
(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*E
llipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-133839250
43170*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1
/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x)
)^(1/2),1/39*I*897^(1/2))*x^3-68265884552712*(-253*(7+5*x)/(-2+3*x))^(1/2)*
13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2
)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+596611
03904094*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))
^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(
-2+3*x))^(1/2),1/39*I*897^(1/2))-22097627369664*(-253*(7+5*x)/(-2+3*x))^(1/
2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(
1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+19312
264847568*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x)
)^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-
2+3*x))^(1/2),1/39*I*897^(1/2))+25780565264608*(-253*(7+5*x)/(-2+3*x))^(1/2
)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1
/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-22530975
655496*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))-5327107465724080*x^5-12112832818781292*x^4-37899
804984700580*x^3+20239739893753473l*x^2+121375813944103236*x+17144045099357
810)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+19
7*x+70)/(7+5*x)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3125*x^5
 + 21875*x^4 + 61250*x^3 + 85750*x^2 + 60025*x + 16807), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2), x)
```

3.85 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=429

$$\frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}}+\frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{27648}+\frac{1445}{576}\sqrt{2-3x}\,\sqrt{-}$$

```
[Out] 1445/576*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/8*(7+5*
x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+331574321009/711659520*
(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I
*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5+
2*x)^(1/2)/(1+4*x)^(1/2)+2466927/4096*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(
1/2)/(-5+2*x)^(1/2)+1561915/27648*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)+861015607/7630848*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4
*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*
x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/
((7+5*x)/(5-2*x))^(1/2)-2466927/8192*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)
/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))
^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {168, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$-\frac{2466927\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)\middle|-\frac{23}{39}\right)}{8192\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{331574321009(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)\middle|-\frac{23}{39}\right)}{1658880\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}+\frac{861015607\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)\middle|-\frac{39}{23}\right)}{331776\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{8}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}+\frac{1445}{576}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}+\frac{1561915\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}{27648}+\frac{2466927\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4096\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] (2466927*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4096*Sqrt[-5 + 2*x]) +
 (1561915*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/27648 +
 (1445*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/576 + (S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/8 - (2466927*Sqr
t[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23
]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8192*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (861015607*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(331776*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)]) + (331574321009*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqr
```

```
t[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5
*x])/Sqrt[2 - 3*x]], -23/39])/(1658880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*
x])
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
```

```
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \frac{1}{8}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2} - \frac{1}{16}\int \frac{(7+5x)^{3/2}\,(-6}{\sqrt{2-3x}\,\sqrt{}}$$

$$= \frac{1445}{576}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\,\sqrt{-5+}$$

$$= \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{27648} + \frac{1445}{576}\sqrt{2-3x}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2}}{27648}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2}}{27648}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2}}{27648}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2}}{27648}$$

Mathematica [A]

time = 40.17, size = 345, normalized size = 0.80

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(-12388907394\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+10666876180\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(186(-5752341805-26349657233x-12645389558x^2+3088122056x^3+1004819520x^4+439372800x^5+82944000x^6)+10695945839\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)\right)}{41140224\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -1/41140224*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-12388907394*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 10666876180*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186
*(-5752341805 - 26349657233*x - 12645389558*x^2 + 3088122056*x^3 + 10048195
20*x^4 + 439372800*x^5 + 82944000*x^6) + 10695945839*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt
[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(392) = 784.
time = 0.14, size = 836, normalized size = 1.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32736337920*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(85
2405450930*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x
))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)
/(-2+3*x))^(1/2),1/39*I*897^(1/2))+5968337778162*(-253*(7+5*x)/(-2+3*x))^(1
/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^
(1/2)*x^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(
1/2))-3857546084535*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x
)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-1136540601240*(-253*(7+5*x)/(-2
+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(
-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1
/2))-7957783704216*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)
/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*
(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+5143394779380*(-253*(7+5*x
)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4
*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*8
97^(1/2))+378846867080*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253
*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+2652594568072*(-253*(7+5*x)/(-2+
3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-
```

```
2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*8
97^(1/2))-1714464926460*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5
+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-12276126720000*x^6-650293712640
00*x^5-148718313057600*x^4-457057504898280*x^3+1871580881531790*x^2+3899881
018770165*x+851375348849025)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sq
rt(2*x - 5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{\left(5\,x+7\right)}^{5/2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(5/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(5/2))/(2*x - 5)^(1/2), x)
```

3.86 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=391

$$\frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}}+\frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}+\frac{1}{6}\sqrt{2-3x}\,\sqrt{-5+2x}\,\surd$$

```
[Out] 1/6*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+963142751/3706
5600*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1
/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)
/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+66377/1920*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x
)^(1/2)/(-5+2*x)^(1/2)+977/288*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(
7+5*x)^(1/2)+2824441/397440*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2
-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-
3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)-66377/3840*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x
)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/(
(2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {168, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$-\frac{66377\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{1280\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{963142751(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{86400\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}+\frac{2824441\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{17280\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{6}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{977}{288}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}+\frac{66377\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1920\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] (66377*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1920*Sqrt[-5 + 2*x]) + (
977*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/288 + (Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/6 - (66377*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1280*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) + (2824441*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(17280*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) + (963142751*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*
x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
 - 3*x]], -23/39])/(86400*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 168

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^m*Sqrt[c + d
*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
```

```
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \frac{1}{6}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2} - \frac{1}{12}\int \frac{\sqrt{7+5x}\,(-465}{\sqrt{2-3x}\,\sqrt{-5}}$$

$$= \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

Mathematica [A]
time = 38.23, size = 340, normalized size = 0.87

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(-37038366\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+31389484\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(186(-17232355-79187903x-38640362x^2+10641080x^3+4555200x^4+1152000x^5)+31069121\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)\right)}{2142720\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -1/2142720*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-37038366*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 31389484*Sqrt[682]*Sqrt[(-5 - 18*x
+ 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-1723
2355 - 79187903*x - 38640362*x^2 + 10641080*x^3 + 4555200*x^4 + 1152000*x^5
) + 31069121*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 1
1*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(
-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(362) = 724$.
time = 0.14, size = 831, normalized size = 2.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/1705017600*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(279
6196590*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^
(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-
2+3*x))^(1/2),1/39*I*897^(1/2))+17336569518*(-253*(7+5*x)/(-2+3*x))^(1/2)*1
3^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)
*x^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))
-11532671865*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3
*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*
x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-3728262120*(-253*(7+5*x)/(-2+3*x))^(1/
2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(
1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-23115
426024*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+
3*x))^(1/2),-69/55,1/39*I*897^(1/2))+15376895820*(-253*(7+5*x)/(-2+3*x))^(1
/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^
(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+1242
754040*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))+7705142008*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2
)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ellip
ticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-512563194
0*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*
23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1
/2),1/39*I*897^(1/2))-170501760000*x^5-674192376000*x^4-1574933045400*x^3+5
718966777810*x^2+11720205583515*x+2550474701775)/(120*x^4-182*x^3-385*x^2+1
97*x+70)
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{\left(5\,x+7\right)}^{3/2}}{\sqrt{2\,x-5}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2), x)
```

3.87 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=351

$$\frac{509\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{240\sqrt{-5+2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}-\frac{509\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{7+5}{5-2}}}{160\sqrt{}}$$

```
[Out] 2198489/1544400*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+509/240*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+1/4*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/
2)*(7+5*x)^(1/2)+8959/16560*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2
-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-
3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)-509/480*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(
1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-
3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {167, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$-\frac{509\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{160\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{2198489(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{3600\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}+\frac{8959\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{720\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}+\frac{509\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{240\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (509*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(240*Sqrt[-5 + 2*x]) + (Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/4 - (509*Sqrt[143/3]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(160*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (8959*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(S
qrt[2]*Sqrt[2 - 3*x])], -39/23])/(720*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*
x)]) + (2198489*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3
*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]],
 -23/39])/(3600*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b
*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*
g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx &= \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}+\frac{1}{8}\int \frac{309-410x}{\sqrt{2-3x}\,\sqrt{-5+2x}} \\
&= \frac{509\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{240\sqrt{-5+2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\surd \\
&= \frac{509\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{240\sqrt{-5+2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\surd \\
&= \frac{509\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{240\sqrt{-5+2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\surd \\
&= \frac{509\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{240\sqrt{-5+2x}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\surd
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 30.22, size = 347, normalized size = 0.99

$$
\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(66960(2-3x)-\frac{3\left(94674\sqrt{682}(2-3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+76756\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(284022(-35-151x-34x^2+40x^3)+70919\sqrt{682}(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)\right)}{(2-3x)\left(\frac{7+5x}{-2+3x}\right)^{3/2}(5+18x-8x^2)}\right)}{267840\sqrt{2-3x}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) - (3*(94674*Sq
rt[682]*(2 - 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 76756*Sqrt[682]*
Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSi
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3
*x)]*(284022*(-35 - 151*x - 34*x^2 + 40*x^3) + 70919*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
```

```
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/((2 -
 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(267840*Sqrt[2 - 3
*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(332) = 664$.
time = 0.14, size = 826, normalized size = 2.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/71042400*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)*(-5+2*x)^(1/2)*(88694
10*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))+39572802*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*Ell
ipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-2947873
5*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*
23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x)
)^(1/2),1/39*I*897^(1/2))-11825880*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3
^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Ellipt
icF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-52763736*(-253*(7+
5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((
1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/
55,1/39*I*897^(1/2))+39304980*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2
)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1
/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+3941960*(-253*(7+5*x)/(
-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)
/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1
/2))+17587912*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+
3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)
/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-13101660*(-253*(7+5*x)/(-2+3*x))^
(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x)
)^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-5807
716200*x^3-2131272000*x^4+14521954590*x^2+30627710685*x+6666885225)/(120*x^
4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*sqrt(5*x + 7)/sqrt(2*x - 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{5\,x+7}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2), x)
```

3.88 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,\sqrt{7+5x}}\,dx$

Optimal. Leaf size=365

$$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{5\sqrt{-5+2x}}-\frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}+\frac{7\sqrt{\frac{11}{23}}\,\sqrt{\ }}{}$$

```
[Out] 41/1240*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(1/
2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+2*
x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/2)
/(1+4*x)^(1/2)+943/68200*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5
*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+
506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/2)/
((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+1/5*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5
*x)^(1/2)/(-5+2*x)^(1/2)+7/230*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)-1/10*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(
1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-
3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$,
Rules used = {179, 182, 435, 171, 550, 429, 553, 176}

$$-\frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{7\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{10\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{41\sqrt{\frac{11}{62}}\,\sqrt{2-3x}\,F\left(\text{ArcTan}\left(\frac{\sqrt{\frac{22}{23}}\,\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{4x+1}}\,\sqrt{4x+1}}+\frac{943\sqrt{2-3x}\,\Pi\left(\frac{78}{55};\text{ArcTan}\left(\frac{\sqrt{\frac{22}{23}}\,\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{100\sqrt{682}\,\sqrt{-\frac{2-3x}{4x+1}}\,\sqrt{4x+1}}+\frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{5\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(10*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) + (7*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(10*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
+ (41*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(20*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```

```
+ (943*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/S
qrt[-5 + 2*x]], 39/62])/(100*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 550

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[-f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,\sqrt{7+5x}}\,dx = \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{41}{20}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx + \frac{77}{20}\int$$

$$= \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\left(1599\sqrt{-\frac{2-3x}{-5+2x}}\,(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)}{10\sqrt{7}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}}{\sqrt{-5}}\right.\right.}{10\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}}{\sqrt{-5}}\right.\right.}{10\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}$$

Mathematica [A]
time = 4.81, size = 318, normalized size = 0.87

$$\frac{\sqrt{2-3x}\left(-3410\sqrt{682}\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{1+4x}{7+5x}}(-14+11x+15x^2)E\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{77+55x}}\right)\middle|\frac{23}{62}\right)+1984\sqrt{682}\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{1+4x}{7+5x}}(-14+11x+15x^2)F\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{77+55x}}\right)\middle|\frac{23}{62}\right)+\sqrt{\frac{-2+3x}{7+5x}}\left(17050(10+21x-70x^2+24x^3)-1599\sqrt{682}\sqrt{\frac{1+4x}{7+5x}}(7+5x)^2\sqrt{\frac{-10+19x-6x^2}{(7+5x)^2}}\,\Pi\left(-\frac{55}{62};\sin^{-1}\left(\sqrt{\frac{155-62x}{77+55x}}\right)\middle|\frac{23}{62}\right)\right)\right)}{34100\sqrt{-5+2x}\sqrt{1+4x}\left(\frac{-2+3x}{7+5x}\right)^{3/2}(7+5x)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*(-3410*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7
 + 5*x)]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*
x)]], 23/62] + 1984*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7 +
 5*x)]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)
]], 23/62] + Sqrt[(-2 + 3*x)/(7 + 5*x)]*(17050*(10 + 21*x - 70*x^2 + 24*x^3
) - 1599*Sqrt[682]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^2*Sqrt[(-10 + 19*x -
 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x
)]], 23/62])))/(34100*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*((-2 + 3*x)/(7 + 5*x))^(
3/2)*(7 + 5*x)^(3/2))
```

Maple [A]
time = 0.13, size = 821, normalized size = 2.25

method	result
elliptic	$\frac{\sqrt{-\left(7+5x\right)\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\left(\frac{4\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^{2}\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)\left(x\right.}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-5+2x}\,\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/1480050*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)*(-5+2*x)^(1/2)*(30690*
(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23
^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^
(1/2),1/39*I*897^(1/2))+22878*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2
)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticP
i(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-57915*(-253*(
7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*
((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1
/39*I*897^(1/2))-40920*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-2
53*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-30504*(-253*(7+5*x)/(-2+3*x))^
(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x)
)^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(
1/2))+77220*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*
x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/
(-2+3*x))^(1/2),1/39*I*897^(1/2))+13640*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1
/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ell
ipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+10168*(-253*(7+
5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((
1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55
,1/39*I*897^(1/2))-25740*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-
5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-2
53*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-17760600*x^3+15096510*x^2+6704
6265*x+15540525)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 -
 11*x - 35), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*sqrt(5*x + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

3.89 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=279

$$\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}}-\frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}}+\frac{2\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\sqrt{\right.\right.}{5\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}$$

```
[Out] -69/8525*(1+4*x)*EllipticPi(1/39*858^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2),78/5
5,1/62*2418^(1/2))*682^(1/2)*((-2+3*x)/(1+4*x))^(1/2)*((-5+2*x)/(1+4*x))^(1
/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)+2/39*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)/(7+5*x)^(1/2)-4/195*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x
)^(1/2)+2/195*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*
897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x)
)^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {170, 1600, 1609, 171, 551, 182, 435}

$$\frac{2\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-\frac{69\sqrt{\frac{2}{341}}\,\sqrt{-\frac{2-3x}{4x+1}}\,\sqrt{-\frac{5-2x}{4x+1}}\,(4x+1)\Pi\left(\frac{78}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{22}{39}}\,\sqrt{5x+7}}{\sqrt{4x+1}}\right)|\frac{39}{62}\right)}{25\sqrt{2-3x}\,\sqrt{2x-5}}-\frac{4\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{195\sqrt{2x-5}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{39\sqrt{5x+7}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*Sqrt[7 + 5*x]) - (4*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(195*Sqrt[-5 + 2*x]) + (2*Sqrt[11/39
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) - (69*Sqrt[2/341]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[-((5 - 2*x)/(1 +
 4*x))]*(1 + 4*x)*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt
[1 + 4*x]], 39/62])/(25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
```

```
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1609

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*
B*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*Sqrt[a + b*x])), x] + (
-Dist[B*((b*g - a*h)/(2*f*h)), Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[g + h*x]), x], x] + Dist[B*(b*e - a*f)*((b*g - a*h)/(2*d*f*h)), Int
[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{3/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{-25+130x-48x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7-}}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{(5-24x)\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{3}{5}\int \frac{}{\sqrt{2}}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{\left(46\sqrt{\frac{}{4}}\right.}{}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}} + \frac{2\sqrt{\frac{11}{39}}}{}$$

Mathematica [A]
time = 17.62, size = 326, normalized size = 1.17

$$\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(-62\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+23\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)-2\sqrt{\frac{7+5x}{-2+3x}}\left(-961(-5-18x+8x^2)+39\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)\right)}{6045\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/
(-2 + 3*x)]], 39/62] + 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(
```

```
-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x
)]], 39/62] - 2*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-961*(-5 - 18*x + 8*x^2) + 39*S
qrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/
(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*
x)]], 39/62])))/(6045*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)
]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(216) = 432$.
time = 0.13, size = 816, normalized size = 2.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/246675*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)*(-5+2*x)^(1/2)*(495*(-25
3*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/
2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2
),1/39*I*897^(1/2))-1116*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-
5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticPi(1/2
3*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-495*(-253*(7+5*x)/
(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x
)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*8
97^(1/2))-660*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+
3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x
)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+1488*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(
1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*
EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+660*
(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23
^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1
/2),1/39*I*897^(1/2))+220*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((
-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-
253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-496*(-253*(7+5*x)/(-2+3*x))^(
1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))
^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2
))-220*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))-313720*x^2+705870*x+196075)/(120*x^4-182*x^3-385
*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 +
 15*x^2 - 252*x - 245), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

3.90 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}}-\frac{9350\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3253419\sqrt{7+5x}}+\frac{3740\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3253419\sqrt{-5+2x}}-$$

```
[Out] 2/117*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-9350/3253419
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+3740/3253419*(2-3
*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+44/61893*(1/(4+2*(1+4*
x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1
/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(
7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-1870/3253419*EllipticE(
1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-
3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {170, 1613, 1616, 12, 176, 429, 182, 435}

$$-\frac{1870\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{83421\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{44\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{2691\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{3740\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{3253419\sqrt{2x-5}}-\frac{9350\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{3253419\sqrt{5x+7}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(117*(7 + 5*x)^(3/2)) - (935
0*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(3253419*Sqrt[7 + 5*x]) + (37
40*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3253419*Sqrt[-5 + 2*x]) - (1
870*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(S
qrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(83421*Sqrt[(2 - 3*x)/(
5 - 2*x)]*Sqrt[7 + 5*x]) + (44*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[S
qrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(2691*Sqrt[-5 + 2*x]*Sqrt[(
7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
```

```
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{1}{117}\int \frac{-33+110x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7-$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3253419\sqrt{7+5x}} - \frac{\int}{}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{37}{}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{37}{}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{37}{}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{37}{}$$

Mathematica [A]
time = 27.22, size = 246, normalized size = 0.85

$$-\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}\,(-23755-122348x-94580x^2+58928x^3)-935\sqrt{682}\,(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+506\sqrt{682}\,(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{3253419\sqrt{2-3x}\,(7+5x)^{3/2}\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-23755 - 1
22348*x - 94580*x^2 + 58928*x^3) - 935*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqr
t[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62] + 506*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[(-5 -
```

```
18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-
2 + 3*x)]], 39/62]))/(3253419*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(281) = 562$.
time = 0.13, size = 738, normalized size = 2.54

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{2925\left(x+\frac{7}{5}\right)^2}-\frac{\quad}{3253419}\right.}{}$

default	$-\frac{2\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-42075\sqrt{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/74828637*(30690*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)
/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+
5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-42075*(-253*(7+5*x)/(-2+3*x))^(1
/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^
(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3+20
46*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))-2805*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1
/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*Ellipti
cE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-43648*(-253*(7+5*x)
/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*
x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*89
7^(1/2))+59840*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2
+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*
x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+19096*(-253*(7+5*x)/(-2+3*x))^(1/2)*13
^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*
EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-26180*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39
*I*897^(1/2))-42015664*x^3+67435540*x^2+87234124*x+16937315)*(-5+2*x)^(1/2)
*(1+4*x)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(1/
2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4
+ 425*x^3 - 1155*x^2 - 2989*x - 1715), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**(5/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

3.91 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{7/2}}\,dx$

Optimal. Leaf size=330

$$\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}}-\frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}}-\frac{20464840\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}{90467822133\sqrt{7+5x}}$$

```
[Out] 2/195*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3646/1626709
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-20464840/9046782
2133*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+8185936/90467
822133*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+111628/1721
058651*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*Elliptic
F((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*89
7^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-409
2968/90467822133*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39
*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2
*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {170, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$-\frac{4092968\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{2319687747\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{111628\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{74828637\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{8185936\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{90467822133\sqrt{2x-5}}-\frac{20464840\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{90467822133\sqrt{5x+7}}-\frac{3646\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{16267095(5x+7)^{3/2}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(195*(7 + 5*x)^(5/2)) - (364
6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(16267095*(7 + 5*x)^(3/2)) -
(20464840*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(90467822133*Sqrt[7 +
 5*x]) + (8185936*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(90467822133*S
qrt[-5 + 2*x]) - (4092968*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x
)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(
2319687747*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (111628*Sqrt[11/23]*S
qrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/2
3])/(74828637*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{7/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{1}{195}\int \frac{-41+90x+48x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} -$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - 2$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - 2$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - 2$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - 2$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - 2$$

<u>**Mathematica**</u> [A]
time = 32.11, size = 251, normalized size = 0.76

$$-\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}\,(-374624540-2271416114x-2953846743x^2+643813106x^3+370051256x^4)-2046484\sqrt{682}\,(-2+3x)(7+5x)^3\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+958111\sqrt{682}\,(-2+3x)(7+5x)^3\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{90467822133\sqrt{2-3x}\,(7+5x)^{5/2}\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-374624540
 - 2271416114*x - 2953846743*x^2 + 643813106*x^3 + 370051256*x^4) - 2046484
```

```
*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ell
ipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 958111*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
cF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(90467822133*S
qrt[2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2)
)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(311) = 622$.
time = 0.13, size = 913, normalized size = 2.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/2080759909059*(460458900*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*(
(-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(
-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^4-389302650*(-253*(7+5*x)/
(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x
)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(
1/2))*x^4+675339720*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x
)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7
+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-570977220*(-253*(7+5*x)/(-2+3*x
))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3
*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x
^3-611898716*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3
*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*
x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+517339966*(-253*(7+5*x)/(-2+3*x))^(1/2
)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1
/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-6303
17072*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1
/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*
x))^(1/2),1/39*I*897^(1/2))+532912072*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2
)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Ell
ipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+401110864*(-253
*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2
)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/3
9*I*897^(1/2))-339125864*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-
5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-2
53*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+263846545528*x^4+459038744578*
x^3-2106092727759*x^2-1619519689282*x-267107297020)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1250*x^5
 + 3875*x^4 - 2800*x^3 - 23030*x^2 - 29498*x - 12005), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)), x)
```

3.92 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{9/2}}\,dx$

Optimal. Leaf size=370

$$\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}}+\frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}}-\frac{3217468\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{50259901185(7+5x)^{3/2}}$$

```
[Out] 2/273*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+98/1807455*(
2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3217468/50259901185
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-40944441340/19566
07901151813*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+163777
76536/1956607901151813*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(
1/2)+258506776/37222482817611*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)-8188888268/1956607901151813*EllipticE(1/23*897^(1/2)*(1
+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*
x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {170, 1618, 1613, 1616, 12, 176, 429, 182, 435}

$$-\frac{8188888268\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{50169433362867\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{258506776\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1618368818157\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{16377776536\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1956607901151813\sqrt{2x-5}}-\frac{40944441340\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1956607901151813\sqrt{5x+7}}-\frac{3217468\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{50259901185(5x+7)^{3/2}}+\frac{98\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1807455(5x+7)^{5/2}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(273*(7 + 5*x)^(7/2)) + (98*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1807455*(7 + 5*x)^(5/2)) - (32
17468*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(50259901185*(7 + 5*x)^(3
/2)) - (40944441340*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1956607901
151813*Sqrt[7 + 5*x]) + (16377776536*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5
*x])/(1956607901151813*Sqrt[-5 + 2*x]) - (8188888268*Sqrt[11/39]*Sqrt[2 - 3
*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/
Sqrt[-5 + 2*x]], -23/39])/(50169433362867*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (258506776*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
```

```
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1618368818157*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a + b*x)^(m + 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1618

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{9/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} - \frac{1}{273}\int \frac{-49+70x+96x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \int$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - 321$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - 321$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - 321$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - 321$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - 321$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - 321$$

Mathematica [A]
time = 28.13, size = 258, normalized size = 0.70

$$\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(\frac{(-2+3x)(2552362046246+19165803061167x+12313608173580x^2+2559027583750x^3)}{(7+5x)^4} - \frac{22\left(558333291\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)-186111097\sqrt{682}(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+71545594\sqrt{682}(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)}\right)}{1956607901151813\sqrt{2-3x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(((-2 + 3*x)*(2552362046246 +
 19165803061167*x + 12313608173580*x^2 + 2559027583750*x^3))/(7 + 5*x)^4 -
(22*(558333291*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 186111097*S
qrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[
Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 71545594*Sqrt[682]*(-2 +
 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
 + 8*x^2))))/(1956607901151813*Sqrt[2 - 3*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(341) = 682$.
time = 0.13, size = 1088, normalized size = 2.94

method	result

elliptic	$$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{18911307184\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\sqrt{2139}}{7772485129618352013\sqrt{-30\left(x+\frac{7}{5}\right)\left(x\right.}}\right.}{}$$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/45001981726491699*(4507711906500*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ellipti
cF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^5-4606249650750*(
-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^
```

```
(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2)
,1/39*I*897^(1/2))*x^5+12922107465300*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2
)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ellip
ticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^4-1320458233215
0*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*
23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1
/2),1/39*I*897^(1/2))*x^4+3265586847820*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1
/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*Ell
ipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-33369719692
10*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(
1/2),1/39*I*897^(1/2))*x^3-14556904316724*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^
(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x
^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+148751155
38822*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1
/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+
3*x))^(1/2),1/39*I*897^(1/2))-4712061512928*(-253*(7+5*x)/(-2+3*x))^(1/2)*1
3^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)
*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+481506630
1584*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/
2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))+5497405098416*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(
1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*El
lipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-5617577351848*
(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23
^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2
),1/39*I*897^(1/2))-2055076505891840*x^5-5939941120321416*x^4+3904489979900
882*x^3+4782097220989241 3*x^2+27541050878584656*x+3978655596814810)*(-5+2*x
)^(1/2)*(1+4*x)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(7+5
*x)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(6250*x^6
 + 28125*x^5 + 13125*x^4 - 134750*x^3 - 308700*x^2 - 266511*x - 84035), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)), x)
```

3.93 $\int \frac{\sqrt{2-3x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=391

$$\frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}}+\frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152}+\frac{5}{24}\sqrt{2-3x}\,\sqrt{-5+2x}$$

```
[Out] 5/24*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+295576909/593
0496*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1
/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)
/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+102487/1536*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*
x)^(1/2)/(-5+2*x)^(1/2)+6955/1152*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)+5241511/317952*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)-102487/3072*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5
+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/
2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {180, 1614, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$-\frac{102487\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{1024\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{295576909(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{13824\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}+\frac{5241511\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{13824\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{5}{24}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{6955\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1152}+\frac{102487\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1536\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (102487*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1536*Sqrt[-5 + 2*x]) +
(6955*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/1152 + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/24 - (102487*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1024*Sqrt[(2 - 3*x)/(5 - 2*x)
]*Sqrt[7 + 5*x]) + (5241511*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(13824*Sqrt[-5 + 2*x]*Sqrt[(7
+ 5*x)/(5 - 2*x)]) + (295576909*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-(
(1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])
/Sqrt[2 - 3*x]], -23/39])/(13824*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2))]]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1614

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
```

```
c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2} - \frac{1}{48}\int \frac{\sqrt{7+5x}\,(-6189+313}{\sqrt{2-3x}\,\sqrt{-5+2}}$$

$$= \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1-}$$

$$= \frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{}}{1152}$$

$$= \frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{}}{1152}$$

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Mathematica [A]
time = 31.26, size = 340, normalized size = 0.87

$$-\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(-57187746\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+46704724\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{7+5x}{-2+3x}}\left(186(-27447805-124999073x-56065622x^2+20626760x^3+6542400x^4+1152000x^5)+47673695\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)\right)}{1714176\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/1714176*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-57187746*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 46704724*Sqrt[682]*Sqrt[(-5 - 18*x
+ 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-2744
7805 - 124999073*x - 56065622*x^2 + 20626760*x^3 + 6542400*x^4 + 1152000*x^
5) + 47673695*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 -
11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5
+ 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(362) = 724$.
time = 0.14, size = 831, normalized size = 2.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/272802816*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(1037
819178*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2
+3*x))^(1/2),1/39*I*897^(1/2))+5320384362*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^
(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x
^2*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-3
561320763*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x)
)^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/
(-2+3*x))^(1/2),1/39*I*897^(1/2))-1383758904*(-253*(7+5*x)/(-2+3*x))^(1/2)*
13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2
)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-70938458
16*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x)
)^(1/2),-69/55,1/39*I*897^(1/2))+4748427684*(-253*(7+5*x)/(-2+3*x))^(1/2)*1
3^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)
*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+461252968
*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*2
3^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/
2),1/39*I*897^(1/2))+2364615272*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1
/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(
1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-1582809228*(-25
3*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/
2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/
39*I*897^(1/2))-34100352000*x^5-193661582400*x^4-610572722760*x^3+165959847
6822*x^2+3700097559873*x+812482475805)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,{\left(5\,x+7\right)}^{5/2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.94 $\int \frac{\sqrt{2-3x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=351

$$\frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}}+\frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}-\frac{785\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{7+}{5-}}}{128\sqrt{}}$$

```
[Out] 3730013/1235520*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+785/192*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+5/16*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)*(7+5*x)^(1/2)+17515/13248*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)-785/384*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {180, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$-\frac{785\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{3730013(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{2880\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}+\frac{17515\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{576\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{5}{16}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}+\frac{785\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{192\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (785*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/16 - (785*Sqrt[143
/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*S
qrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt
[7 + 5*x]) + (17515*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (3730013*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(2880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 180

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*(a + b*x)^(m - 1)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{1}{32}\int \frac{-4121+4074x+}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}$$

<u>Mathematica</u> [A]
time = 24.05, size = 349, normalized size = 0.99

$$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(200880+\frac{(2-3x)\left(-\frac{1314090\sqrt{682}\,(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)}{(2-3x)^2}+\frac{998820\sqrt{682}\,(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)}{(2-3x)^2}+\sqrt{\frac{7+5x}{-2+3x}}\left(\frac{3942270(-35-151x-34x^2+40x^3)}{(-2+3x)^3}+\frac{1082907\sqrt{682}\left(\frac{1+4x}{-2+3x}\right)^{3/2}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)}{1+4x}\right)\right)}{\left(\frac{7+5x}{-2+3x}\right)^{3/2}(5+18x-8x^2)}\right)}{642816}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(200880 + ((2 - 3
*x)*((-1314090*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*El
lipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^
2 + (998820*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellip
ticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^2 +
```

```
Sqrt[(7 + 5*x)/(-2 + 3*x)]*((3942270*(-35 - 151*x - 34*x^2 + 40*x^3))/(-2
+ 3*x)^3 + (1082907*Sqrt[682]*((1 + 4*x)/(-2 + 3*x))^(3/2)*Sqrt[(-35 - 11*x
 + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])/(1 + 4*x))))/(((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18
*x - 8*x^2))))/642816
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(332) = 664$.
time = 0.14, size = 826, normalized size = 2.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/56833920*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(17339
850*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2
)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*
x))^(1/2),1/39*I*897^(1/2))+67140234*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)
*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*El
lipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-454632
75*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))-23119800*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Ellip
ticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-89520312*(-253*(7
+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*(
(1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69
/55,1/39*I*897^(1/2))+60617700*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/
2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(
1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+7706600*(-253*(7+5*x)/
(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x
)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(
1/2))+29840104*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2
+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x
)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-20205900*(-253*(7+5*x)/(-2+3*x))
^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x
))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-107
09641800*x^3-2131272000*x^4+18688591350*x^2+49132479825*x+10956070125)/(120
*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2
- 18*x - 5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)^{\frac{3}{2}}}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**(3/2)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,{\left(5\,x+7\right)}^{3/2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.95 $\int \frac{\sqrt{2-3x}\,\sqrt{7+5x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=365

$$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4\sqrt{-5+2x}}-\frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}-\frac{39\sqrt{\frac{11}{23}}\,\sqrt{}$$

```
[Out] 179/992*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(1/
2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+2*
x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/2)
/(1+4*x)^(1/2)+4117/54560*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+
5*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529
+506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/2)
/((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+1/4*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+
5*x)^(1/2)/(-5+2*x)^(1/2)-39/184*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*
x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x
)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(
(7+5*x)/(5-2*x))^(1/2)-1/8*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^
(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2
-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {179, 182, 435, 171, 550, 429, 553, 176}

$$-\frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-\frac{39\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{179\sqrt{\frac{11}{62}}\,\sqrt{2-3x}\,F\left(\text{ArcTan}\left(\frac{\sqrt{\frac{22}{23}}\,\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{4x+1}}\,\sqrt{4x+1}}+\frac{4117\sqrt{2-3x}\,\Pi\left(\frac{78}{55};\text{ArcTan}\left(\frac{\sqrt{\frac{22}{23}}\,\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{80\sqrt{682}\,\sqrt{-\frac{2-3x}{4x+1}}\,\sqrt{4x+1}}+\frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{4\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
5*x]) - (39*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) +
 (179*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(16*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```

```
+ (4117*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/
Sqrt[-5 + 2*x]], 39/62])/(80*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 550

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[-f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{7+5x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{179}{16}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx - \frac{429}{16}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\left(6981\sqrt{-\frac{2-3x}{-5+2x}}\,(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)}{8\sqrt{71}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}}{\sqrt{-5}}\right.\right.}{8\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}$$

$$= \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}}{\sqrt{-5}}\right.\right.}{8\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{7+5x}}$$

Mathematica [A]
time = 6.21, size = 347, normalized size = 0.95

$$-\frac{6820\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5-18x+8x^2)E\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)-1265\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5-18x+8x^2)F\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{-5+2x}{1+4x}}\left(13640\sqrt{2}(70-83x-53x^2+30x^3)+4117\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}(1+4x)^2\sqrt{\frac{-35-11x+10x^2}{(1+4x)^2}}\,\Pi\left(\frac{78}{55};\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)\right)}{27280\sqrt{2-3x}\sqrt{-10+4x}\sqrt{\frac{-5+2x}{1+4x}}\sqrt{1+4x}\sqrt{7+5x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/27280*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 + 4*x
)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4*x
)]], 39/62] - 1265*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 +
 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 +
 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83*x - 53
*x^2 + 30*x^3) + 4117*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*Sqrt
[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22/39]*Sq
rt[(7 + 5*x)/(1 + 4*x)]], 39/62]))/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x]*Sqrt[(-5
+ 2*x)/(1 + 4*x)]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
```

Maple [A]
time = 0.13, size = 821, normalized size = 2.25

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{28\sqrt{-\frac{3795(x+\frac{7}{5})}{x-\frac{2}{3}}}\,(x-\frac{2}{3})^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)\left(x\right.}}\right.}{}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/1184040*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(30690*
(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23
^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^
(1/2),1/39*I*897^(1/2))+99882*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2
)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticP
i(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))-57915*(-253*(
7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*
((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1
/39*I*897^(1/2))-40920*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-2
53*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-133176*(-253*(7+5*x)/(-2+3*x))
^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x
))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^
(1/2))+77220*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3
*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)
/(-2+3*x))^(1/2),1/39*I*897^(1/2))+13640*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(
1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*El
lipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+44392*(-253*(7
+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*(
(1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/5
5,1/39*I*897^(1/2))-25740*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((
-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-
253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-17760600*x^3+15096510*x^2+670
46265*x+15540525)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 -
18*x - 5), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{5x+7}}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(1/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{5\,x+7}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.96 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx$

Optimal. Leaf size=101

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{1+4x}{2-3x}}\,\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$$

```
[Out] 62/2145*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/5
5,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1
/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {171, 551}

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (62*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*Ellipt
icPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(5
*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{\left(62(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-\frac{11x^2}{23}}\sqrt{}}\right.}{\sqrt{897}\sqrt{-5+2x}\sqrt{1+4x}}$$

$$= \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right.\right.}{5\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

Mathematica [A]
time = 4.18, size = 170, normalized size = 1.68

$$\frac{\sqrt{\frac{1+4x}{7+5x}}(7+5x)^{3/2}\left(-62\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{-2+3x}{7+5x}}F\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{77+55x}}\right)|\frac{23}{62}\right)+117\sqrt{\frac{-10+19x-6x^2}{(7+5x)^2}}\,\Pi\left(-\frac{55}{62};\sin^{-1}\left(\sqrt{\frac{155-62x}{77+55x}}\right)|\frac{23}{62}\right)\right)}{5\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^(3/2)*(-62*Sqrt[(5 - 2*x)/(7 + 5*x)]*S
qrt[(-2 + 3*x)/(7 + 5*x)]*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)]],
 23/62] + 117*Sqrt[(-10 + 19*x - 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, Arc
Sin[Sqrt[(155 - 62*x)/(77 + 55*x)]], 23/62]))/(5*Sqrt[682]*Sqrt[2 - 3*x]*Sq
rt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Maple [A]
time = 0.13, size = 134, normalized size = 1.33

method	result

default	$-\frac{62\,\mathrm{EllipticPi}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\frac{i\sqrt{897}}{39}\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{3}\,\sqrt{13}\,(-2+3x)\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{1}}{49335(40x^{3}-34x^{2}-151x-35)}$
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{4\sqrt{-\frac{3795(x+\frac{7}{5})}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^{2}\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)\left(x-\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -62/49335*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(
1/2))*((1+4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*1
3^(1/2)*(-2+3*x)*(-253*(7+5*x)/(-2+3*x))^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)
*(7+5*x)^(1/2)*(2-3*x)^(1/2)/(40*x^3-34*x^2-151*x-35)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 -
 34*x^2 - 151*x - 35), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

3.97 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2\sqrt{\frac{11}{39}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{22}}\,\sqrt{1+4x}}{\sqrt{7+5x}}\right)\middle|\frac{62}{39}\right)}{23\sqrt{-5+2x}}$$

[Out] 2/897*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/39*2418^(1/2))*429^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(60) = 120. time = 0.09, antiderivative size = 195, normalized size of antiderivative = 3.25, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {182, 433, 429, 506, 422}

$$-\frac{\sqrt{\frac{22}{31}}\,\sqrt{4x+1}\,F\left(\text{ArcTan}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{2x-5}}{\sqrt{5x+7}}\right)\middle|\frac{39}{62}\right)}{39\sqrt{2-3x}\,\sqrt{-\frac{4x+1}{2-3x}}}+\frac{2\sqrt{682}\,\sqrt{4x+1}\,E\left(\text{ArcTan}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{2x-5}}{\sqrt{5x+7}}\right)\middle|\frac{39}{62}\right)}{897\sqrt{2-3x}\,\sqrt{-\frac{4x+1}{2-3x}}}-\frac{62\sqrt{2x-5}\,\sqrt{4x+1}}{897\sqrt{2-3x}\,\sqrt{5x+7}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]

[Out] (-62*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (2*Sqrt[682]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) - (Sqrt[22/31]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x]], 39/62])/(39*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}\,dx = \frac{\left(\sqrt{2}\,\sqrt{2-3x}\,\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x, \frac{\sqrt{-5+2 \cdots}}{\sqrt{7+5 \cdots}}\right.}{39\sqrt{1+4x}\,\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= \frac{\left(\sqrt{2}\,\sqrt{2-3x}\,\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{23x^2}{22}}\,\sqrt{1+\frac{31x^2}{11}}}\,dx \cdots\right.}{39\sqrt{1+4x}\,\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= -\frac{62\sqrt{-5+2x}\,\sqrt{1+4x}}{897\sqrt{2-3x}\,\sqrt{7+5x}} - \frac{\sqrt{\frac{22}{31}}\,\sqrt{1+4x}\,F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{- \cdots}}{\sqrt{7+ \cdots}}\right.\right.}{39\sqrt{2-3x}\,\sqrt{-\frac{1+4x}{2-3x}}}$$

$$= -\frac{62\sqrt{-5+2x}\,\sqrt{1+4x}}{897\sqrt{2-3x}\,\sqrt{7+5x}} + \frac{2\sqrt{682}\,\sqrt{1+4x}\,E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{\cdots}}{\sqrt{7- \cdots}}\right.\right.}{897\sqrt{2-3x}\,\sqrt{-\frac{1+4 \cdots}{2-3 \cdots}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(60) = 120.
time = 29.67, size = 237, normalized size = 3.95

$$\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\left(-1922\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)+62\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)-23\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)}{27807\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-1922*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
+ 8*x^2) + 62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x +
 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]
- 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*
```

```
EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(27807*
Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 563, normalized size = 9.38

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{34\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}}{24942879\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)\left(x\right.}}\right.}{}$
default	$-\frac{2\sqrt{2-3x}\sqrt{7+5x}\sqrt{-5+2x}\sqrt{1+4x}\left(9\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/20631*(2-3*x)^(1/2)*(7+5*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(9*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),
1/39*I*897^(1/2))-9*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x
)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-25
3*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-12*(-253*(7+5*x)/(-2+3*x))^(1/2
)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1
/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+12*(-2
53*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1
/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2)
,1/39*I*897^(1/2))+4*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*
x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(
7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-4*(-253*(7+5*x)/(-2+3*x))^(1/2)*13
^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*
EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-5704*x^2+128
34*x+3565)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4
+ 110*x^3 - 993*x^2 - 1232*x - 245), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

3.98 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=290

$$-\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}}-\frac{98330\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{74828637\sqrt{7+5x}}+\frac{39332\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{74828637\sqrt{-5+2x}}$$

```
[Out] -10/2691*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-98330/748
28637*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+39332/748286
37*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+716/1423539*(1/
(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^
(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*2
53^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-19666/7482863
7*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*4
29^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5
*x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {183, 1613, 1616, 12, 176, 429, 182, 435}

$$-\frac{19666\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{1918683\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{716\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{61893\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{39332\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{74828637\sqrt{2x-5}}-\frac{98330\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{74828637\sqrt{5x+7}}-\frac{10\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-10*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2691*(7 + 5*x)^(3/2)) - (
98330*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(74828637*Sqrt[7 + 5*x])
+ (39332*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(74828637*Sqrt[-5 + 2*x
]) - (19666*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[A
rcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1918683*Sqrt[(
2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (716*Sqrt[11/23]*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(61893*Sqrt[-5 +
 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 183

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h)))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}\,dx = -\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{\int \frac{-771+854x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}}{2691}$$

$$= -\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\,\sqrt{-5+2x}}{74828637\sqrt{7+5x}}$$

$$= -\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\,\sqrt{-5+2x}}{74828637\sqrt{7+5x}}$$

$$= -\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\,\sqrt{-5+2x}}{74828637\sqrt{7+5x}}$$

$$= -\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\,\sqrt{-5+2x}}{74828637\sqrt{7+5x}}$$

$$= -\frac{10\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\,\sqrt{-5+2x}}{74828637\sqrt{7+5x}}$$

<u>Mathematica</u> **[A]**
time = 31.91, size = 248, normalized size = 0.86

$$-\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\left(-9833\sqrt{682}\,(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+31\left(\sqrt{\frac{7+5x}{-2+3x}}\,(-389005-1578968x-20372x^2+285680x^3)+92\sqrt{682}\,(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)\right)}{74828637\sqrt{2-3x}\,(7+5x)^{3/2}\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-9833*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sq
rt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62] + 31*(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-389005 - 1578
968*x - 20372*x^2 + 285680*x^3) + 92*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[
```

```
(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])))/(74828637*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7
+ 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(281) = 562$.

time = 0.13, size = 738, normalized size = 2.54

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{13455\left(x+\frac{7}{5}\right)^2}-\frac{\cdots}{74828637\cdots}\right.}{\cdots}$

default	$-\frac{2\left(499410\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-4424$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/1721058651*(499410*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2
*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*
(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-442485*(-253*(7+5*x)/(-2+3*x)
)^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*
x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^
3+33294*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^
(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-
2+3*x))^(1/2),1/39*I*897^(1/2))-29499*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2
)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*E
llipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-710272*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/
39*I*897^(1/2))+629312*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-2
53*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+310744*(-253*(7+5*x)/(-2+3*x))
^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x
))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-275
324*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2
)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^
(1/2),1/39*I*897^(1/2))-203689840*x^3+14525236*x^2+1125804184*x+277360565)*
(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+7
0)/(7+5*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5
 + 1950*x^4 - 4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

3.99 $\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=721

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{g+hx}}{h\sqrt{e+fx}}-\frac{\sqrt{dg-ch}\,\sqrt{fg-eh}\,\sqrt{a+bx}\,\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}}{\sqrt{dg-ch}}\right.\right.}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\,\sqrt{g+hx}}$$

```
[Out] (a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x+g
)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d*e
)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-a*h+b*g)^(1/2)*((-e*h+f*g)*(b*
x+a)/(-a*h+b*g)/(f*x+e))^(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2
)/f^2/h^2/(-a*f+b*e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+(b*x+a)^(1/2)*(d*x+c
)^(1/2)*(h*x+g)^(1/2)/h/(f*x+e)^(1/2)+(-c*f+d*e)*(-2*a*f*h+b*e*h+b*f*g)*Ell
ipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*
d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/f^2/h/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d
*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-EllipticE((-e*h+
f*g)^(1/2)*(d*x+c)^(1/2)/(-c*h+d*g)^(1/2)/(f*x+e)^(1/2),((-a*f+b*e)*(-c*h+d
*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)
^(1/2)*((-c*f+d*e)*(h*x+g)/(-c*h+d*g)/(f*x+e))^(1/2)/f/h/(-(-c*f+d*e)*(b*x+
a)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 721, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {179, 182, 435, 171, 551, 176, 430}

$$\frac{(e+fx)\sqrt{bg-ah}\,\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\,\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\,(adfh-b(-cfh+deh+dfg))\Pi\left(\frac{f(bg-ah)}{(be-af)h};\text{ArcSin}\left(\frac{\sqrt{be-af}\,\sqrt{g+hx}}{\sqrt{bg-ah}\,\sqrt{e+fx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{f^2h^2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{be-af}}+\frac{\sqrt{g+hx}\,(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{bg-ah}\,\sqrt{e+fx}}{\sqrt{fg-eh}\,\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{c+dx}\,\sqrt{bg-ah}\,\sqrt{fg-eh}\,\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{\sqrt{a+bx}\,\sqrt{dg-ch}\,\sqrt{fg-eh}\,\sqrt{\frac{(g+hx)(de-cf)}{(e+fx)(dg-ch)}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{fg-eh}\,\sqrt{c+dx}}{\sqrt{dg-ch}\,\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{g+hx}\,\sqrt{-\frac{(a+bx)(de-cf)}{(e+fx)(bc-ad)}}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{g+hx}}{h\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[d*g -
 c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*
h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt[d*g -
 c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqrt[g + h*
x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqr
t[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
```

```
(d*e - c*f)*(b*g - a*h)))])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c +
 d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[b*g
 - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*h)*(a + b*x))/
((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x
))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e
- a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
 a*h))/((b*e - a*f)*(d*g - c*h))])/(f^2*Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*S
qrt[c + d*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
```

```
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{((de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,(e+fx)^{3/2}\sqrt{g \cdots}}}{2fh}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{g+hx}}{h\sqrt{e+fx}} + \frac{\left((adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(\cdots}{(bg-ah)(\cdots}}\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{\sqrt{dg-ch}\,\sqrt{fg-eh}\,\sqrt{a+bx}\,\sqrt{\frac{(de-cf)\cdots}{(dg-ch)\cdots}}\cdots}{fh\sqrt{-\frac{(de\cdots}{(bc\cdots}}\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6667 vs. $2(721) = 1442$.
time = 48.81, size = 6667, normalized size = 9.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15273 vs. $2(656) = 1312$.
time = 0.13, size = 15274, normalized size = 21.18

method	result	size
elliptic	Expression too large to display	1544
default	Expression too large to display	15274

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)),x)
[Out] int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)), x)
```

3.100 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{c+dx}\,E\left(\tan^{-1}\left(\frac{\sqrt{-be+af}\,\sqrt{g+hx}}{\sqrt{bg-ah}\,\sqrt{e+fx}}\right)|\frac{(-bc+ad)(fg-eh)}{(-be+af)(dg-ch)}\right)}{\sqrt{-be+af}\,\sqrt{bg-ah}\,\sqrt{a+bx}\,\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}$$

```
[Out] -2*(1/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e)))^(1/2)*(1+(a*f-b*e)*(h*x+g)/
(-a*h+b*g)/(f*x+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*(h*x+g)^(1/2)/(-a*h+b*g
)^(1/2)/(f*x+e)^(1/2)/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2),((a*d-
b*c)*(-e*h+f*g)/(a*f-b*e)/(-c*h+d*g))^(1/2))*(d*x+c)^(1/2)/(a*f-b*e)^(1/2)/
(-a*h+b*g)^(1/2)/(b*x+a)^(1/2)/((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 208, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {182, 435}

$$-\frac{2\sqrt{c+dx}\,\sqrt{fg-eh}\,\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{bg-ah}\,\sqrt{e+fx}}{\sqrt{fg-eh}\,\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}\,(be-af)\sqrt{bg-ah}\,\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*
h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g
- e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h
)))])/((b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]*Sqrt[g + h*x])
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
```

```
, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{\left(2\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(bc-ad}{de-c}}}{\sqrt{1-\frac{(bg-ah}{fg-e}}}\right.}{(be-af)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bg-}}{\sqrt{fg-}}\right.\right.}{(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}}$$

Mathematica [A]
time = 23.74, size = 206, normalized size = 1.28

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right)\middle|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(-fg+eh)(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{-\frac{(be-af)(bg-ah)(e+fx)(g+hx)}{(fg-eh)^2(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*EllipticE[ArcSin[Sqrt[((-(b*e)
 + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h))/((b
*e - a*f)*(d*g - c*h))])/((-(f*g) + e*h)*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*
x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4552 vs. $2(251) = 502$.
time = 0.12, size = 4553, normalized size = 28.28

method	result	size
elliptic	Expression too large to display	1948
default	Expression too large to display	4553

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 2*(-((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e
)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*
h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h
-d*g))^(1/2))*a^2*c^2*f*h-((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d
-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c)
)^(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a
*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*b^2*c^2*e*g+((c*h-d*g)*(b*x+a)/(a*h-b*g
)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*
x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c
))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a^2*c^2*f*h+((c*h
-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c)
)^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b
*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1
/2))*b^2*c^2*e*g+((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*
x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*E
llipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(
a*f-b*e)/(c*h-d*g))^(1/2))*a*b*d^2*e*h*x^2+((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*
x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/
(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1
/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*b*d^2*f*g*x^2-((c*h-
d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))
^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b*
x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/
2))*a*b*d^2*e*h*x^2-((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*
(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2
)*EllipticE(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g
)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*b*d^2*f*g*x^2-2*((c*h-d*g)*(b*x+a)/(a*h-b*g
)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*
x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c
))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a^2*c*d*f*h*x-2*(
(c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*
x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-d*g
```

```
)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g)
)^(1/2))*b^2*c*d*e*g*x+2*((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-
b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))
^(1/2)*EllipticE(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*
h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a^2*c*d*f*h*x+2*((c*h-d*g)*(b*x+a)/(a*h-
b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*
(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*
x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*b^2*c*d*e*g*x+
a^2*d^2*f*h*x^2+b^2*c^2*f*h*x^2+a^2*d^2*e*h*x+a^2*d^2*f*g*x+b^2*c^2*e*h*x+b
^2*c^2*f*g*x-2*a*b*c*d*e*g+a^2*d^2*e*g+b^2*c^2*e*g+((c*h-d*g)*(b*x+a)/(a*h-
b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*
(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*
x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*b^2*d^2*e*g*x^
2+((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/
(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-
d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d
*g))^(1/2))*a*b*c^2*e*h+((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b
*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^
(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h
-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*b*c^2*f*g-((c*h-d*g)*(b*x+a)/(a*h-b*g)/
(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+
g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))
^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*b*c^2*e*h-((c*h-d
*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^
(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticE(((c*h-d*g)*(b*x
+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2
))*a*b*c^2*f*g+2*((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*
x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*E
llipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(
a*f-b*e)/(c*h-d*g))^(1/2))*a*b*c*d*e*h*x+2*((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*
x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a*d-b*c)*(h*x+g)/
(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1
/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*b*c*d*f*g*x-2*((c*h-
d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))
^(1/2)*((a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*f*h*x
^4 + a^2*f*g*x + (b^2*f*g + 2*a*b*f*h)*x^3 + (2*a*b*f*g + a^2*f*h)*x^2 + (b
^2*h*x^3 + a^2*g + (b^2*g + 2*a*b*h)*x^2 + (2*a*b*g + a^2*h)*x)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/((a + b*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)),x)
```

```
[Out] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)), x)
```

3.101 $\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=351

$$-\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}}-\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}+\frac{2135\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\ }}{12}$$

```
[Out] -3431855/247104*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-2135/192*(2-3*x)^(1/2)*(1+4*x)^(1/
2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-25/48*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)*(7+5*x)^(1/2)+29047/6336*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)+2135/384*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*
x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/
((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {173, 1616, 1612, 176, 429, 171, 551, 182, 435}

$$\frac{2135\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{128\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{3431855(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\Pi\left(-\frac{69}{55};\text{ArcSin}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{576\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}+\frac{29047\sqrt{\frac{23}{11}}\sqrt{5x+7}F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{2135\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) - (2
5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/48 + (2135*Sqrt
[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/2
3]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (29047*Sqrt[23/11]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)
/(5 - 2*x)]) - (3431855*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(576*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 173

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b^2*(a + b*x)^(m - 2)*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1612

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = -\frac{25}{48}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{1}{96}\int \frac{28003+}{\sqrt{2-3x}\,\sqrt{-5}}$$

$$= -\frac{2135\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= -\frac{2135\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= -\frac{2135\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

$$= -\frac{2135\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}$$

<u>Mathematica</u> [A]
time = 23.51, size = 347, normalized size = 0.99

$$\frac{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(1227600(-2+3x)+\frac{-13104630\sqrt{682}\,(-2+3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)+17113116\sqrt{682}\,(-2+3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)-385\sqrt{\frac{7+5x}{-2+3x}}\left(-102114(-35-151x-34x^2+40x^3)-47445\sqrt{682}\,(2-3x)^2\sqrt{\frac{1+4x}{-2+3x}}\sqrt{\frac{-35-11x+10x^2}{(2-3x)^2}}\,\Pi\left(\frac{117}{62};\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)\middle|\frac{39}{62}\right)\right)}{(2-3x)\left(\frac{7+5x}{-2+3x}\right)^{3/2}(5+18x-8x^2)}\right)}{2356992\sqrt{2-3x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(1227600*(-2 + 3*x) + (-1310463
0*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elli
pticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 17113116*Sq
rt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elliptic
F[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 385*Sqrt[(7 + 5
*x)/(-2 + 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) - 47445*Sqrt[682]*
(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)
```

```
^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/
62]))/((2 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(235699
2*Sqrt[2 - 3*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(332) = 664.
time = 0.14, size = 826, normalized size = 2.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/11366784*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(12025
458*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2
)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*
x))^(1/2),1/39*I*897^(1/2))-61773390*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)
*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*El
lipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+247297
05*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)
*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x
))^(1/2),1/39*I*897^(1/2))-16033944*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*
3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Ellip
ticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+82364520*(-253*(7
+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*(
(1+4*x)/(-2+3*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69
/55,1/39*I*897^(1/2))-32972940*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/
2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(
1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+5344648*(-253*(7+5*x)/
(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x
)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(
1/2))-27454840*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2
+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x
)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+10990980*(-253*(7+5*x)/(-2+3*x))
^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x
))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+710
424000*x^4+6506299800*x^3-8725486770*x^2-27462475755*x-6221390175)/(120*x^4
-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sq
rt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5\,x+7)^{5/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.102 $\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=469

$$-\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}+\frac{65\sqrt{}}{}$$

```
[Out] -895/2976*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(
1/2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+
2*x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/
2)/(1+4*x)^(1/2)-4117/32736*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(
7+5*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(5
29+506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/
2)/((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+23/132*(7+5*x)*EllipticPi(1/11*34
1^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/124,1/62*2418^(1/2))*682^(1/2)*((2-3
*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)-5/1
2*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+65/184*(1/(4+2*(
1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*
2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/
2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+5/24*EllipticE(1/23
*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)
^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {172, 179, 182, 435, 171, 550, 429, 553, 176, 551}

$$\frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{6\sqrt{2-3x}\sqrt{2x-5}}+\frac{65\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}\,F\left(\text{ArcTan}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\frac{4117\sqrt{2-3x}\,\Pi\left(\frac{78}{55};\text{ArcTan}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\frac{39}{62}\right)}{48\sqrt{682}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\frac{5\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{12\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(12*Sqrt[-5 + 2*x]) + (5*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[7 + 5*x]) + (65*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 -
2*x)]) - (895*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[
```

```
7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(48*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x]) + (23*Sqrt[31/22]*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x
)]*(7 + 5*x)*EllipticPi[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 +
 5*x]], 39/62])/(6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]) - (4117*Sqrt[2 - 3*x]*Elli
pticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(
48*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*x]*
(Sqrt[c + d*x]/(Sqrt[e + f*x]*Sqrt[g + h*x])), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 550

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[-f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = -\left(\frac{5}{3}\int \frac{\sqrt{2-3x}\,\sqrt{7+5x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx\right)+\frac{31}{3}\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1 \ldots}}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{895}{48}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+ \ldots}}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{23\sqrt{\frac{31}{22}}\,\sqrt{\frac{2-3x}{7+5x}}\,\sqrt{\frac{5-2x}{7+5x}}\,(7 \ldots}{6\sqrt{2 \ldots}}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{5\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\Big(\ldots}{8\sqrt{\frac{2-3x}{5-2x}} \ldots}$$

$$= -\frac{5\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{12\sqrt{-5+2x}}+\frac{5\sqrt{\frac{143}{3}}\,\sqrt{2-3x}\,\sqrt{\frac{7+5x}{5-2x}}\,E\Big(\ldots}{8\sqrt{\frac{2-3x}{5-2x}} \ldots}$$

Mathematica [**A**]
time = 9.12, size = 347, normalized size = 0.74

$$\frac{\sqrt{-5+2x}\left(6820\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}\left(-5-18x+8x^2\right)E\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)-6969\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}\left(-5-18x+8x^2\right)F\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)+\sqrt{\frac{-5+2x}{1+4x}}\left(13640\sqrt{2}\left(70-83x-53x^2+30x^3\right)+9821\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}(1+4x)^2\sqrt{\frac{-35-11x+10x^2}{(1+4x)^2}}\,\Pi\left(\frac{78}{55};\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)\right)\right)}{16368\sqrt{4-6x}\left(\frac{-5+2x}{1+4x}\right)^{3/2}(1+4x)^{3/2}\sqrt{7+5x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(
1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(
1 + 4*x)]], 39/62] - 6969*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*
```

```
x)/(1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*
x)/(1 + 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83
*x - 53*x^2 + 30*x^3) + 9821*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)
^2*Sqrt[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22
/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/62]))/(16368*Sqrt[4 - 6*x]*((-5 + 2*x)
/(1 + 4*x))^(3/2)*(1 + 4*x)^(3/2)*Sqrt[7 + 5*x])
```

Maple [A]
time = 0.13, size = 821, normalized size = 1.75

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{98\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\sqrt{2139}\sqrt{\frac{x+\ldots}{x-\ldots}}\ldots}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)(x\ldots}}\ldots\right.}{\sqrt{7+5x}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$
default	$-\frac{\sqrt{7+5x}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\left(107694\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/710424*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(107694*
(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23
^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^
(1/2),1/39*I*897^(1/2))-238266*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/
2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*Elliptic
Pi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2))+57915*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
```

```
*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),
1/39*I*897^(1/2))-143592*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-
5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(
-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+317688*(-253*(7+5*x)/(-2+3*x
))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3
*x))^(1/2)*x*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*89
7^(1/2))-77220*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2
+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*
x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+47864*(-253*(7+5*x)/(-2+3*x))^(1/2)*13
^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*
EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-105896*(-253
*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2
)*((1+4*x)/(-2+3*x))^(1/2)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-6
9/55,1/39*I*897^(1/2))+25740*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)
*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23
*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+17760600*x^3-15096510*x^2-
67046265*x-15540525)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^
3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{3}{2}}}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**(3/2)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(5\,x+7\right)}^{3/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.103 $\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=100

$$\frac{23\sqrt{\frac{2-3x}{7+5x}}\,\sqrt{\frac{5-2x}{7+5x}}\,(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\,\sqrt{2-3x}\,\sqrt{-5+2x}}$$

```
[Out] 23/1364*(7+5*x)*EllipticPi(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/12
4,1/62*2418^(1/2))*682^(1/2)*((2-3*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2
)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {171, 551}

$$\frac{23\sqrt{\frac{2-3x}{5x+7}}\,\sqrt{\frac{5-2x}{5x+7}}\,(5x+7)\Pi\left(\frac{55}{124};\text{ArcSin}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\,\sqrt{2-3x}\,\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (23*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*EllipticP
i[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 39/62])/(2*Sqr
t[682]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{\left(23\sqrt{2}\,\sqrt{\frac{2-3x}{7+5x}}\,\sqrt{-\frac{-5+2x}{7+5x}}\,(7+5x)\right)\text{Subst}\left(\int \frac{}{(4-5x^2)\sqrt{1-}}\right.}{11\sqrt{2-3x}\,\sqrt{-5+2x}}$$

$$= \frac{23\sqrt{\frac{2-3x}{7+5x}}\,\sqrt{\frac{5-2x}{7+5x}}\,(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{1+4x}}{\sqrt{7+5x}}\right)\right|}{2\sqrt{682}\,\sqrt{2-3x}\,\sqrt{-5+2x}}$$

Mathematica [A]
time = 3.82, size = 95, normalized size = 0.95

$$-\frac{62\sqrt{1+4x}\,\sqrt{\frac{5-2x}{7+5x}}\,\Pi\left(-\frac{55}{69};\sin^{-1}\left(\frac{\sqrt{\frac{23}{11}}\,\sqrt{2-3x}}{\sqrt{7+5x}}\right)\middle|-\frac{39}{23}\right)}{3\sqrt{253}\,\sqrt{-5+2x}\,\sqrt{\frac{1+4x}{7+5x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-62*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticPi[-55/69, ArcSin[(Sqr
t[23/11]*Sqrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(3*Sqrt[253]*Sqrt[-5 + 2*x
]*Sqrt[(1 + 4*x)/(7 + 5*x)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 162, normalized size = 1.62

method	result

default	$-\frac{62\left(\text{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-\text{EllipticPi}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\frac{i\sqrt{897}}{39}\right)\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2\cdots}{-2+3\cdots}}\cdots}{29601(40x^3-34x^2-151x-35)\cdots}$
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{14\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}\cdots}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)(x-\cdots}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -62/29601*(EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-E
llipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2)))*((1+
4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*13^(1/2)*(-
2+3*x)*(-253*(7+5*x)/(-2+3*x))^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(
1/2)*(7+5*x)^(1/2)/(40*x^3-34*x^2-151*x-35)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3
- 70*x^2 + 21*x + 10), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{5\,x+7}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

3.104 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal. Leaf size=71

$$\frac{2\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

```
[Out] 2/253*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF
((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897
^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {176, 429}

$$\frac{2\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (2*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -
39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{\left(\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\ \sqrt{}}\right.}{\sqrt{-5+2x}\sqrt{\frac{7+5x}{2-3x}}}$$

$$= \frac{2\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

Mathematica [A]
time = 3.20, size = 90, normalized size = 1.27

$$-\frac{2\sqrt{1+4x}\sqrt{\frac{5-2x}{7+5x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{7+5x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{1+4x}{7+5x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (-2*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticF[ArcSin[(Sqrt[23/11]*S
qrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(1 +
4*x)/(7 + 5*x)])
```

Maple [A]
time = 0.13, size = 133, normalized size = 1.87

method	result
default	$-\frac{2\,\text{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\sqrt{\frac{1+4x}{-2+3x}}\sqrt{23}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{3}\sqrt{13}(-2+3x)\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{1+}}{9867(40x^3-34x^2-151x-35)}$

elliptic	$\frac{2\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\,\left(x-\frac{2}{3}\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}\ \mathrm{E}}{305877\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)\left(x-\frac{5}{2}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/9867*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*((1+
4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*13^(1/2)*(-
2+3*x)*(-253*(7+5*x)/(-2+3*x))^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(
1/2)*(7+5*x)^(1/2)/(40*x^3-34*x^2-151*x-35)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4
 - 182*x^3 - 385*x^2 + 197*x + 70), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

3.105 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}} dx$

Optimal. Leaf size=195

$$\frac{10\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{22}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{62}{39}\right)}{713\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}+\frac{2\sqrt{\frac{3}{143}}(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}F}{31\sqrt{-5+2x}\sqrt{}}$$

```
[Out] 2/4433*(2-3*x)*EllipticF(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),1/39*I*
897^(1/2))*429^(1/2)*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)/(-5+2
*x)^(1/2)/(1+4*x)^(1/2)+10/27807*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+
5*x)^(1/2),1/39*2418^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)
/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 270, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {177, 176, 429, 182, 433, 506, 422}

$$\frac{6\sqrt{5x+7}F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{5\sqrt{\frac{22}{31}}\sqrt{4x+1}F\left(\text{ArcTan}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{1209\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}+\frac{10\sqrt{\frac{22}{31}}\sqrt{4x+1}E\left(\text{ArcTan}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}-\frac{10\sqrt{2x-5}\sqrt{4x+1}}{897\sqrt{2-3x}\sqrt{5x+7}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-10*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (10*
Sqrt[22/31]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqr
t[7 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) + (6*
Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/
23])/(31*Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (5*Sqrt[22/3
1]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x
]], 39/62])/(1209*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
h}, x]
```

Rule 177

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}\,dx = \frac{3}{31}\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx + \frac{5}{31}\int$$

$$= \frac{\left(5\sqrt{2}\,\sqrt{2-3x}\,\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x\right.}{1209\sqrt{1+4x}\,\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= \frac{6\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\,\sqrt{-5+2x}\,\sqrt{\frac{7+5x}{5-2x}}} + \frac{\left(5\sqrt{2}\,\sqrt{\right.}{}$$

$$= -\frac{10\sqrt{-5+2x}\,\sqrt{1+4x}}{897\sqrt{2-3x}\,\sqrt{7+5x}} + \frac{6\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1}}{\sqrt{2}\,\sqrt{}}\right.\right.}{31\sqrt{253}\,\sqrt{-5+2x}\,\sqrt{}}$$

$$= -\frac{10\sqrt{-5+2x}\,\sqrt{1+4x}}{897\sqrt{2-3x}\,\sqrt{7+5x}} + \frac{10\sqrt{\frac{22}{31}}\,\sqrt{1+4x}\,E\left(\tan^{-1}\right.}{897\sqrt{2-3x}}$$

Mathematica [A]
time = 16.30, size = 237, normalized size = 1.22

$$-\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\left(1705\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)-55\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-23\sqrt{682}\,\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\,(-14+11x+15x^2)\,F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{305877\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{\frac{7+5x}{-2+3x}}\,(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*
x + 8*x^2) - 55*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x
 + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62
] - 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2
)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(3058
77*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^
2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(155) = 310.
time = 0.13, size = 563, normalized size = 2.89

method	result

elliptic	$\frac{\sqrt{-\left(7+5x\right)\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(\frac{7252\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{x-\frac{2}{3}}}\left(x-\frac{2}{3}\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{x-\frac{2}{3}}}}{8505521739\sqrt{-30\left(x+\frac{7}{5}\right)\left(x-\frac{2}{3}\right)\left(x\right.}}\right.}{}$
default	$-\frac{2\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(1116\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/7035171*(7+5*x)^(1/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(1116*(
```

```
-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^
(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(
1/2),1/39*I*897^(1/2))-495*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*(
(-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticE(1/
23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-1488*(-253*(7+5*x)/(-2+3
*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2
+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2
))+660*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticE(1/23*(-253*(7+5*x)/(-2+3
*x))^(1/2),1/39*I*897^(1/2))+496*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(
1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(
1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-220*(-253*(7+5*x)/(-2+
3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-
2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2)
)-313720*x^2+705870*x+196075)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5
 - 70*x^4 - 3199*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

3.106 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}}-\frac{895300\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2319687747\sqrt{7+5x}}+\frac{358120\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7}}{2319687747\sqrt{-5+2x}}$$

```
[Out] -50/83421*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-895300/2
319687747*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+358120/2
319687747*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+103964/4
85426799*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*Ellipt
icF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*
897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-1
79060/2319687747*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39
*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2
*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {178, 1613, 1616, 12, 176, 429, 182, 435}

$$-\frac{179060\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{59479173\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}+\frac{103964\sqrt{5x+7}\,F\left(\text{ArcTan}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{1918683\sqrt{253}\,\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{358120\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{2319687747\sqrt{2x-5}}-\frac{895300\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{2319687747\sqrt{5x+7}}-\frac{50\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-50*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) -
(895300*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5*
x]) + (358120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2319687747*Sqrt[-
5 + 2*x]) - (179060*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*Ell
ipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(594791
73*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (103964*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1918683*Sqrt[25
3]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 178

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b^2*(a + b*x)^(m + 1)*Sq
rt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(
b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1613

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1616

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Dist[1/(2*b*d*f*h), Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*f
*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Dist[C*(d*e -
 c*f)*((d*g - c*h)/(2*b*d*f*h)), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}\,dx = -\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}} + \frac{\int \frac{119}{\sqrt{2-3x}\,\sqrt{-5+}}}{}$$

$$= -\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\,\sqrt{}}{231968774}$$

$$= -\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\,\sqrt{}}{231968774}$$

$$= -\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\,\sqrt{}}{231968774}$$

$$= -\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\,\sqrt{}}{231968774}$$

$$= -\frac{50\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\,\sqrt{}}{231968774}$$

Mathematica [A]
time = 32.32, size = 246, normalized size = 0.85

$$-\frac{2\sqrt{-5+2x}\,\sqrt{1+4x}\left(1705\sqrt{\frac{7+5x}{-2+3x}}(-671560-2797991x-294854x^2+608600x^3)-984830\sqrt{682}(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}E\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)-28819\sqrt{682}(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}F\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)\right)}{25516565217\sqrt{2-3x}(7+5x)^{3/2}\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-671560
- 2797991*x - 294854*x^2 + 608600*x^3) - 984830*Sqrt[682]*(-2 + 3*x)*(7 + 5
*x)^2*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 28819*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2
```

```
*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62]))/(25516565217*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*
Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(281) = 562$.
time = 0.14, size = 738, normalized size = 2.56

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{82359956\sqrt{-\frac{3795(x+\frac{7}{5})}{x-\frac{2}{3}}}\,(x-\frac{2}{3})^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{x-\frac{2}{3}}}\,\sqrt{2139}\,\sqrt{}}{709539128989119\sqrt{-30\,(x+\frac{7}{5})\,(x-}}\right.}{}$

default	$-\frac{2\left(72514890\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-4431\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/586880999991*(72514890*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((
-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-
253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))*x^3-44317350*(-253*(7+5*x)/(-
2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/
(-2+3*x))^(1/2)*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/
2))*x^3+4834326*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-
2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x^2*EllipticF(1/23*(-253*(7
+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-2954490*(-253*(7+5*x)/(-2+3*x))^(1/
2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(
1/2)*x^2*EllipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-103
132288*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(
1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*EllipticF(1/23*(-253*(7+5*x)/(-2+3
*x))^(1/2),1/39*I*897^(1/2))+63029120*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2
)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*x*Ell
ipticE(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))+45120376*(-253*
(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*23^(1/2)
*((1+4*x)/(-2+3*x))^(1/2)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39
*I*897^(1/2))-27575240*(-253*(7+5*x)/(-2+3*x))^(1/2)*13^(1/2)*3^(1/2)*((-5+
2*x)/(-2+3*x))^(1/2)*23^(1/2)*((1+4*x)/(-2+3*x))^(1/2)*EllipticE(1/23*(-253
*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-23866249000*x^3+11562699610*x^2+
109723217065*x+26335225400)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(1/2)/(120
*x^4-182*x^3-385*x^2+197*x+70)/(7+5*x)^(1/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^
6 + 3850*x^5 - 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

3.107 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=968

$$\frac{b\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{g+hx}}{dh\sqrt{e+fx}}-\frac{b\sqrt{dg-ch}\,\sqrt{fg-eh}\,\sqrt{a+bx}\,\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{fg-e}}{\sqrt{dg-ch}}\right.\right.}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\,\sqrt{g+hx}}$$

[Out] b*(a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-a*h+b*g)^(1/2)*((-e*h+f*g)*(b*x+a)/(-a*h+b*g)/(f*x+e))^(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2)/d/f^2/h^2/(-a*f+b*e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)-2*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-a*d+b*c)^(1/2)*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/d/h/(d*x+c)^(1/2)/(f*x+e)^(1/2)+b*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(h*x+g)^(1/2)/d/h/(f*x+e)^(1/2)+b*(-c*f+d*e)*(-2*a*f*h+b*e*h+b*f*g)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/d/f^2/h/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-b*EllipticE((-e*h+f*g)^(1/2)*(d*x+c)^(1/2)/(-c*h+d*g)^(1/2)/(f*x+e)^(1/2),((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*((-c*f+d*e)*(h*x+g)/(-c*h+d*g)/(f*x+e))^(1/2)/d/f/h/(-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/2)

Rubi [A]
time = 0.63, antiderivative size = 968, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {172, 179, 182, 435, 171, 551, 176, 430}

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]

[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(d*h*Sqrt[e + f*x]) - (b*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt

```
[d*g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g -
e*h))])/(d*f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqr
t[g + h*x]) + (b*(d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
- a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g
 - e*h))/((d*e - c*f)*(b*g - a*h)))])/(d*f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e
*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
 + (b*Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*
h)*(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g -
 c*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), Arc
Sin[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e
 - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(d*f^2*Sqrt[b*e - a*f]*h^2
*Sqrt[a + b*x]*Sqrt[c + d*x]) - (2*Sqrt[b*c - a*d]*Sqrt[-(d*g) + c*h]*(a +
b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h
)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c -
a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*h*Sqrt[
c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*x]*
(Sqrt[c + d*x]/(Sqrt[e + f*x]*Sqrt[g + h*x])), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 179

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[Sqrt[a + b*x]*Sqrt[c +
d*x]*(Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Dist[(d*e - c*f)*((f*g - e*h
)/(2*f*h)), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e
 + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Dist[(d*e - c
*f)*((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h)), Int[1/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+}}}{d}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{(b(de-cf)(fg-eh))\int \frac{}{\sqrt{c+dx}}}{2dfh}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{2\sqrt{bc-ad}\sqrt{-dg+ch}(a+bx)}{}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6638 vs. $2(968) = 1936$.
time = 29.87, size = 6638, normalized size = 6.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 17030 vs. $2(884) = 1768$.
time = 0.12, size = 17031, normalized size = 17.59

method	result	size
elliptic	Expression too large to display	1541

default	Expression too large to display	17031

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.108 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=228

$$\frac{2\sqrt{-dg+ch}\,(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\,\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{g+hx}}{\sqrt{-dg+ch}\,\sqrt{a+b}}\right.\right.}{\sqrt{bc-ad}\,h\sqrt{c+dx}\,\sqrt{e+fx}}$$

```
[Out] 2*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)
^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f
*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*(
(-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/h/(-a*d+b*c)^(1/2)/(d*x+c)^(1/
2)/(f*x+e)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {171, 551}

$$\frac{2(a+bx)\sqrt{ch-dg}\,\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\,\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\text{ArcSin}\left(\frac{\sqrt{bc-ad}\,\sqrt{g+hx}}{\sqrt{ch-dg}\,\sqrt{a+bx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d
)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 171

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*(a + b*x)*Sqrt[(b*g - a
*h)*((c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g
 - e*h)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x])), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g -
 e*h))]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \frac{\left(2(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\,\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt{c+dx}\,\sqrt{\cdots}}$$

$$= \frac{2\sqrt{-dg+ch}\,(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\,\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\Pi\cdots}{\sqrt{bc-ad}\,h\sqrt{c+dx\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 584 vs. 2(228) = 456.

time = 28.32, size = 584, normalized size = 2.56

$$-\frac{2\sqrt{\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}}(c+dx)^{3/2}\left(\frac{ad\sqrt{\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}(g+hx)F\left(\sin^{-1}\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)}{(dg-ch)(c+dx)\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}}+\frac{bc\sqrt{\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}(g+hx)F\left(\sin^{-1}\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)}{(-dg+ch)(c+dx)\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}}+\frac{b(fg-eh)\sqrt{-\frac{(de-cf)(dg-ch)(e+fx)(g+hx)}{(fg-eh)^2(c+dx)^2}}\,\Pi\left(\frac{d(-fg+eh)}{(de-cf)h};\sin^{-1}\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)}{(de-cf)h}\right)}{d\sqrt{a+bx}\,\sqrt{e+fx}\,\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]*(c + d*x)^(3/2)*(
(a*d*Sqrt[((d*g - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*Ellipt
icF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c
 - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((d*g - c*h)*(c + d*x)*
Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*c*Sqrt[((d*g
 - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*EllipticF[ArcSin[Sqrt
[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g)
 + e*h))/((d*e - c*f)*(b*g - a*h))])/((-(d*g) + c*h)*(c + d*x)*Sqrt[((-(d*e
) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*(f*g - e*h)*Sqrt[-(((d*e
 - c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(c + d*x)^2))]*Elli
pticPi[(d*(-(f*g) + e*h))/((d*e - c*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g +
 h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)
```

```
*(b*g - a*h))])/((d*e - c*f)*h)))/(d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1249 vs. $2(209) = 418$.
time = 0.12, size = 1250, normalized size = 5.48

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)} \left(\frac{2a\left(-\frac{a}{b}+\frac{g}{h}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)(}{\left(-\frac{g}{h}+\frac{a}{b}\right)(}}}{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)(x}} \right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)/d*((c*h-d*g)*(b*
x+a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*(
(a*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*(EllipticF(((c*h-d*g)*(b*x+a)/(a
*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*d
^2*h*x^2-EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(
a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*b*d^2*g*x^2-EllipticPi(((c*h-d*g)*(b*x
+a)/(a*h-b*g)/(d*x+c))^(1/2),(a*h-b*g)*d/b/(c*h-d*g),((c*f-d*e)*(a*h-b*g)/(
a*f-b*e)/(c*h-d*g))^(1/2))*a*d^2*h*x^2+EllipticPi(((c*h-d*g)*(b*x+a)/(a*h-b
*g)/(d*x+c))^(1/2),(a*h-b*g)*d/b/(c*h-d*g),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(
c*h-d*g))^(1/2))*b*d^2*g*x^2+2*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+
c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*c*d*h*x-2*Elli
pticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f
-b*e)/(c*h-d*g))^(1/2))*b*c*d*g*x-2*EllipticPi(((c*h-d*g)*(b*x+a)/(a*h-b*g)
/(d*x+c))^(1/2),(a*h-b*g)*d/b/(c*h-d*g),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h
-d*g))^(1/2))*a*c*d*h*x+2*EllipticPi(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^
(1/2),(a*h-b*g)*d/b/(c*h-d*g),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/
2))*b*c*d*g*x+EllipticF(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d
*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*c^2*h-EllipticF(((c*h-d*g)*(b*x
```

```
+a)/(a*h-b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2
))*b*c^2*g-EllipticPi(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),(a*h-b*g)
*d/b/(c*h-d*g),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*a*c^2*h+Ell
ipticPi(((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2),(a*h-b*g)*d/b/(c*h-d*g)
,((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*b*c^2*g)/(c*h-d*g)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.109 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}\sqrt{e+fx}\,F\left(\tan^{-1}\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{fg-eh}\sqrt{c+dx}}$$

```
[Out] -2*(1/(1+(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a)))^(1/2)*(1+(-a*f+b*e)*(h*x+g
)/(-e*h+f*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-e*h+
f*g)^(1/2)/(b*x+a)^(1/2)/(1+(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2),((
-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*((-e*h+f*g)*(d*x+c)/(-c*
h+d*g)/(f*x+e))^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+
c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {176, 430}

$$\frac{2\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Elli
pticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x]
)], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[b*g - a*
h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x)))])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\left(2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+\frac{(bc-}{de-}}}\right.}{(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-}{(fg-}}}$$

$$= \frac{2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e}}{\sqrt{fg-eh}\sqrt{a}}\right.\right.}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af}{(fg-eh}}}$$

Mathematica [**A**]
time = 24.27, size = 227, normalized size = 1.41

$$-\frac{2\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\sqrt{g+hx}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right)\middle|\frac{(-bc+ad)(-fg+eh)}{(be-af)(dg-ch)}\right)}{(bg-ah)\sqrt{c+dx}\sqrt{e+fx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqr
t[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
+ a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])
```

Maple [**A**]
time = 0.12, size = 270, normalized size = 1.68

method	result
default	$-\frac{2\sqrt{\frac{(ch-dg)(bx+a)}{(ah-bg)(dx+c)}}\,\sqrt{\frac{(ad-bc)(fx+e)}{(af-be)(dx+c)}}\,\sqrt{\frac{(ad-bc)(hx+g)}{(ah-bg)(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(ch-dg)(bx+a)}{(ah-bg)(dx+c)}},\sqrt{\frac{(cf-de)(ah-bg)}{(af-be)(ch-dg)}}\right)(a\,d^2h\,x}{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,\sqrt{bx+a}\,(ch-dg)(ad-bc)}$
elliptic	$\frac{2\sqrt{(bx+a)(dx+c)(fx+e)(hx+g)}\,\left(-\frac{a}{b}+\frac{g}{h}\right)\sqrt{\frac{\left(-\frac{g}{h}+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{g}{h}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\,\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{a}{b}\right)\left(x+\frac{c}{d}\right)}}\,\sqrt{\frac{\left(-\frac{c}{d}+\frac{a}{b}\right)(}{\left(-\frac{g}{h}+\frac{a}{b}\right)(}}}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\left(-\frac{g}{h}+\frac{c}{d}\right)\left(-\frac{c}{d}+\frac{a}{b}\right)\sqrt{bdfh\left(x+\frac{a}{b}\right)(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*((c*h-d*g)*(b*x+
a)/(a*h-b*g)/(d*x+c))^(1/2)*((a*d-b*c)*(f*x+e)/(a*f-b*e)/(d*x+c))^(1/2)*((a
*d-b*c)*(h*x+g)/(a*h-b*g)/(d*x+c))^(1/2)*EllipticF(((c*h-d*g)*(b*x+a)/(a*h-
b*g)/(d*x+c))^(1/2),((c*f-d*e)*(a*h-b*g)/(a*f-b*e)/(c*h-d*g))^(1/2))*(a*d^2
*h*x^2-b*d^2*g*x^2+2*a*c*d*h*x-2*b*c*d*g*x+a*c^2*h-b*c^2*g)/(c*h-d*g)/(a*d-
b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*d*f*h*x
^4 + a*c*f*g*x + (b*d*f*g + (b*c + a*d)*f*h)*x^3 + (a*c*f*h + (b*c + a*d)*f
```

```
*g)*x^2 + (b*d*h*x^3 + a*c*g + (b*d*g + (b*c + a*d)*h)*x^2 + (a*c*h + (b*c
+ a*d)*g)*x)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.110 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=429

$$-\frac{2b\sqrt{fg-eh}\,\sqrt{c+dx}\,\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\,\sqrt{e+fx}}{\sqrt{fg-eh}\,\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(bc-ad)(be-af)\sqrt{bg-ah}\,\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\,\sqrt{g+hx}}-\frac{2d\sqrt{\ldots}}{\ldots}$$

```
[Out] -2*d*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2
),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)
/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+
f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*b
*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-
(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-e*h+f*g)^(1/2)*(d*x+c
)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)/(-a*d+b*c)/(-a*f+b*e
)/(-a*h+b*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1
/2)
```

Rubi [A]

time = 0.18, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {177, 176, 430, 182, 435}

$$-\frac{2d\sqrt{g+hx}\,\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{bg-ah}\,\sqrt{e+fx}}{\sqrt{fg-eh}\,\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\,(bc-ad)\sqrt{bg-ah}\,\sqrt{fg-eh}\,\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\,\sqrt{fg-eh}\,\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{bg-ah}\,\sqrt{e+fx}}{\sqrt{fg-eh}\,\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}\,(bc-ad)(be-af)\sqrt{bg-ah}\,\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a
*h)))])/((b*c - a*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (2*d*Sqrt[((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*
h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*
h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 176

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[2*Sqrt[g + h*x]*(Sqrt[(
```

```
b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*
Sqrt[(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))])), Subst[Int[1/(Sq
rt[1 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 177

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 182

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2*Sqrt[c + d*x]*(Sqrt[
(-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h
*x]*Sqrt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])), Subst[Int[Sqrt
[1 + (b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} - \frac{d\int \frac{}{\sqrt{a+bx}\sqrt{c+}}}{}$$

$$= -\frac{\left(2d\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{(bc-ad)(fg-eh)\sqrt{c+dx}}$$

$$= -\frac{2b\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}E\left(\sin^{-}\right.}{(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(be}{(de}}}$$

Mathematica [A]
time = 35.39, size = 462, normalized size = 1.08

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(-b^2+\frac{bd(fg-eh)(a+bx)\sqrt{-\frac{(de-cf)(dg-ch)(e+fx)(g+hx)}{(fg-eh)^2(c+dx)^2}}-b(de-cf)(bg-ah)\sqrt{\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}}E\left(\sin^{-1}\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)+d(be-af)(bg-ah)\sqrt{\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}}F\left(\sin^{-1}\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)}{(fg-eh)(c+dx)\sqrt{-\frac{(de-cf)(dg-ch)(e+fx)(g+hx)}{(fg-eh)^2(c+dx)^2}}}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*(-b^2 + (b*d*(f*g - e*h)*(a +
b*x)*Sqrt[-(((d*e - c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(c
 + d*x)^2))] - b*(d*e - c*f)*(b*g - a*h)*Sqrt[((d*g - c*h)*(a + b*x))/((b*g
 - a*h)*(c + d*x))]*EllipticE[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g
- e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))
] + d*(b*e - a*f)*(b*g - a*h)*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c
+ d*x))]*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c +
 d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((f*g -
e*h)*(c + d*x)*Sqrt[-(((d*e - c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g -
 e*h)^2*(c + d*x)^2))])))/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9325 vs. 2(391) = 782.
time = 0.12, size = 9326, normalized size = 21.74

method	result	size
elliptic	Expression too large to display	2200
default	Expression too large to display	9326

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d*f*h
*x^5 + a^2*c*f*g*x + (b^2*d*f*g + (b^2*c + 2*a*b*d)*f*h)*x^4 + ((b^2*c + 2*
a*b*d)*f*g + (2*a*b*c + a^2*d)*f*h)*x^3 + (a^2*c*f*h + (2*a*b*c + a^2*d)*f*
g)*x^2 + (b^2*d*h*x^4 + a^2*c*g + (b^2*d*g + (b^2*c + 2*a*b*d)*h)*x^3 + ((b
^2*c + 2*a*b*d)*g + (2*a*b*c + a^2*d)*h)*x^2 + (a^2*c*h + (2*a*b*c + a^2*d)
*g)*x)*e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

3.111 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=786

$$-\frac{2d^3\sqrt{a+bx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}}-\frac{2b^3\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{(bc-ad)^2(be-af)(bg-ah)\sqrt{a+bx}}+\frac{2b(a^2d^2fh-abd^2(fg+\cdots}{(bc-\cdots}$$

```
[Out] -2*d^3*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-
c*h+d*g)/(d*x+c)^(1/2)-2*b^3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*
d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*b*(a^2*d^2*f*h-a*b*d^2*(e*h+
f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*
x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(-a*h+b*g)/(-c*h+d*g)/(b*x+a)
^(1/2)-4*b*d*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x
+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)
*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*h+b*g)^(1
/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))
^(1/2)-2*(a^2*d^2*f*h-a*b*d^2*(e*h+f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g
)))*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2)
,(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-e*h+f*g)^(1/2)*(d*
x+c)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)/(-a*d+b*c)^2/(-a*
f+b*e)/(-c*f+d*e)/(-c*h+d*g)/(-a*h+b*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-c*f+d*e
)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Rubi [F]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Defer[Int][1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx=\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7075 vs. $2(786) = 1572$.
time = 35.30, size = 7075, normalized size = 9.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 22969 vs. $2(724) = 1448$.
time = 0.19, size = 22970, normalized size = 29.22

method	result	size
elliptic	Expression too large to display	7103
default	Expression too large to display	22970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d^2*f
*h*x^6 + a^2*c^2*f*g*x + (b^2*d^2*f*g + 2*(b^2*c*d + a*b*d^2)*f*h)*x^5 + (2
*(b^2*c*d + a*b*d^2)*f*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f*h)*x^4 + ((b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*f*g + 2*(a*b*c^2 + a^2*c*d)*f*h)*x^3 + (a^2*c^2
*f*h + 2*(a*b*c^2 + a^2*c*d)*f*g)*x^2 + (b^2*d^2*h*x^5 + a^2*c^2*g + (b^2*d
^2*g + 2*(b^2*c*d + a*b*d^2)*h)*x^4 + (2*(b^2*c*d + a*b*d^2)*g + (b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*h)*x^3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g + 2*(a*b*
c^2 + a^2*c*d)*h)*x^2 + (a^2*c^2*h + 2*(a*b*c^2 + a^2*c*d)*g)*x)*e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)), x)
```

3.112 $\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{e^2(e+fx)^{1+n}}{bdf^3(1+n)}+\frac{(bc+ad)e(e+fx)^{1+n}}{b^2d^2f^2(1+n)}+\frac{(b^2c^2+abcd+a^2d^2)\,(e+fx)^{1+n}}{b^3d^3f(1+n)}-\frac{2e(e+fx)^{2+n}}{bdf^3(2+n)}-\frac{(bc+ad)(e+\ldots}{b^2d^2f^2(2+\ldots}$$

```
[Out] e^2*(f*x+e)^(1+n)/b/d/f^3/(1+n)+(a*d+b*c)*e*(f*x+e)^(1+n)/b^2/d^2/f^2/(1+n)
+(a^2*d^2+a*b*c*d+b^2*c^2)*(f*x+e)^(1+n)/b^3/d^3/f/(1+n)-2*e*(f*x+e)^(2+n)/
b/d/f^3/(2+n)-(a*d+b*c)*(f*x+e)^(2+n)/b^2/d^2/f^2/(2+n)+(f*x+e)^(3+n)/b/d/f
^3/(3+n)-a^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/b
^3/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d
*(f*x+e)/(-c*f+d*e))/d^3/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A]
time = 0.19, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {186, 45, 70}

$$-\frac{a^4(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)}+\frac{(a^2d^2+abcd+b^2c^2)\,(e+fx)^{n+1}}{b^3d^3f(n+1)}+\frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)}-\frac{(ad+bc)(e+fx)^{n+2}}{b^2d^2f^2(n+2)}+\frac{c^4(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^3(n+1)(bc-ad)(de-cf)}+\frac{e^2(e+fx)^{n+1}}{bdf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bdf^3(n+2)}+\frac{(e+fx)^{n+3}}{bdf^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*d*f^3*(1 + n)) + ((b*c + a*d)*e*(e + f*x)^(1 + n
))/(b^2*d^2*f^2*(1 + n)) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(e + f*x)^(1 + n)
)/(b^3*d^3*f*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*d*f^3*(2 + n)) - ((b*c +
 a*d)*(e + f*x)^(2 + n))/(b^2*d^2*f^2*(2 + n)) + (e + f*x)^(3 + n)/(b*d*f^3
*(3 + n)) - (a^4*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
 + f*x))/(b*e - a*f)])/(b^3*(b*c - a*d)*(b*e - a*f)*(1 + n)) + (c^4*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(
d^3*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{(b^2c^2+abcd+a^2d^2)(e+fx)^n}{b^3d^3} - \frac{(bc+ad)x(e+fx)^n}{b^2d^2} + \frac{x^2(e+fx)^n}{bd} + \frac{a}{b^3(bc}\right.$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} + \frac{\int x^2(e+fx)^n\,dx}{bd} + \frac{a^4\int\frac{(e+fx)^n}{a+bx}\,dx}{b^3(bc-ad)} - \frac{c^4\int\frac{(e+f}{c+}}{d^3(bc-}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{a^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)} +$$

$$= \frac{e^2(e+fx)^{1+n}}{bdf^3(1+n)} + \frac{(bc+ad)e(e+fx)^{1+n}}{b^2d^2f^2(1+n)} + \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{2e(}{bd}$$

Mathematica [A]
time = 1.02, size = 285, normalized size = 0.89

$$\frac{(e+fx)^{1+n}\left(-\frac{a^4d^3\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)}+\frac{-((bc-ad)(-de+cf)(a^2d^2f^2(6+5n+n^2)+abdf(3+n)(cf(2+n)+d(e-f(1+n)x))+b^2(c^2f^2(6+5n+n^2)+cdf(3+n)(e-f(1+n)x)+d^2(2e^2-2ef(1+n)x+f^2(2+3n+n^2)x^2))))+b^3c^4f^3(6+5n+n^2)\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(-bc+ad)f^3(-de+cf)(2+n)(3+n)}\right)}{b^3d^3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((a^4*d^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f
*x))/(b*e - a*f)])/((b*c - a*d)*(b*e - a*f))) + (-((b*c - a*d)*(-(d*e) + c*
f)*(a^2*d^2*f^2*(6 + 5*n + n^2) + a*b*d*f*(3 + n)*(c*f*(2 + n) + d*(e - f*(
1 + n)*x)) + b^2*(c^2*f^2*(6 + 5*n + n^2) + c*d*f*(3 + n)*(e - f*(1 + n)*x)
 + d^2*(2*e^2 - 2*e*f*(1 + n)*x + f^2*(2 + 3*n + n^2)*x^2)))) + b^3*c^4*f^3
*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*
f)])/((-(b*c) + a*d)*f^3*(-(d*e) + c*f)*(2 + n)*(3 + n))))/(b^3*d^3*(1 + n)
)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^4(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 (e+f x)^n}{(a+b x) (c+d x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.113 $\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=216

$$-\frac{e(e+fx)^{1+n}}{bdf^2(1+n)}-\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)}+\frac{(e+fx)^{2+n}}{bdf^2(2+n)}+\frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)}-\frac{c^3(e+\ldots}{}$$

```
[Out] -e*(f*x+e)^(1+n)/b/d/f^2/(1+n)-(a*d+b*c)*(f*x+e)^(1+n)/b^2/d^2/f/(1+n)+(f*x
+e)^(2+n)/b/d/f^2/(2+n)+a^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e
)/(-a*f+b*e))/b^2/(-a*d+b*c)/(-a*f+b*e)/(1+n)-c^3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A]

time = 0.11, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {186, 45, 70}

$$\frac{a^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)}-\frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)}-\frac{c^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)}-\frac{e(e+fx)^{n+1}}{bdf^2(n+1)}+\frac{(e+fx)^{n+2}}{bdf^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*d*f^2*(1 + n))) - ((b*c + a*d)*(e + f*x)^(1 + n)
)/(b^2*d^2*f*(1 + n)) + (e + f*x)^(2 + n)/(b*d*f^2*(2 + n)) + (a^3*(e + f*x
)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b
^2*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (c^3*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*(b*c - a*d)*(d*e - c*
f)*(1 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{(-bc-ad)(e+fx)^n}{b^2d^2} + \frac{x(e+fx)^n}{bd} - \frac{a^3(e+fx)^n}{b^2(bc-ad)(a+bx)} - \frac{c^3(e+fx)^n}{d^2(-bc+ad)(c+\cdots}\right.$$

$$= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{\int x(e+fx)^n\,dx}{bd} - \frac{a^3\int\frac{(e+fx)^n}{a+bx}\,dx}{b^2(bc-ad)} + \frac{c^3\int\frac{(e+fx)^n}{c+dx}\,dx}{d^2(bc-ad)}$$

$$= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} - \frac{c^3(e+fx\cdots}{d^2(\cdots}$$

$$= -\frac{e(e+fx)^{1+n}}{bdf^2(1+n)} - \frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{(e+fx)^{2+n}}{bdf^2(2+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+\cdots\right.}{b^2(bc-ad)(be-\cdots}$$

Mathematica [A]
time = 0.54, size = 174, normalized size = 0.81

$$\frac{(e+fx)^{1+n}\left(\frac{a^3\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{be-af} + \frac{(bc-ad)(-de+cf)(bcf(2+n)+adf(2+n)+bd(e-f(1+n)x))-b^2c^3f^2(2+n)\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d^2f^2(de-cf)(2+n)}\right)}{b^2(bc-ad)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((a^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(b*e - a*f) + ((b*c - a*d)*(-(d*e) + c*f)*(b*c*f*(2 + n) + a*d
*f*(2 + n) + b*d*(e - f*(1 + n)*x)) - b^2*c^3*f^2*(2 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*f^2*(d*e - c*f)*(2 + n))
))/(b^2*(b*c - a*d)*(1 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^3(fx+e)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^3/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x**3*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(e+f\,x\right)}^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.114 $\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{(e+fx)^{1+n}}{bdf(1+n)}-\frac{a^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)}+\frac{c^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{d(bc-ad)(de-cf)(1+n)}$$

```
[Out] (f*x+e)^(1+n)/b/d/f/(1+n)-a^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x
+e)/(-a*f+b*e))/b/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^2*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A]

time = 0.08, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {186, 70}

$$-\frac{a^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)}+\frac{c^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)}+\frac{(e+fx)^{n+1}}{bdf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*d*f*(1 + n)) - (a^2*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b*(b*c - a*d)*(b*e - a*f)*(
1 + n)) + (c^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e +
 f*x))/(d*e - c*f)])/(d*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{(e+fx)^n}{bd} + \frac{a^2(e+fx)^n}{b(bc-ad)(a+bx)} + \frac{c^2(e+fx)^n}{d(-bc+ad)(c+dx)}\right)dx$$
$$= \frac{(e+fx)^{1+n}}{bdf(1+n)} + \frac{a^2\int\frac{(e+fx)^n}{a+bx}\,dx}{b(bc-ad)} - \frac{c^2\int\frac{(e+fx)^n}{c+dx}\,dx}{d(bc-ad)}$$
$$= \frac{(e+fx)^{1+n}}{bdf(1+n)} - \frac{a^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)} + \frac{c^2(e+fx)^{1+n}\,{}_2F_1\left(\right.}{d(bc-ad)(}$$

Mathematica [A]
time = 0.18, size = 153, normalized size = 0.97

$$\frac{(e+fx)^{1+n}\left(a^2df(-de+cf)\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)+(be-af)\left(-((bc-ad)(-de+cf))+bc^2f\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)\right)\right)}{bd(bc-ad)f(be-af)(de-cf)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a^2*d*f*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 +
n, (b*(e + f*x))/(b*e - a*f)] + (b*e - a*f)*(-((b*c - a*d)*(-(d*e) + c*f))
+ b*c^2*f*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])))/
(b*d*(b*c - a*d)*f*(b*e - a*f)*(d*e - c*f)*(1 + n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2(fx+e)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^2/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x**2*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(e+f\,x\right)}^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.115 $\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}-\frac{c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

```
[Out] a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/(
-a*f+b*e)/(1+n)-c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*
e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {162, 70}

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}-\frac{c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e
- a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n)) - (c*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e - c
*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = -\frac{a\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad} + \frac{c\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}$$
$$= \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} - \frac{c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+}{de-}\right.}{(bc-ad)(de-cf)(1+n)}$$

Mathematica [A]
time = 0.13, size = 116, normalized size = 0.94

$$\frac{(e+fx)^{1+n}\left(a(-de+cf)\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)+c(be-af)\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)\right)}{(bc-ad)(be-af)(-de+cf)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
[Out] ((e + f*x)^(1 + n)*(a*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*
(e + f*x))/(b*e - a*f)] + c*(b*e - a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(fx+e)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
[Out] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.116 $\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}+\frac{d(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

```
[Out] -b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/
(-a*f+b*e)/(1+n)+d*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d
*e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {88, 70}

$$\frac{d(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}-\frac{b(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n))) + (d*(e + f*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e
 - c*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{b\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}$$
$$= -\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} + \frac{d(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{(bc-ad)(de-cf)(1+n)}$$

Mathematica [A]
time = 0.11, size = 116, normalized size = 0.94

$$\frac{(e+fx)^{1+n}\left(b(de-cf)\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)+d(-be+af)\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)\right)}{(bc-ad)(be-af)(-de+cf)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(b*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
+ f*x))/(b*e - a*f)] + d*(-(b*e) + a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral((e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((e + f*x)^n/((a + b*x)*(c + d*x)), x)
```

3.117 $\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{b^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)}-\frac{d^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{c(bc-ad)(de-cf)(1+n)}-\frac{(e+fx)^{1+n}\,{}_2F_1(}{ac}$$

```
[Out] b^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a/(-a*d+b*
c)/(-a*f+b*e)/(1+n)-d^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-
c*f+d*e))/c/(-a*d+b*c)/(-c*f+d*e)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2
+n],1+f*x/e)/a/c/e/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {186, 67, 70}

$$\frac{b^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)}-\frac{d^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{ace(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] (b^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/(a*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (d^2*(e + f*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c*(b*c - a*d)*
(d*e - c*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, 1 + (f*x)/e])/(a*c*e*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
```

```
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx &= \int \left(\frac{(e+fx)^n}{acx}+\frac{b^2(e+fx)^n}{a(-bc+ad)(a+bx)}+\frac{d^2(e+fx)^n}{c(bc-ad)(c+dx)}\right)dx\\ &= \frac{\int \frac{(e+fx)^n}{x}\,dx}{ac}-\frac{b^2\int \frac{(e+fx)^n}{a+bx}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{(e+fx)^n}{c+dx}\,dx}{c(bc-ad)}\\ &= \frac{b^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)}-\frac{d^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{c(bc-ad)(de-cf)(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 177, normalized size = 1.01

$$(e+fx)^n\left(\frac{b^2(e+fx)\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a(-bc+ad)(-be+af)(1+n)}+\frac{\frac{d^2(e+fx)\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(-de+cf)(1+n)}+\frac{\left(1+\frac{e}{fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{e}{fx}\right)}{an}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] (e + f*x)^n*((b^2*(e + f*x)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x)
)/(b*e - a*f)])/(a*(-(b*c) + a*d)*(-(b*e) + a*f)*(1 + n)) + ((d^2*(e + f*x)
*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d
)*(-(d*e) + c*f)*(1 + n)) + Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*x))]/(a
*n*(1 + e/(f*x))^n))/c)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{x\,(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)^n}{x\,(a + bx)\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral((e + f*x)**n/(x*(a + b*x)*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e+f\,x\right)}^n}{x\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x*(a + b*x)*(c + d*x)),x)
```

```
[Out] int((e + f*x)^n/(x*(a + b*x)*(c + d*x)), x)
```

3.118 $\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=222

$$-\frac{b^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)}+\frac{d^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{c^2(bc-ad)(de-cf)(1+n)}+\frac{(bc+ad)(e+fx}{}$$

```
[Out] -b^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a^2/(-a*d
+b*c)/(-a*f+b*e)/(1+n)+d^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)
/(-c*f+d*e))/c^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)+(a*d+b*c)*(f*x+e)^(1+n)*hyperg
eom([1, 1+n],[2+n],1+f*x/e)/a^2/c^2/e/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1
+n],[2+n],1+f*x/e)/a/c/e^2/(1+n)
```

Rubi [A]
time = 0.11, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {186, 67, 70}

$$-\frac{b^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)}+\frac{(ad+bc)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{a^2c^2e(n+1)}+\frac{d^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{c^2(n+1)(bc-ad)(de-cf)}+\frac{f(e+fx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{ace^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] -((b^3*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (d^3*(e + f*x)^(1 + n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c^2*(b*c
- a*d)*(d*e - c*f)*(1 + n)) + ((b*c + a*d)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*c^2*e*(1 + n)) + (f*(e + f*x)^(1 +
n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*c*e^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \left(\frac{(e+fx)^n}{acx^2} + \frac{(-bc-ad)(e+fx)^n}{a^2c^2x} - \frac{b^3(e+fx)^n}{a^2(-bc+ad)(a+bx)} - \frac{d^3(e+f}{c^2(bc-ad)(}\right.$$

$$= \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{ac} + \frac{b^3\int \frac{(e+fx)^n}{a+bx}\,dx}{a^2(bc-ad)} - \frac{d^3\int \frac{(e+fx)^n}{c+dx}\,dx}{c^2(bc-ad)} - \frac{(bc+ad)\int \frac{(e+fx)^n}{x}\,dx}{a^2c^2}$$

$$= -\frac{b^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)} + \frac{d^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+\right.}{c^2(bc-ad)(de-cf)(1+}$$

Mathematica [A]
time = 0.60, size = 228, normalized size = 1.03

$$(e+fx)^n\left(-\frac{b^3(e+fx)\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)}+\frac{-\frac{d^3(e+fx)\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(-de+cf)(1+n)}+\frac{\left(1+\frac{e}{fx}\right)^{-n}\left(acn\,{}_2F_1\left(1-n,-n;2-n;-\frac{e}{fx}\right)-(bc+ad)(-1+n)x\,{}_2F_1\left(-n,-n;1-n;-\frac{e}{fx}\right)\right)}{a^2(-1+n)nx}}{c^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] (e + f*x)^n*(-((b^3*(e + f*x)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*
x))/(b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (-((d^3*(e + f*x
)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*
d)*(-(d*e) + c*f)*(1 + n))) + (a*c*n*Hypergeometric2F1[1 - n, -n, 2 - n, -(
e/(f*x))] - (b*c + a*d)*(-1 + n)*x*Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*
x))])/(a^2*(-1 + n)*n*(1 + e/(f*x))^n*x))/c^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{x^2\,(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(b*d*x^4 + a*c*x^2 + (b*c + a*d)*x^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x^2\,(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x)
[Out] int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)), x)
```

3.119 $\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=167

$$\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)}+\frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))\,(a+bx}{b^4(2+m)}$$

```
[Out] (-a*d+b*c)*(-a*f+b*e)*(-a*h+b*g)*(b*x+a)^(1+m)/b^4/(1+m)+(3*a^2*d*f*h+b^2*(
c*e*h+c*f*g+d*e*g)-2*a*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(2+m)/b^4/(2+m)-(3*a*
d*f*h-b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(3+m)/b^4/(3+m)+d*f*h*(b*x+a)^(4+m)/b^
4/(4+m)
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {147}

$$\frac{(a+bx)^{m+2}\,(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg))}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-\frac{(a+bx)^{m+3}(3adfh-b(cfh+deh+dfg))}{b^4(m+3)}+\frac{dfh(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((3
*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(
a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*
(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx &= \int \left(\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^m}{b^3}+\frac{(3a^2dfh+b^2(deg\right.\\ &= \frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)}+\frac{(3a^2dfh+b^2(deg+\end{aligned}$$

Mathematica [A]
time = 0.39, size = 241, normalized size = 1.44

$$\frac{(a+bx)^{1+m}\,(-6a^3dfh+2a^2b(cfh(4+m)+d(fg(4+m)+eh(4+m)+3fh(1+m)x))-ab^2(c(4+m)(fg(3+m)+eh(3+m)+2fh(1+m)x)+d(e(4+m)(g(3+m)+2h(1+m)x)+f(1+m)x(2g(4+m)+3h(2+m)x)))+b^3(c(4+m)(e(3+m)(g(2+m)+h(1+m)x)+f(1+m)x(g(3+m)+h(2+m)x))+d(1+m)x(e(4+m)(g(3+m)+h(2+m)x)+f(2+m)x(g(4+m)+h(3+m)x))))}{b^4(1+m)(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-6*a^3*d*f*h + 2*a^2*b*(c*f*h*(4 + m) + d*(f*g*(4 + m)
+ e*h*(4 + m) + 3*f*h*(1 + m)*x)) - a*b^2*(c*(4 + m)*(f*g*(3 + m) + e*h*(3
+ m) + 2*f*h*(1 + m)*x) + d*(e*(4 + m)*(g*(3 + m) + 2*h*(1 + m)*x) + f*(1 +
 m)*x*(2*g*(4 + m) + 3*h*(2 + m)*x))) + b^3*(c*(4 + m)*(e*(3 + m)*(g*(2 + m
) + h*(1 + m)*x) + f*(1 + m)*x*(g*(3 + m) + h*(2 + m)*x)) + d*(1 + m)*x*(e*
(4 + m)*(g*(3 + m) + h*(2 + m)*x) + f*(2 + m)*x*(g*(4 + m) + h*(3 + m)*x)))
))/(b^4*(1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(167) = 334$.
time = 0.10, size = 726, normalized size = 4.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(-b^3*d*f*h*m^3*x^3-b^3*c*f*h*m^3*x^2-b^3*d*e*h*m^3*x^2-b^3*
d*f*g*m^3*x^2-6*b^3*d*f*h*m^2*x^3+3*a*b^2*d*f*h*m^2*x^2-b^3*c*e*h*m^3*x-b^3
*c*f*g*m^3*x-7*b^3*c*f*h*m^2*x^2-b^3*d*e*g*m^3*x-7*b^3*d*e*h*m^2*x^2-7*b^3*
d*f*g*m^2*x^2-11*b^3*d*f*h*m*x^3+2*a*b^2*c*f*h*m^2*x+2*a*b^2*d*e*h*m^2*x+2*
a*b^2*d*f*g*m^2*x+9*a*b^2*d*f*h*m*x^2-b^3*c*e*g*m^3-8*b^3*c*e*h*m^2*x-8*b^3
*c*f*g*m^2*x-14*b^3*c*f*h*m*x^2-8*b^3*d*e*g*m^2*x-14*b^3*d*e*h*m*x^2-14*b^3
*d*f*g*m*x^2-6*b^3*d*f*h*x^3-6*a^2*b*d*f*h*m*x+a*b^2*c*e*h*m^2+a*b^2*c*f*g*
m^2+10*a*b^2*c*f*h*m*x+a*b^2*d*e*g*m^2+10*a*b^2*d*e*h*m*x+10*a*b^2*d*f*g*m*
x+6*a*b^2*d*f*h*x^2-9*b^3*c*e*g*m^2-19*b^3*c*e*h*m*x-19*b^3*c*f*g*m*x-8*b^3
*c*f*h*x^2-19*b^3*d*e*g*m*x-8*b^3*d*e*h*x^2-8*b^3*d*f*g*x^2-2*a^2*b*c*f*h*m
-2*a^2*b*d*e*h*m-2*a^2*b*d*f*g*m-6*a^2*b*d*f*h*x+7*a*b^2*c*e*h*m+7*a*b^2*c*
f*g*m+8*a*b^2*c*f*h*x+7*a*b^2*d*e*g*m+8*a*b^2*d*e*h*x+8*a*b^2*d*f*g*x-26*b^
3*c*e*g*m-12*b^3*c*e*h*x-12*b^3*c*f*g*x-12*b^3*d*e*g*x+6*a^3*d*f*h-8*a^2*b*
c*f*h-8*a^2*b*d*e*h-8*a^2*b*d*f*g+12*a*b^2*c*e*h+12*a*b^2*c*f*g+12*a*b^2*d*
e*g-24*b^3*c*e*g)/b^4/(m^4+10*m^3+35*m^2+50*m+24)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(173) = 346$.
time = 0.35, size = 484, normalized size = 2.90

$$\frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m cfg}{(m^2+3m+2)b^2}+\frac{(bx+a)^{m+1}cge}{b(m+1)}+\frac{((m^2+3m+2)b^3x^3+(m^2+m)ab^2x^2-2a^2bmx+2a^3)(bx+a)^m dfg}{(m^3+6m^2+11m+6)b^3}+\frac{((m^2+3m+2)b^3x^3+(m^2+m)ab^2x^2-2a^2bmx+2a^3)(bx+a)^m cfh}{(m^3+6m^2+11m+6)b^3}+\frac{(b^2(m+1)x^2+abmx-a^2)dge^{(m\log(bx+a)+1)}}{(m^2+3m+2)b^2}+\frac{(b^2(m+1)x^2+abmx-a^2)che^{(m\log(bx+a)+1)}}{(m^2+3m+2)b^2}+\frac{((m^3+6m^2+11m+6)b^4x^4+(m^3+3m^2+2m)ab^3x^3-3(m^2+m)a^2b^2x^2+6a^3bmx-6a^4)(bx+a)^m dfh}{(m^4+10m^3+35m^2+50m+24)b^4}+\frac{((m^2+3m+2)b^3x^3+(m^2+m)ab^2x^2-2a^2bmx+2a^3)dhe^{(m\log(bx+a)+1)}}{(m^3+6m^2+11m+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*f*g/((m^2 + 3*m + 2)*b^2) +
 (b*x + a)^(m + 1)*c*g*e/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)
*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d*f*g/((m^3 + 6*m^2 + 11*m +
```

```
6)*b^3) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*
a^3)*(b*x + a)^m*c*f*h/((m^3 + 6*m^2 + 11*m + 6)*b^3) + (b^2*(m + 1)*x^2 +
a*b*m*x - a^2)*d*g*e^(m*log(b*x + a) + 1)/((m^2 + 3*m + 2)*b^2) + (b^2*(m +
 1)*x^2 + a*b*m*x - a^2)*c*h*e^(m*log(b*x + a) + 1)/((m^2 + 3*m + 2)*b^2) +
 ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m^2 + 2*m)*a*b^3*x^3 - 3*(m^2
 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x + a)^m*d*f*h/((m^4 + 10*m^3 +
 35*m^2 + 50*m + 24)*b^4) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2
- 2*a^2*b*m*x + 2*a^3)*d*h*e^(m*log(b*x + a) + 1)/((m^3 + 6*m^2 + 11*m + 6)
*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(173) = 346$.
time = 1.08, size = 892, normalized size = 5.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*c*f*g*m^2 - (b^4*d*f*h*m^3 + 6*b^4*d*f*h*m^2 + 11*b^4*d*f*h*m + 6
*b^4*d*f*h)*x^4 - (8*b^4*d*f*g + 8*b^4*c*f*h + (b^4*d*f*g + (b^4*c + a*b^3*
d)*f*h)*m^3 + (7*b^4*d*f*g + (7*b^4*c + 3*a*b^3*d)*f*h)*m^2 + 2*(7*b^4*d*f*
g + (7*b^4*c + a*b^3*d)*f*h)*m)*x^3 + 4*(3*a^2*b^2*c - 2*a^3*b*d)*f*g - 2*(
4*a^3*b*c - 3*a^4*d)*f*h - (12*b^4*c*f*g + (a*b^3*c*f*h + (b^4*c + a*b^3*d)
*f*g)*m^3 + ((8*b^4*c + 5*a*b^3*d)*f*g + (5*a*b^3*c - 3*a^2*b^2*d)*f*h)*m^2
 + ((19*b^4*c + 4*a*b^3*d)*f*g + (4*a*b^3*c - 3*a^2*b^2*d)*f*h)*m)*x^2 - (2
*a^3*b*c*f*h - (7*a^2*b^2*c - 2*a^3*b*d)*f*g)*m - (a*b^3*c*f*g*m^3 - (2*a^2
*b^2*c*f*h - (7*a*b^3*c - 2*a^2*b^2*d)*f*g)*m^2 + 2*(2*(3*a*b^3*c - 2*a^2*b
^2*d)*f*g - (4*a^2*b^2*c - 3*a^3*b*d)*f*h)*m)*x - (a*b^3*c*g*m^3 + (b^4*d*h
*m^3 + 7*b^4*d*h*m^2 + 14*b^4*d*h*m + 8*b^4*d*h)*x^3 - (a^2*b^2*c*h - (9*a*
b^3*c - a^2*b^2*d)*g)*m^2 + (12*b^4*d*g + 12*b^4*c*h + (b^4*d*g + (b^4*c +
a*b^3*d)*h)*m^3 + (8*b^4*d*g + (8*b^4*c + 5*a*b^3*d)*h)*m^2 + (19*b^4*d*g +
 (19*b^4*c + 4*a*b^3*d)*h)*m)*x^2 + 12*(2*a*b^3*c - a^2*b^2*d)*g - 4*(3*a^2
*b^2*c - 2*a^3*b*d)*h + ((26*a*b^3*c - 7*a^2*b^2*d)*g - (7*a^2*b^2*c - 2*a^
3*b*d)*h)*m + (24*b^4*c*g + (a*b^3*c*h + (b^4*c + a*b^3*d)*g)*m^3 + ((9*b^4
*c + 7*a*b^3*d)*g + (7*a*b^3*c - 2*a^2*b^2*d)*h)*m^2 + 2*((13*b^4*c + 6*a*b
^3*d)*g + 2*(3*a*b^3*c - 2*a^2*b^2*d)*h)*m)*x)*e)*(b*x + a)^m/(b^4*m^4 + 10
*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8221 vs. $2(160) = 320$.
time = 1.92, size = 8221, normalized size = 49.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)*(h*x+g),x)

[Out] Piecewise((a**m*(c*e*g*x + c*e*h*x**2/2 + c*f*g*x**2/2 + c*f*h*x**3/3 + d*e
*g*x**2/2 + d*e*h*x**3/3 + d*f*g*x**3/3 + d*f*h*x**4/4), Eq(b, 0)), (6*a**3
*d*f*h*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) + 11*a**3*d*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) - 2*a**2*b*c*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) - 2*a**2*b*d*e*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) - 2*a**2*b*d*f*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*f*h*x*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*f*h*x/(6*a**3*b**4
+ 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*e*h/(6*a**3*b**
4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*f*g/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*c*f*h*x/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*d*e*g/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*d*e
*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b*
*2*d*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*f*h*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c*e*g/(6*a**3*b**4 + 18*a**2*b**5*x
 + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*e*h*x/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*f*g*x/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*c*f*h*x**2/(6*a**3*b**4 +
 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*d*e*g*x/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*e*h*x**2/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*f*g*x
**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*
d*f*h*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*
b**7*x**3), Eq(m, -4)), (-6*a**3*d*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5
*x + 2*b**6*x**2) - 9*a**3*d*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) +
 2*a**2*b*c*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a
**2*b*c*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*e*h*log(a
/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*e*h/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*f*g*log(a/b + x)/(2*a**2*b**4
+ 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*f*g/(2*a**2*b**4 + 4*a*b**5*x + 2*
b**6*x**2) - 12*a**2*b*d*f*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) - 12*a**2*b*d*f*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*
b**2*c*e*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b**2*c*f*g/(2*a**2*
b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a**2*b*
*4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - a*b**2*d*e*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4
*a*b**2*d*e*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a
*b**2*d*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x*l
og(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x/(2*
```

```
a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*f*h*x**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c*e*g/(2*a**2*b**4 + 4*a*b**5
*x + 2*b**6*x**2) - 2*b**3*c*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2)
 - 2*b**3*c*f*g*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*c*f*h*x
**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*d*e*g*x/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*e*h*x**2*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*g*x**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*h*x**3/(2*a**2*b**4 + 4
*a*b**5*x + 2*b**6*x**2), Eq(m, -3)), (6*a**3*d*f*h*log(a/b + x)/(2*a*b**4
+ 2*b**5*x) + 6*a**3*d*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h*log(a/b +
 x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b
*d*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h/(2*a*b**4 + 2*b*
*5*x) - 4*a**2*b*d*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*f*g/
(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
 + 2*a*b**2*c*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*e*h/(2*a*
b**4 + 2*b**5*x) + 2*a*b**2*c*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*
b**2*c*f*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a*b**4
+ 2*b**5*x) + 2*a*b**2*d*e*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*
d*e*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*d*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b
**5*x) - 4*a*b**2*d*f*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*f
*h*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c*e*g/(2*a*b**4 + 2*b**5*x) + 2*b**3
*c*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*g*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*h*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e
*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1665 vs. $2(173) = 346$.
time = 0.80, size = 1665, normalized size = 9.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d*f*h*m^3*x^4 + (b*x + a)^m*b^4*d*f*g*m^3*x^3 + (b*x + a)^
m*b^4*c*f*h*m^3*x^3 + (b*x + a)^m*a*b^3*d*f*h*m^3*x^3 + 6*(b*x + a)^m*b^4*d
*f*h*m^2*x^4 + (b*x + a)^m*b^4*d*h*m^3*x^3*e + (b*x + a)^m*b^4*c*f*g*m^3*x^
2 + (b*x + a)^m*a*b^3*d*f*g*m^3*x^2 + (b*x + a)^m*a*b^3*c*f*h*m^3*x^2 + 7*(
b*x + a)^m*b^4*d*f*g*m^2*x^3 + 7*(b*x + a)^m*b^4*c*f*h*m^2*x^3 + 3*(b*x + a
)^m*a*b^3*d*f*h*m^2*x^3 + 11*(b*x + a)^m*b^4*d*f*h*m*x^4 + (b*x + a)^m*b^4*
d*g*m^3*x^2*e + (b*x + a)^m*b^4*c*h*m^3*x^2*e + (b*x + a)^m*a*b^3*d*h*m^3*x
^2*e + 7*(b*x + a)^m*b^4*d*h*m^2*x^3*e + (b*x + a)^m*a*b^3*c*f*g*m^3*x + 8*
(b*x + a)^m*b^4*c*f*g*m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*f*g*m^2*x^2 + 5*(b*x
+ a)^m*a*b^3*c*f*h*m^2*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m^2*x^2 + 14*(b*x
+ a)^m*b^4*d*f*g*m*x^3 + 14*(b*x + a)^m*b^4*c*f*h*m*x^3 + 2*(b*x + a)^m*a*b
```

```
^3*d*f*h*m*x^3 + 6*(b*x + a)^m*b^4*d*f*h*x^4 + (b*x + a)^m*b^4*c*g*m^3*x*e
+ (b*x + a)^m*a*b^3*d*g*m^3*x*e + (b*x + a)^m*a*b^3*c*h*m^3*x*e + 8*(b*x +
a)^m*b^4*d*g*m^2*x^2*e + 8*(b*x + a)^m*b^4*c*h*m^2*x^2*e + 5*(b*x + a)^m*a*
b^3*d*h*m^2*x^2*e + 14*(b*x + a)^m*b^4*d*h*m*x^3*e + 7*(b*x + a)^m*a*b^3*c*
f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*c*f*h
*m^2*x + 19*(b*x + a)^m*b^4*c*f*g*m*x^2 + 4*(b*x + a)^m*a*b^3*d*f*g*m*x^2 +
 4*(b*x + a)^m*a*b^3*c*f*h*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m*x^2 + 8*(b
*x + a)^m*b^4*d*f*g*x^3 + 8*(b*x + a)^m*b^4*c*f*h*x^3 + (b*x + a)^m*a*b^3*c
*g*m^3*e + 9*(b*x + a)^m*b^4*c*g*m^2*x*e + 7*(b*x + a)^m*a*b^3*d*g*m^2*x*e
+ 7*(b*x + a)^m*a*b^3*c*h*m^2*x*e - 2*(b*x + a)^m*a^2*b^2*d*h*m^2*x*e + 19*
(b*x + a)^m*b^4*d*g*m*x^2*e + 19*(b*x + a)^m*b^4*c*h*m*x^2*e + 4*(b*x + a)^
m*a*b^3*d*h*m*x^2*e + 8*(b*x + a)^m*b^4*d*h*x^3*e - (b*x + a)^m*a^2*b^2*c*f
*g*m^2 + 12*(b*x + a)^m*a*b^3*c*f*g*m*x - 8*(b*x + a)^m*a^2*b^2*d*f*g*m*x -
 8*(b*x + a)^m*a^2*b^2*c*f*h*m*x + 6*(b*x + a)^m*a^3*b*d*f*h*m*x + 12*(b*x
+ a)^m*b^4*c*f*g*x^2 + 9*(b*x + a)^m*a*b^3*c*g*m^2*e - (b*x + a)^m*a^2*b^2*
d*g*m^2*e - (b*x + a)^m*a^2*b^2*c*h*m^2*e + 26*(b*x + a)^m*b^4*c*g*m*x*e +
12*(b*x + a)^m*a*b^3*d*g*m*x*e + 12*(b*x + a)^m*a*b^3*c*h*m*x*e - 8*(b*x +
a)^m*a^2*b^2*d*h*m*x*e + 12*(b*x + a)^m*b^4*d*g*x^2*e + 12*(b*x + a)^m*b^4*
c*h*x^2*e - 7*(b*x + a)^m*a^2*b^2*c*f*g*m + 2*(b*x + a)^m*a^3*b*d*f*g*m + 2
*(b*x + a)^m*a^3*b*c*f*h*m + 26*(b*x + a)^m*a*b^3*c*g*m*e - 7*(b*x + a)^m*a
^2*b^2*d*g*m*e - 7*(b*x + a)^m*a^2*b^2*c*h*m*e + 2*(b*x + a)^m*a^3*b*d*h*m*
e + 24*(b*x + a)^m*b^4*c*g*x*e - 12*(b*x + a)^m*a^2*b^2*c*f*g + 8*(b*x + a)
^m*a^3*b*d*f*g + 8*(b*x + a)^m*a^3*b*c*f*h - 6*(b*x + a)^m*a^4*d*f*h + 24*(
b*x + a)^m*a*b^3*c*g*e - 12*(b*x + a)^m*a^2*b^2*d*g*e - 12*(b*x + a)^m*a^2*
b^2*c*h*e + 8*(b*x + a)^m*a^3*b*d*h*e)/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 +
 50*b^4*m + 24*b^4)
```

Mupad [B]
time = 2.95, size = 819, normalized size = 4.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x),x)
```

```
[Out] (x*(a + b*x)^m*(24*b^4*c*e*g + 9*b^4*c*e*g*m^2 + b^4*c*e*g*m^3 + 26*b^4*c*e
*g*m + 12*a*b^3*c*e*h*m + 12*a*b^3*c*f*g*m + 12*a*b^3*d*e*g*m + 6*a^3*b*d*f
*h*m + 7*a*b^3*c*e*h*m^2 + 7*a*b^3*c*f*g*m^2 + 7*a*b^3*d*e*g*m^2 + a*b^3*c*
e*h*m^3 + a*b^3*c*f*g*m^3 + a*b^3*d*e*g*m^3 - 8*a^2*b^2*c*f*h*m - 8*a^2*b^2
*d*e*h*m - 8*a^2*b^2*d*f*g*m - 2*a^2*b^2*c*f*h*m^2 - 2*a^2*b^2*d*e*h*m^2 -
2*a^2*b^2*d*f*g*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x
)^m*(6*a^4*d*f*h + 12*a^2*b^2*c*e*h + 12*a^2*b^2*c*f*g + 12*a^2*b^2*d*e*g -
 24*a*b^3*c*e*g - 8*a^3*b*c*f*h - 8*a^3*b*d*e*h - 8*a^3*b*d*f*g - 26*a*b^3*
c*e*g*m - 2*a^3*b*c*f*h*m - 2*a^3*b*d*e*h*m - 2*a^3*b*d*f*g*m - 9*a*b^3*c*e
*g*m^2 - a*b^3*c*e*g*m^3 + 7*a^2*b^2*c*e*h*m + 7*a^2*b^2*c*f*g*m + 7*a^2*b^
```

```
2*d*e*g*m + a^2*b^2*c*e*h*m^2 + a^2*b^2*c*f*g*m^2 + a^2*b^2*d*e*g*m^2))/(b^
4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(3*m + m^2 + 2)*(
4*b*c*f*h + 4*b*d*e*h + 4*b*d*f*g + a*d*f*h*m + b*c*f*h*m + b*d*e*h*m + b*d
*f*g*m))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^2*(m + 1)*(a + b*x)^m
*(12*b^2*c*e*h + 12*b^2*c*f*g + 12*b^2*d*e*g + b^2*c*e*h*m^2 + b^2*c*f*g*m^
2 + b^2*d*e*g*m^2 + 7*b^2*c*e*h*m + 7*b^2*c*f*g*m + 7*b^2*d*e*g*m - 3*a^2*d
*f*h*m + a*b*c*f*h*m^2 + a*b*d*e*h*m^2 + a*b*d*f*g*m^2 + 4*a*b*c*f*h*m + 4*
a*b*d*e*h*m + 4*a*b*d*f*g*m))/(b^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (
d*f*h*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)
```

3.120 $\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=134

$$-\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)}+\frac{(dg-ch)(fg-eh)(a+bx)^{1+m}\,{}_2F_1\Big(1$$

$$h^2(bg-ah)(1+$$

```
[Out] -(b*x+a)^(1+m)*(a*d*f*h+b*(-c*f*h-d*e*h+d*f*g)*(2+m)-b*d*f*h*(1+m)*x)/b^2/h
^2/(1+m)/(2+m)+(-c*h+d*g)*(-e*h+f*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m]
,-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+b*g)/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$,
Rules used = {152, 70}

$$\frac{(a+bx)^{m+1}(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}-\frac{(a+bx)^{m+1}(adfh-bh(m+2)(cf+de)+bdfg(m+2)-bdfh(m+1)x)}{b^2h^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*d*f*h + b*d*f*g*(2 + m) - b*(d*e + c*f)*h*(2 + m) -
 b*d*f*h*(1 + m)*x))/(b^2*h^2*(1 + m)*(2 + m))) + ((d*g - c*h)*(f*g - e*h)*
(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
 a*h))])/(h^2*(b*g - a*h)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} +$$

$$= -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.34, size = 165, normalized size = 1.23

$$\frac{1}{6}(a+bx)^m\left(\frac{3(de+cf)x^2\left(1+\frac{bx}{a}\right)^{-m}F_1\left(2;-m,1;3;-\frac{bx}{a},-\frac{hx}{g}\right)}{g}+\frac{2dfx^3\left(1+\frac{bx}{a}\right)^{-m}F_1\left(3;-m,1;4;-\frac{bx}{a},-\frac{hx}{g}\right)}{g}+\frac{6ce\left(\frac{h(a+bx)}{b(g+hx)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{bg-ah}{bg+bhx}\right)}{hm}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((a + b*x)^m*((3*(d*e + c*f)*x^2*AppellF1[2, -m, 1, 3, -((b*x)/a), -((h*x)/
g)])/(g*(1 + (b*x)/a)^m) + (2*d*f*x^3*AppellF1[3, -m, 1, 4, -((b*x)/a), -((
h*x)/g)])/(g*(1 + (b*x)/a)^m) + (6*c*e*Hypergeometric2F1[-m, -m, 1 - m, (b*
g - a*h)/(b*g + b*h*x)])/(h*m*((h*(a + b*x))/(b*(g + h*x)))^m)))/6
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m\,(dx+c)\,(fx+e)}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
[Out] integral((d*f*x^2 + c*f*x + (d*x + c)*e)*(b*x + a)^m/(h*x + g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m (c+dx) (e+fx)}{g+hx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)/(h*x+g),x)
[Out] Integral((a + b*x)**m*(c + d*x)*(e + f*x)/(g + h*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e+f\,x)\,(a+b\,x)^m\,(c+d\,x)}{g+h\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x),x)
[Out] int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x), x)
```

3.121 $\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=140

$$-\frac{(de-cf)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)}+\frac{(dg-ch)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(fg-eh)(1+m)}$$

```
[Out] -(-c*f+d*e)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*x+a)/(-a*f+b*e))/(
-a*f+b*e)/(-e*h+f*g)/(1+m)+(-c*h+d*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m
],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-e*h+f*g)/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {162, 70}

$$\frac{(a+bx)^{m+1}(dg-ch)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] -(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a
 + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)*(
a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = -\frac{(de-cf)\int \frac{(a+bx)^m}{e+fx}\,dx}{fg-eh} + \frac{(dg-ch)\int \frac{(a+bx)^m}{g+hx}\,dx}{fg-eh}$$
$$= -\frac{(de-cf)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)} + \frac{(dg-ch)(a+bx)^{1+m}}{(bg-ah}$$

Mathematica [A]
time = 0.21, size = 115, normalized size = 0.82

$$\frac{(a+bx)^{1+m}\left(-\frac{(de-cf)\,{}_2F_1\left(1,1+m;2+m;\frac{f(a+bx)}{-be+af}\right)}{be-af}+\frac{(dg-ch)\,{}_2F_1\left(1,1+m;2+m;\frac{h(a+bx)}{-bg+ah}\right)}{bg-ah}\right)}{(fg-eh)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(((d*e - c*f)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(a
 + b*x))/(-(b*e) + a*f)])/(b*e - a*f)) + ((d*g - c*h)*Hypergeometric2F1[1,
1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/(b*g - a*h)))/((f*g - e*h)*(1
+ m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m\,(dx+c)}{(fx+e)\,(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
[Out] integral((d*x + c)*(b*x + a)^m/(f*h*x^2 + f*g*x + (h*x + g)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m (c+dx)}{(e+fx)(g+hx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)/(f*x+e)/(h*x+g),x)
[Out] Integral((a + b*x)**m*(c + d*x)/((e + f*x)*(g + h*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^m\,(c+d\,x)}{(e+f\,x)\,(g+h\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x)
[Out] int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)), x)
```

3.122 $\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=224

$$\frac{d^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)}-\frac{f^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(de-cf)(fg-eh)(1+m)}+\frac{h^2(a+bx)^{1}\cdots}{(bg-a\cdots}$$

```
[Out] d^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c
)/(-c*f+d*e)/(-c*h+d*g)/(1+m)-f^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f
*(b*x+a)/(-a*f+b*e))/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)/(1+m)+h^2*(b*x+a)^(1+
m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-c*h+d*g)/(-
e*h+f*g)/(1+m)
```

Rubi [A]
time = 0.13, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {186, 70}

$$\frac{d^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])
/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)
*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \left(\frac{d^2(a+bx)^m}{(de-cf)(dg-ch)(c+dx)} + \frac{f^2(a+bx)^m}{(de-cf)(-fg+eh)(e+fx)} + \frac{}{(dg}\right.$$

$$= \frac{d^2 \int \frac{(a+bx)^m}{c+dx}\,dx}{(de-cf)(dg-ch)} - \frac{f^2 \int \frac{(a+bx)^m}{e+fx}\,dx}{(de-cf)(fg-eh)} + \frac{h^2 \int \frac{(a+bx)^m}{g+hx}\,dx}{(dg-ch)(fg-eh)}$$

$$= \frac{d^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)} - \frac{f^2(a+bx)^{1+m}\,{}_2F_1\left(1,1\right.}{(be-af)(de-cf}$$

Mathematica [A]
time = 0.58, size = 229, normalized size = 1.02

$$\frac{(a+bx)^m \left(d(fg-eh)\left(\frac{d(a+bx)}{b(c+dx)}\right)^{-m} {}_2F_1\left(-m,-m;1-m;\frac{bc-ad}{bc+bdx}\right) - f(dg-ch)\left(\frac{f(a+bx)}{b(e+fx)}\right)^{-m} {}_2F_1\left(-m,-m;1-m;\frac{be-af}{be+bfx}\right) + (de-cf)h\left(\frac{h(a+bx)}{b(g+hx)}\right)^{-m} {}_2F_1\left(-m,-m;1-m;\frac{bg-ah}{bg+bhx}\right)\right)}{(de-cf)(dg-ch)(fg-eh)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^m*((d*(f*g - e*h)*Hypergeometric2F1[-m, -m, 1 - m, (b*c - a*d)/(
b*c + b*d*x)])/((d*(a + b*x))/(b*(c + d*x)))^m - (f*(d*g - c*h)*Hypergeomet
ric2F1[-m, -m, 1 - m, (b*e - a*f)/(b*e + b*f*x)])/((f*(a + b*x))/(b*(e + f*
x)))^m + ((d*e - c*f)*h*Hypergeometric2F1[-m, -m, 1 - m, (b*g - a*h)/(b*g +
 b*h*x)])/((h*(a + b*x))/(b*(g + h*x)))^m))/((d*e - c*f)*(d*g - c*h)*(f*g -
 e*h)*m)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)\,(fx+e)\,(hx+g)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*f*h*x^3 + c*f*g*x + (d*f*g + c*f*h)*x^2 + (d*h*x^2
+ c*g + (d*g + c*h)*x)*e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^m}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

```
[Out] int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)), x)
```

3.123 $\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)}-\frac{dx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n\right.}{c(bc-ad)(1+m)}$$

```
[Out] b*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b*x/a,-f*x/e)/a/(-a*d+b*c)/(1+m)
/((1+f*x/e)^n)-d*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-d*x/c,-f*x/e)/c/(
-a*d+b*c)/(1+m)/((1+f*x/e)^n)
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {186, 140, 138}

$$\frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{dx}{c}\right)}{c(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (b*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b*x)/
a)])/(a*(b*c - a*d)*(1 + m)*(1 + (f*x)/e)^n) - (d*x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)])/(c*(b*c - a*d)*(1 + m)*
(1 + (f*x)/e)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{bx^m(e+fx)^n}{(bc-ad)(a+bx)} - \frac{dx^m(e+fx)^n}{(bc-ad)(c+dx)}\right)dx$$
$$= \frac{b\int \frac{x^m(e+fx)^n}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{x^m(e+fx)^n}{c+dx}\,dx}{bc-ad}$$
$$= \frac{\left(b(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{a+bx}\,dx}{bc-ad} - \frac{\left(d(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{c+dx}}{bc-ad}$$
$$= \frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}(e+fx)^n}{}$$

Mathematica [A]
time = 0.25, size = 104, normalized size = 0.74

$$\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\left(-bcF_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{bx}{a}\right)+adF_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{dx}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*(-(b*c*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((
b*x)/a)]) + a*d*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)]))/(a*
c*(-(b*c) + a*d)*(1 + m)*(1 + (f*x)/e)^n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(e+f\,x\right)}^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

3.124 $\int (a+bx)^m (c+dx)^n (e+fx)(g+hx)\,dx$

Optimal. Leaf size=266

$$-\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3+m+n)-bdfh(2+m+n)x)}{b^2d^2(2+m+n)(3+m+n)}+\frac{(a^2 \ldots}{}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(1+n)*(b*c*f*h*(2+m)+a*d*f*h*(2+n)-b*d*(e*h+f*g)*(3+
m+n)-b*d*f*h*(2+m+n)*x)/b^2/d^2/(2+m+n)/(3+m+n)+(a^2*d^2*f*h*(1+n)*(2+n)+a*
b*d*(1+n)*(2*c*f*h*(1+m)-d*(e*h+f*g)*(3+m+n))+b^2*(c^2*f*h*(1+m)*(2+m)-c*d*
(e*h+f*g)*(1+m)*(3+m+n)+d^2*e*g*(2+m+n)*(3+m+n)))*(b*x+a)^(1+m)*(d*x+c)^n*h
ypergeom([-n, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^3/d^2/(1+m)/(2+m+n)/(3+m+
n)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A]
time = 0.11, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {152, 72, 71}

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d(m+n+3)(eh+fg))+b^2(c^2fh(m+1)(m+2)-cd(m+1)(m+n+3)(eh+fg)+d^2eg(m+n+2)(m+n+3))\right)}{b^3d^2(m+1)(m+n+2)(m+n+3)}-\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n) -
 b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m + n)
*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*(1
+ m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*(f*g
 + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b*x)^(
1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b*c - a
*d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(f}{b^2d^2(2+m+n)(3+m}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(f}{b^2d^2(2+m+n)(3+m}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(f}{b^2d^2(2+m+n)(3+m}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.45, size = 197, normalized size = 0.74

$$\frac{1}{6}(a+bx)^m(c+dx)^n\left(3(fg+eh)x^2\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^{-n}F_1\left(2;-m,-n;3;-\frac{bx}{a},-\frac{dx}{c}\right)+2fhx^3\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^{-n}F_1\left(3;-m,-n;4;-\frac{bx}{a},-\frac{dx}{c}\right)+\frac{6eg\left(\frac{d(a+bx)}{-bc+ad}\right)^{-m}(c+dx)\,{}_2F_1\left(-m,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^m*(c + d*x)^n*((3*(f*g + e*h)*x^2*AppellF1[2, -m, -n, 3, -((b*x)
/a), -((d*x)/c)])/((1 + (b*x)/a)^m*(1 + (d*x)/c)^n) + (2*f*h*x^3*AppellF1[3
, -m, -n, 4, -((b*x)/a), -((d*x)/c)])/((1 + (b*x)/a)^m*(1 + (d*x)/c)^n) + (
6*e*g*(c + d*x)*Hypergeometric2F1[-m, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*
d)])/(d*(1 + n)*((d*(a + b*x))/(-(b*c) + a*d))^m)))/6
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^m (dx+c)^n (fx+e) (hx+g) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="fricas")
[Out] integral((f*h*x^2 + f*g*x + (h*x + g)*e)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)\,\left(g+h\,x\right)\,{\left(a+b\,x\right)}^m\,{\left(c+d\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

3.125 $\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=245

$$\frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+m)+3bdfhx)}{12b^2d^2}+\frac{(bc-ad)\left(a^2d^2fh(6-5m\right.}{}$$

```
[Out] 1/12*(b*x+a)^(1+m)*(d*x+c)^(2-m)*(4*b*d*(e*h+f*g)-a*d*f*h*(3-m)-b*c*f*h*(2+
m)+3*b*d*f*h*x)/b^2/d^2+1/12*(-a*d+b*c)*(a^2*d^2*f*h*(m^2-5*m+6)-2*a*b*d*(2
-m)*(2*d*(e*h+f*g)-c*f*h*(1+m))+b^2*(12*d^2*e*g-4*c*d*(e*h+f*g)*(1+m)+c^2*f
*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([-1+m, 1+
m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^4/d^2/(1+m)/((d*x+c)^m)
```

Rubi [A]
time = 0.11, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$,
Rules used = {152, 72, 71}

$$\frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m-1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(m^2-5m+6)-2abd(2-m)(2d(eh+fg)-cfh(m+1))+b^2(c^2fh(m^2+3m+2)-4cd(m+1)(eh+fg)+12d^2eg)\right)}{12b^4d^2(m+1)}+\frac{(a+bx)^{m+1}(c+dx)^{2-m}(-adfh(3-m)-bcfh(m+2)+4bd(eh+fg)+3bdfhx)}{12b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(2 - m)*(4*b*d*(f*g + e*h) - a*d*f*h*(3 - m) -
 b*c*f*h*(2 + m) + 3*b*d*f*h*x))/(12*b^2*d^2) + ((b*c - a*d)*(a^2*d^2*f*h*(
6 - 5*m + m^2) - 2*a*b*d*(2 - m)*(2*d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(1
2*d^2*e*g - 4*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)
^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[-1 + m, 1 + m, 2 +
 m, -((d*(a + b*x))/(b*c - a*d))])/(12*b^4*d^2*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{1-m} (e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bc}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bc}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bc}{12b^2d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.55, size = 264, normalized size = 1.08

$$\frac{1}{12}(a+bx)^m(c+dx)^{-m}\left(6(deg+cfg+ceh)x^2\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^m F_1\left(2;-m,m;3;-\frac{bx}{a},-\frac{dx}{c}\right)+4(dfg+deh+cfh)x^3\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^m F_1\left(3;-m,m;4;-\frac{bx}{a},-\frac{dx}{c}\right)+3dfhx^4\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^m F_1\left(4;-m,m;5;-\frac{bx}{a},-\frac{dx}{c}\right)-\frac{12ceg\left(\frac{d(a+bx)}{-bc+ad}\right)^{-m}(c+dx)\,{}_2F_1\left(1-m,-m;2-m;\frac{b(c+dx)}{bc-ad}\right)}{d(-1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^m*((6*(d*e*g + c*f*g + c*e*h)*x^2*(1 + (d*x)/c)^m*AppellF1[2, -m
, m, 3, -((b*x)/a), -((d*x)/c)])/(1 + (b*x)/a)^m + (4*(d*f*g + d*e*h + c*f*
h)*x^3*(1 + (d*x)/c)^m*AppellF1[3, -m, m, 4, -((b*x)/a), -((d*x)/c)])/(1 +
(b*x)/a)^m + (3*d*f*h*x^4*(1 + (d*x)/c)^m*AppellF1[4, -m, m, 5, -((b*x)/a),
 -((d*x)/c)])/(1 + (b*x)/a)^m - (12*c*e*g*(c + d*x)*Hypergeometric2F1[1 - m
, -m, 2 - m, (b*(c + d*x))/(b*c - a*d)])/(d*(-1 + m)*((d*(a + b*x))/(-(b*c)
 + a*d))^m)))/(12*(c + d*x)^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{m} \left(dx+c\right)^{1-m} \left(fx+e\right) \left(hx+g\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + f*g*x + (h*x + g)*e)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)\,\left (g+h\,x\right)\,{\left (a+b\,x\right)}^m\,{\left (c+d\,x\right)}^{1-m} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m),x)
[Out] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m), x)
```

3.126 $\int (a+bx)^m (c+dx)^{-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=235

$$\frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)+2bdfhx)}{6b^2d^2}+\frac{(a^2d^2fh(2-3m+m^2)-$$

```
[Out] 1/6*(b*x+a)^(1+m)*(d*x+c)^(1-m)*(3*b*d*(e*h+f*g)-a*d*f*h*(2-m)-b*c*f*h*(2+m
)+2*b*d*f*h*x)/b^2/d^2+1/6*(a^2*d^2*f*h*(m^2-3*m+2)-a*b*d*(1-m)*(3*d*(e*h+f
*g)-2*c*f*h*(1+m))+b^2*(6*d^2*e*g-3*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)
))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([m, 1+m],[2+m],-d*(b*x+
a)/(-a*d+b*c))/b^3/d^2/(1+m)/((d*x+c)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {152, 72, 71}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(m^2-3m+2)-abd(1-m)(3d(eh+fg)-2cfh(m+1))+b^2(c^2fh(m^2+3m+2)-3cd(m+1)(eh+fg)+6d^2eg)\right)}{6b^3d^2(m+1)}+\frac{(a+bx)^{m+1}(c+dx)^{1-m}(-adfh(2-m)-bcfh(m+2)+3bd(eh+fg)+2bdfhx)}{6b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*(3*b*d*(f*g + e*h) - a*d*f*h*(2 - m) -
 b*c*f*h*(2 + m) + 2*b*d*f*h*x))/(6*b^2*d^2) + ((a^2*d^2*f*h*(2 - 3*m + m^2
) - a*b*d*(1 - m)*(3*d*(f*g + e*h) - 2*c*f*h*(1 + m)) + b^2*(6*d^2*e*g - 3*
c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*((b*(
c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(6*b^3*d^2*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcf}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcf}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcf}{6b^2d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.38, size = 195, normalized size = 0.83

$$\frac{1}{6}(a+bx)^m(c+dx)^{-m}\left(3(fg+eh)x^2\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^m F_1\left(2;-m,m;3;-\frac{bx}{a},-\frac{dx}{c}\right)+2fhx^3\left(1+\frac{bx}{a}\right)^{-m}\left(1+\frac{dx}{c}\right)^m F_1\left(3;-m,m;4;-\frac{bx}{a},-\frac{dx}{c}\right)-\frac{6eg\left(\frac{d(a+bx)}{-bc+ad}\right)^{-m}(c+dx)\,{}_2F_1\left(1-m,-m;2-m;\frac{b(c+dx)}{bc-ad}\right)}{d(-1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^m*((3*(f*g + e*h)*x^2*(1 + (d*x)/c)^m*AppellF1[2, -m, m, 3, -((b
*x)/a), -((d*x)/c)])/(1 + (b*x)/a)^m + (2*f*h*x^3*(1 + (d*x)/c)^m*AppellF1[
3, -m, m, 4, -((b*x)/a), -((d*x)/c)])/(1 + (b*x)/a)^m - (6*e*g*(c + d*x)*Hy
pergeometric2F1[1 - m, -m, 2 - m, (b*(c + d*x))/(b*c - a*d)])/(d*(-1 + m)*(
(d*(a + b*x))/(-(b*c) + a*d))^m)))/(6*(c + d*x)^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(fx+e)\,(hx+g)\,(dx+c)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

```
[Out] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + f*g*x + (h*x + g)*e)*(b*x + a)^m/(d*x + c)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(f*x+e)*(h*x+g)/((d*x+c)**m),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,\,(g+h\,x)\,\,(a+b\,x)^m}{(c+d\,x)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m,x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m, x)
```

3.127 $\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=261

$$\frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh)+afhm)+d(bc-ad)fhmx\right)}{2bd^2(bc-ad)m}-\frac{(b^2c^2fh(1$$

```
[Out] 1/2*(b*x+a)^(1+m)*(2*b*d^2*e*g+b*c^2*f*h*(2+m)-c*d*(2*b*(e*h+f*g)+a*f*h*m)+
d*(-a*d+b*c)*f*h*m*x)/b/d^2/(-a*d+b*c)/m/((d*x+c)^m)-1/2*(b^2*c^2*f*h*(1+m)
*(2+m)-2*b*c*d*(1+m)*(a*f*h*m+b*e*h+b*f*g)+d^2*(2*b^2*e*g+2*a*b*(e*h+f*g)*m
-a^2*f*h*(1-m)*m))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([m, 1+m
],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/m/(1+m)/((d*x+c)^m)
```

Rubi [A]
time = 0.13, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {151, 72, 71}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(d^2(a^2(-f)h(1-m)m+2abm(eh+fg)+2b^2eg)-2bcd(m+1)(afhm+beh+bfg)+b^2c^2fh(m+1)(m+2)\right)}{2b^2d^2m(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(2*b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*(2*b*(f*g + e*h)
+ a*f*h*m) + d*(b*c - a*d)*f*h*m*x))/(2*b*d^2*(b*c - a*d)*m*(c + d*x)^m) -
((b^2*c^2*f*h*(1 + m)*(2 + m) - 2*b*c*d*(1 + m)*(b*f*g + b*e*h + a*f*h*m) +
 d^2*(2*b^2*e*g + 2*a*b*(f*g + e*h)*m - a^2*f*h*(1 - m)*m))*(a + b*x)^(1 +
m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a
 + b*x))/(b*c - a*d))])/(2*b^2*d^2*(b*c - a*d)*m*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(\right.}{2bd^2(bc-ad)m}$$

Mathematica [A]
time = 0.27, size = 221, normalized size = 0.85

$$\frac{(a+bx)^{1+m}(c+dx)^{-m}\left(b(adfhm(c+dx)-b(2d^2eg+c^2fh(2+m)+cd(-2fg-2eh+fhmx)))+\frac{\left(a^2d^2fh(-1+m)m+2abdm(d(fg+eh)-cfh(1+m))+b^2\left(2d^2eg-2cd(fg+eh)(1+m)+c^2fh\left(2+3m+m^2\right)\right)\right)\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,1+m;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{1+m}\right)}{2b^2d^2(-bc+ad)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(b*(a*d*f*h*m*(c + d*x) - b*(2*d^2*e*g + c^2*f*h*(2 + m)
 + c*d*(-2*f*g - 2*e*h + f*h*m*x))) + ((a^2*d^2*f*h*(-1 + m)*m + 2*a*b*d*m*
(d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(2*d^2*e*g - 2*c*d*(f*g + e*h)*(1 + m
) + c^2*f*h*(2 + 3*m + m^2)))*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2
F1[m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(1 + m)))/(2*b^2*d^2*(-(
b*c) + a*d)*m*(c + d*x)^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{m} \left(dx+c\right)^{-1-m} \left(fx+e\right) \left(hx+g\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + f*g*x + (h*x + g)*e)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,\left(g+h\,x\right)\,\left(a+b\,x\right)^m}{\left(c+d\,x\right)^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1), x)
```

3.128 $\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=203

$$\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(bd^2eg+bc^2fh(2+m)-cd(b(fg+eh)+afh(1+m))+d(bc-ad)fh(1+m)x\right)}{bd^2(bc-ad)(1+m)}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^(-1-m)*(b*d^2*e*g+b*c^2*f*h*(2+m)-c*d*(b*(e*h+f*g)+a*
f*h*(1+m))+d*(-a*d+b*c)*f*h*(1+m)*x)/b/d^2/(-a*d+b*c)/(1+m)-(a*d*f*h*m+b*(d
*(e*h+f*g)-c*f*h*(2+m)))*(b*x+a)^m*hypergeom([-m, -m],[1-m],b*(d*x+c)/(-a*d
+b*c))/b/d^3/m/((-d*(b*x+a)/(-a*d+b*c))^m)/((d*x+c)^m)
```

Rubi [A]
time = 0.07, antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {148, 72, 71}

$$-\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b(c^2fh(m+2)-cd(eh+fg)+d^2eg)\right)}{bd^2(m+1)(bc-ad)}-\frac{(a+bx)^m(c+dx)^{-m}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{b(c+dx)}{bc-ad}\right)(adfhm-bcfh(m+2)+bd(eh+fg))}{bd^3m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*(a*c*d*f*h*(1 + m) - b*(d^2*e*g - c
*d*(f*g + e*h) + c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(1 + m)*x))/(b*d^2*(b
*c - a*d)*(1 + m))) - ((b*d*(f*g + e*h) + a*d*f*h*m - b*c*f*h*(2 + m))*(a +
 b*x)^m*Hypergeometric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(b*d^3
*m*(-((d*(a + b*x))/(b*c - a*d)))^m*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b(d^2eg-cd(\right.}{bd^2(bc-ad)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b(d^2eg-cd(\right.}{bd^2(bc-ad)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b(d^2eg-cd(\right.}{bd^2(bc-ad)}$$

Mathematica [A]
time = 0.28, size = 198, normalized size = 0.98

$$\frac{(a+bx)^m(c+dx)^{-m}\left(-\frac{d(a+bx)\left(adfh(1+m)(c+dx)-b\left(d^2eg+c^2fh(2+m)+cd(-fg-eh+fh(1+m)x)\right)\right)}{c+dx}+\frac{(bc-ad)(1+m)(-bd(fg+eh)-adfhm+bcfh(2+m))\left(\frac{d(a+bx)}{-bc+ad}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{b(c+dx)}{bc-ad}\right)}{m}\right)}{bd^3(bc-ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^m*(-((d*(a + b*x)*(a*d*f*h*(1 + m)*(c + d*x) - b*(d^2*e*g + c^2*
f*h*(2 + m) + c*d*(-(f*g) - e*h + f*h*(1 + m)*x))))/(c + d*x)) + ((b*c - a*
d)*(1 + m)*(-(b*d*(f*g + e*h)) - a*d*f*h*m + b*c*f*h*(2 + m))*Hypergeometri
c2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(m*((d*(a + b*x))/(-(b*c) +
 a*d))^m)))/(b*d^3*(b*c - a*d)*(1 + m)*(c + d*x)^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{m}\left(dx+c\right)^{-2-m}\left(fx+e\right)\left(hx+g\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
[Out] integral((f*h*x^2 + f*g*x + (h*x + g)*e)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-2-m)*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,(g+h\,x)\,{(a+b\,x)}^m}{{(c+d\,x)}^{m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2), x)
```

3.129 $\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=246

$$-\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm-a^3dfh(1+m)-b^3ceg(2+m)+ab^2(c(fg+eh)+deg(1+m))-\right.}{b^2(bc-ad)^2(1+m)}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-2-m)*(a^2*b*c*f*h*m-a^3*d*f*h*(1+m)-b^3*c*e*g*(2+m
)+a*b^2*(c*(e*h+f*g)+d*e*g*(1+m))-b*(a^2*d*f*h*(3+2*m)+b^2*(d*e*g+c*(e*h+f*
g)*(1+m))-a*b*(2*c*f*h*(1+m)+d*(e*h+f*g)*(2+m)))*x)/b^2/(-a*d+b*c)^2/(1+m)/
(2+m)+f*h*(b*x+a)^(3+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([3+m, 3+m],[4+m]
,-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(3+m)/((d*x+c)^m)
```

Rubi [A]
time = 0.11, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {150, 72, 71}

$$\frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m+3,m+3;m+4;-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}-\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^3(-d)fh(m+1)-bx(a^2dfh(2m+3)-ab(2cfh(m+1)+d(m+2)(eh+fg))+b^2(c(m+1)(eh+fg)+deg))+a^2bcfhm+ab^2(c(eh+fg)+deg(m+1))-b^3ceg(m+2)\right)}{b^2(m+1)(m+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m)*(a^2*b*c*f*h*m - a^3*d*f*h*(1 + m)
- b^3*c*e*g*(2 + m) + a*b^2*(c*(f*g + e*h) + d*e*g*(1 + m)) - b*(a^2*d*f*h*
(3 + 2*m) + b^2*(d*e*g + c*(f*g + e*h)*(1 + m)) - a*b*(2*c*f*h*(1 + m) + d*
(f*g + e*h)*(2 + m)))*x))/(b^2*(b*c - a*d)^2*(1 + m)*(2 + m))) + (f*h*(a +
b*x)^(3 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[3 + m, 3 + m,
4 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(3 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm-a^3dfh(1+m)-b^3\right.}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm-a^3dfh(1+m)-b^3\right.}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm-a^3dfh(1+m)-b^3\right.}{}$$

Mathematica [A]
time = 0.34, size = 237, normalized size = 0.96

$$-\frac{(a+bx)^m(c+dx)^{-2-m}\left(d^3(a+bx)\left(-a^3dfh(1+m)+a^2bfh(cm-d(3+2m)x)+ab^2(ceh+deg(1+m)+dfg(2+m)x+deh(2+m)x+cf(g+2h(1+m)x))-b^3(degx+c(eg(2+m)+fg(1+m)x+eh(1+m)x))\right)+(bc-ad)^4fh(1+m)\left(\frac{d(a+bx)}{-bc+ad}\right)^{-m}{}_2F_1\left(-2-m,-2-m;-1-m;\frac{b(c+dx)}{bc-ad}\right)\right)}{b^2d^3(bc-ad)^2(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^m*(c + d*x)^(-2 - m)*(d^3*(a + b*x)*(-(a^3*d*f*h*(1 + m)) + a^
2*b*f*h*(c*m - d*(3 + 2*m)*x) + a*b^2*(c*e*h + d*e*g*(1 + m) + d*f*g*(2 + m
)*x + d*e*h*(2 + m)*x + c*f*(g + 2*h*(1 + m)*x)) - b^3*(d*e*g*x + c*(e*g*(2
 + m) + f*g*(1 + m)*x + e*h*(1 + m)*x))) + ((b*c - a*d)^4*f*h*(1 + m)*Hyper
geometric2F1[-2 - m, -2 - m, -1 - m, (b*(c + d*x))/(b*c - a*d)])/((d*(a + b
*x))/(-(b*c) + a*d))^m))/(b^2*d^3*(b*c - a*d)^2*(1 + m)*(2 + m)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{m} \left(dx+c\right)^{-3-m} \left(fx+e\right) \left(hx+g\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + f*g*x + (h*x + g)*e)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-3-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,\left(g+h\,x\right)\,{\left(a+b\,x\right)}^m}{{\left(c+d\,x\right)}^{m+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3), x)
```

3.130 $\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=362

$$\frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1+m)+c^2f}{bd^2(bc-ad)^2(2+m)(3+m)}$$

```
[Out] (a^2*d^2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2
*e*g+c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)
/b/d^2/(-a*d+b*c)^2/(2+m)/(3+m)+(a^2*d^2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*
h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2*e*g+c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2
)))*(b*x+a)^(1+m)*(d*x+c)^(-1-m)/d^2/(-a*d+b*c)^3/(1+m)/(2+m)/(3+m)+(b*x+a)
^(1+m)*(d*x+c)^(-3-m)*(a*c*d*f*h*(3+m)+b*(d^2*e*g-c*d*(e*h+f*g)-c^2*f*h*(2+
m))-d*(-a*d+b*c)*f*h*(3+m)*x)/b/d^2/(-a*d+b*c)/(3+m)
```

Rubi [A]

time = 0.24, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {151, 47, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}(a^2d^2fh(m^2+5m+6)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2(c^2fh(m^2+3m+2)+cd(m+1)(eh+fg)+2d^2eg))}{bd^2(m+2)(m+3)(bc-ad)^2}+\frac{(a+bx)^{m+1}(c+dx)^{-m-1}(a^2d^2fh(m^2+5m+6)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2(c^2fh(m^2+3m+2)+cd(m+1)(eh+fg)+2d^2eg))}{d^2(m+1)(m+2)(m+3)(bc-ad)^3}+\frac{(a+bx)^{m+1}(c+dx)^{-m-3}(-dfh(m+3)x(bc-ad)+acdfh(m+3)+b(c^2(-f)h(m+2)-cd(eh+fg)+d^2eg))}{bd^2(m+3)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 +
 m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))
*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(b*d^2*(b*c - a*d)^2*(2 + m)*(3 + m)
) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*
(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^
2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c - a*d)^3*(1 + m)*(2 +
m)*(3 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*
(d^2*e*g - c*d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x
))/(b*d^2*(b*c - a*d)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
```

```
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b(d^2eg-cd(fg\right.}{bd^2(bc-ad)(3\cdot}$$

$$= \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1}{}$$

$$= \frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1}{}$$

Mathematica [A]
time = 0.46, size = 220, normalized size = 0.61

$$\frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(adfh(3+m)(c+dx)+\frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg+cd(fg+eh)(1+m)+c^2fh(2+3m+m^2)))(c+dx)(-ad(1+m)+bc(2+m)+bdx)}{(bc-ad)^2(1+m)(2+m)}+b(d^2eg-c^2fh(2+m)-cd(eh+f(g+h(3+m)x)))\right)}{bd^2(bc-ad)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*d*f*h*(3 + m)*(c + d*x) + ((a^2*d^
2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 + m)) + b
^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(-(a*d*(1 + m)) + b*c*(2 + m) + b*d*x))/((b*c - a*d)^2*(1 + m)*(2 + m))
+ b*(d^2*e*g - c^2*f*h*(2 + m) - c*d*(e*h + f*(g + h*(3 + m)*x)))))/(b*d^2*
(b*c - a*d)*(3 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(362) = 724$.

time = 0.12, size = 894, normalized size = 2.47

method	result
gosper	$-\frac{(dx+c)^{-3-m}(bx+a)^{1+m}(a^2d^2fh\,m^2x^2-2abcdfh\,m^2x^2+b^2c^2fh\,m^2x^2+a^2d^2eh\,m^2x+a^2d^2fg\,m^2x+5a^2d^2fhm\,x^2-2abcdeh\,m^2x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x+c)^(-3-m)*(b*x+a)^(1+m)*(a^2*d^2*f*h*m^2*x^2-2*a*b*c*d*f*h*m^2*x^2+b^
2*c^2*f*h*m^2*x^2+a^2*d^2*e*h*m^2*x+a^2*d^2*f*g*m^2*x+5*a^2*d^2*f*h*m*x^2-2
*a*b*c*d*e*h*m^2*x-2*a*b*c*d*f*g*m^2*x-8*a*b*c*d*f*h*m*x^2-a*b*d^2*e*h*m*x^
2-a*b*d^2*f*g*m*x^2+b^2*c^2*e*h*m^2*x+b^2*c^2*f*g*m^2*x+3*b^2*c^2*f*h*m*x^2
+b^2*c*d*e*h*m*x^2+b^2*c*d*f*g*m*x^2+2*a^2*c*d*f*h*m*x+a^2*d^2*e*g*m^2+4*a^
2*d^2*e*h*m*x+4*a^2*d^2*f*g*m*x+6*a^2*d^2*f*h*x^2-2*a*b*c^2*f*h*m*x-2*a*b*c
*d*e*g*m^2-8*a*b*c*d*e*h*m*x-8*a*b*c*d*f*g*m*x-6*a*b*c*d*f*h*x^2-2*a*b*d^2*
e*g*m*x-3*a*b*d^2*e*h*x^2-3*a*b*d^2*f*g*x^2+b^2*c^2*e*g*m^2+4*b^2*c^2*e*h*m
*x+4*b^2*c^2*f*g*m*x+2*b^2*c^2*f*h*x^2+2*b^2*c*d*e*g*m*x+b^2*c*d*e*h*x^2+b^
2*c*d*f*g*x^2+2*b^2*d^2*e*g*x^2+a^2*c*d*e*h*m+a^2*c*d*f*g*m+6*a^2*c*d*f*h*x
+3*a^2*d^2*e*g*m+3*a^2*d^2*e*h*x+3*a^2*d^2*f*g*x-a*b*c^2*e*h*m-a*b*c^2*f*g*
m-2*a*b*c^2*f*h*x-8*a*b*c*d*e*g*m-10*a*b*c*d*e*h*x-10*a*b*c*d*f*g*x-2*a*b*d
^2*e*g*x+5*b^2*c^2*e*g*m+3*b^2*c^2*e*h*x+3*b^2*c^2*f*g*x+6*b^2*c*d*e*g*x+2*
a^2*c^2*f*h+a^2*c*d*e*h+a^2*c*d*f*g+2*a^2*d^2*e*g-3*a*b*c^2*e*h-3*a*b*c^2*f
*g-6*a*b*c*d*e*g+6*b^2*c^2*e*g)/(a^3*d^3*m^3-3*a^2*b*c*d^2*m^3+3*a*b^2*c^2*
d*m^3-b^3*c^3*m^3+6*a^3*d^3*m^2-18*a^2*b*c*d^2*m^2+18*a*b^2*c^2*d*m^2-6*b^3
*c^3*m^2+11*a^3*d^3*m-33*a^2*b*c*d^2*m+33*a*b^2*c^2*d*m-11*b^3*c^3*m+6*a^3*
d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(370) = 740$.

time = 1.35, size = 1654, normalized size = 4.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] (2*a^3*c^3*f*h + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*f*h*m^2 + (b^3*c*
d^2 - 3*a*b^2*d^3)*f*g + 2*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*f*h +
((b^3*c*d^2 - a*b^2*d^3)*f*g + (3*b^3*c^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*
f*h)*m)*x^4 - (a^2*b*c^3 - a^3*c^2*d)*f*g*m + (4*(b^3*c^2*d - 3*a*b^2*c*d^2
)*f*g + 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 3*a^3*d^3)*f*h + ((b^3
*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*f*g + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*
d^2 + a^3*d^3)*f*h)*m^2 + ((5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*f*g
+ (3*b^3*c^3 - 7*a*b^2*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*f*h)*m)*x^3 - (3*a^
2*b*c^3 - a^3*c^2*d)*f*g + (12*a^3*c*d^2*f*h + 3*(b^3*c^3 - 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*f*g + ((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3
*d^3)*f*g + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f*h)*m^2 + (4*(b^3*c^3
- a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f*g + (a*b^2*c^3 - 8*a^2*b*c^2*d + 7
*a^3*c*d^2)*f*h)*m)*x^2 + (8*a^3*c^2*d*f*h + (a*b^2*c^3 - 2*a^2*b*c^2*d + a
^3*c*d^2)*f*g*m^2 - 4*(3*a^2*b*c^2*d - a^3*c*d^2)*f*g + ((3*a*b^2*c^3 - 8*a
^2*b*c^2*d + 5*a^3*c*d^2)*f*g - 2*(a^2*b*c^3 - a^3*c^2*d)*f*h)*m)*x + ((2*b
^3*d^3*g + (b^3*c*d^2 - a*b^2*d^3)*h*m + (b^3*c*d^2 - 3*a*b^2*d^3)*h)*x^4 +
 (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g*m^2 + (8*b^3*c*d^2*g + (b^3*c^2*
d - 2*a*b^2*c*d^2 + a^2*b*d^3)*h*m^2 + 4*(b^3*c^2*d - 3*a*b^2*c*d^2)*h + (2
*(b^3*c*d^2 - a*b^2*d^3)*g + (5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*h)
*m)*x^3 + (12*b^3*c^2*d*g + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*h)*m^2 + 3*(b^3*c^3 - 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*h + ((7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b
*d^3)*g + 4*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*h)*m)*x^2 + 2*(
3*a*b^2*c^3 - 3*a^2*b*c^2*d + a^3*c*d^2)*g - (3*a^2*b*c^3 - a^3*c^2*d)*h +
((5*a*b^2*c^3 - 8*a^2*b*c^2*d + 3*a^3*c*d^2)*g - (a^2*b*c^3 - a^3*c^2*d)*h)
*m + (((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g + (a*b^2*c^3 - 2*a
^2*b*c^2*d + a^3*c*d^2)*h)*m^2 + 2*(3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + a^3*d^3)*g - 4*(3*a^2*b*c^2*d - a^3*c*d^2)*h + ((5*b^3*c^3 - a*b^2*c^2
*d - 7*a^2*b*c*d^2 + 3*a^3*d^3)*g + (3*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*
d^2)*h)*m)*x)*e)*(b*x + a)^m*(d*x + c)^(-m - 4)/(6*b^3*c^3 - 18*a*b^2*c^2*d
 + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*m^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m^2 +
11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-4-m)*(f*x+e)*(h*x+g),x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Mupad [B]
time = 4.49, size = 1895, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 4),x)
[Out] - ((a + b*x)^m*(2*a^3*c^3*f*h + 6*a*b^2*c^3*e*g - 3*a^2*b*c^3*e*h - 3*a^2*b
*c^3*f*g + 2*a^3*c*d^2*e*g + a^3*c^2*d*e*h + a^3*c^2*d*f*g - 6*a^2*b*c^2*d*
e*g + 5*a*b^2*c^3*e*g*m - a^2*b*c^3*e*h*m - a^2*b*c^3*f*g*m + 3*a^3*c*d^2*e
*g*m + a^3*c^2*d*e*h*m + a^3*c^2*d*f*g*m + a*b^2*c^3*e*g*m^2 + a^3*c*d^2*e*
g*m^2 - 2*a^2*b*c^2*d*e*g*m^2 - 8*a^2*b*c^2*d*e*g*m))/((a*d - b*c)^3*(c + d
*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x^3*(a + b*x)^m*(6*a^3*d^3*f*h + 2
*b^3*c^3*f*h + 8*b^3*c*d^2*e*g + 4*b^3*c^2*d*e*h + 4*b^3*c^2*d*f*g + 5*a^3*
d^3*f*h*m + 3*b^3*c^3*f*h*m + a^3*d^3*f*h*m^2 + b^3*c^3*f*h*m^2 - 12*a*b^2*
c*d^2*e*h - 12*a*b^2*c*d^2*f*g - 6*a*b^2*c^2*d*f*h + 6*a^2*b*c*d^2*f*h - 2*
a*b^2*d^3*e*g*m + 3*a^2*b*d^3*e*h*m + 3*a^2*b*d^3*f*g*m + 2*b^3*c*d^2*e*g*m
 + 5*b^3*c^2*d*e*h*m + 5*b^3*c^2*d*f*g*m + a^2*b*d^3*e*h*m^2 + a^2*b*d^3*f*
g*m^2 + b^3*c^2*d*e*h*m^2 + b^3*c^2*d*f*g*m^2 - 2*a*b^2*c*d^2*e*h*m^2 - 2*a
*b^2*c*d^2*f*g*m^2 - a*b^2*c^2*d*f*h*m^2 - a^2*b*c*d^2*f*h*m^2 - 8*a*b^2*c*
d^2*e*h*m - 8*a*b^2*c*d^2*f*g*m - 7*a*b^2*c^2*d*f*h*m - a^2*b*c*d^2*f*h*m))
/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x*(a + b*x)^
m*(2*a^3*d^3*e*g + 6*b^3*c^3*e*g + 4*a^3*c*d^2*e*h + 4*a^3*c*d^2*f*g + 8*a^
3*c^2*d*f*h + 3*a^3*d^3*e*g*m + 5*b^3*c^3*e*g*m + a^3*d^3*e*g*m^2 + b^3*c^3
*e*g*m^2 + 6*a*b^2*c^2*d*e*g - 6*a^2*b*c*d^2*e*g - 12*a^2*b*c^2*d*e*h - 12*
a^2*b*c^2*d*f*g + 3*a*b^2*c^3*e*h*m + 3*a*b^2*c^3*f*g*m - 2*a^2*b*c^3*f*h*m
 + 5*a^3*c*d^2*e*h*m + 5*a^3*c*d^2*f*g*m + 2*a^3*c^2*d*f*h*m + a*b^2*c^3*e*
h*m^2 + a*b^2*c^3*f*g*m^2 + a^3*c*d^2*e*h*m^2 + a^3*c*d^2*f*g*m^2 - a*b^2*c
^2*d*e*g*m^2 - a^2*b*c*d^2*e*g*m^2 - 2*a^2*b*c^2*d*e*h*m^2 - 2*a^2*b*c^2*d*
f*g*m^2 - a*b^2*c^2*d*e*g*m - 7*a^2*b*c*d^2*e*g*m - 8*a^2*b*c^2*d*e*h*m - 8
*a^2*b*c^2*d*f*g*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 +
 6)) - (x^2*(a + b*x)^m*(3*a^3*d^3*e*h + 3*a^3*d^3*f*g + 3*b^3*c^3*e*h + 3*
b^3*c^3*f*g + 12*b^3*c^2*d*e*g + 12*a^3*c*d^2*f*h + 4*a^3*d^3*e*h*m + 4*a^3
```

```
*d^3*f*g*m + 4*b^3*c^3*e*h*m + 4*b^3*c^3*f*g*m + a^3*d^3*e*h*m^2 + a^3*d^3*
f*g*m^2 + b^3*c^3*e*h*m^2 + b^3*c^3*f*g*m^2 - 9*a*b^2*c^2*d*e*h - 9*a*b^2*c
^2*d*f*g - 9*a^2*b*c*d^2*e*h - 9*a^2*b*c*d^2*f*g + a^2*b*d^3*e*g*m + a*b^2*
c^3*f*h*m + 7*b^3*c^2*d*e*g*m + 7*a^3*c*d^2*f*h*m + a^2*b*d^3*e*g*m^2 + a*b
^2*c^3*f*h*m^2 + b^3*c^2*d*e*g*m^2 + a^3*c*d^2*f*h*m^2 - 2*a*b^2*c*d^2*e*g*
m^2 - a*b^2*c^2*d*e*h*m^2 - a*b^2*c^2*d*f*g*m^2 - a^2*b*c*d^2*e*h*m^2 - a^2
*b*c*d^2*f*g*m^2 - 2*a^2*b*c^2*d*f*h*m^2 - 8*a*b^2*c*d^2*e*g*m - 4*a*b^2*c^
2*d*e*h*m - 4*a*b^2*c^2*d*f*g*m - 4*a^2*b*c*d^2*e*h*m - 4*a^2*b*c*d^2*f*g*m
 - 8*a^2*b*c^2*d*f*h*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m
^3 + 6)) - (x^4*(a + b*x)^m*(2*b^3*d^3*e*g - 3*a*b^2*d^3*e*h - 3*a*b^2*d^3*
f*g + 6*a^2*b*d^3*f*h + b^3*c*d^2*e*h + b^3*c*d^2*f*g + 2*b^3*c^2*d*f*h - 6
*a*b^2*c*d^2*f*h - a*b^2*d^3*e*h*m - a*b^2*d^3*f*g*m + 5*a^2*b*d^3*f*h*m +
b^3*c*d^2*e*h*m + b^3*c*d^2*f*g*m + 3*b^3*c^2*d*f*h*m + a^2*b*d^3*f*h*m^2 +
 b^3*c^2*d*f*h*m^2 - 2*a*b^2*c*d^2*f*h*m^2 - 8*a*b^2*c*d^2*f*h*m))/((a*d -
b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

3.131 $\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=507

$$\frac{(a^2d^2fh(12+7m+m^2) - 2abd(4+m)(d(fg+eh)+cfh(1+m)) + b^2(6d^2eg+2cd(fg+eh)(1+m)+c}{2bd^2(bc-ad)^2(3+m)(4+m)}$$

```
[Out] 1/2*(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*(
6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)
^(-3-m)/b/d^2/(-a*d+b*c)^2/(3+m)/(4+m)+(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4
+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*(6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*
(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)/d^2/(-a*d+b*c)^3/(2+m)/(3+m)/(4+
m)+b*(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*
(6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c
)^(-1-m)/d^2/(-a*d+b*c)^4/(1+m)/(2+m)/(3+m)/(4+m)+1/2*(b*x+a)^(1+m)*(d*x+c)
^(-4-m)*(a*c*d*f*h*(4+m)+b*(2*d^2*e*g-2*c*d*(e*h+f*g)-c^2*f*h*(2+m))-d*(-a*
d+b*c)*f*h*(4+m)*x)/b/d^2/(-a*d+b*c)/(4+m)
```

Rubi [A]

time = 0.36, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {151, 47, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}(a^2d^2fh(m^2+7m+12)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2(c^2fh(m^2+3m+2)+2cd(m+1)(eh+fg)+6d^2eg))}{2bd^2(m+3)(m+4)(bc-ad)^2}+\frac{(a+bx)^{m+1}(c+dx)^{-m-2}(a^2d^2fh(m^2+7m+12)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2(c^2fh(m^2+3m+2)+2cd(m+1)(eh+fg)+6d^2eg))}{d^2(m+2)(m+3)(m+4)(bc-ad)^3}+\frac{b(a+bx)^{m+1}(c+dx)^{-m-1}(a^2d^2fh(m^2+7m+12)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2(c^2fh(m^2+3m+2)+2cd(m+1)(eh+fg)+6d^2eg))}{d^2(m+1)(m+2)(m+3)(m+4)(bc-ad)^4}+\frac{(a+bx)^{m+1}(c+dx)^{-m-4}(-dfh(m+4)x(bc-ad)+acdfh(m+4)+b(c^2(-f)h(m+2)-2cd(eh+fg)+2d^2eg))}{2bd^2(m+4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1
+ m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2
)))*(a + b*x)^(1 + m)*(c + d*x)^(-3 - m))/(2*b*d^2*(b*c - a*d)^2*(3 + m)*(4
 + m)) + ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) +
c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 +
3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(d^2*(b*c - a*d)^3*(2 +
m)*(3 + m)*(4 + m)) + (b*(a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d
*(f*g + e*h) + c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m)
+ c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c
 - a*d)^4*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(
-4 - m)*(a*c*d*f*h*(4 + m) + b*(2*d^2*e*g - 2*c*d*(f*g + e*h) - c^2*f*h*(2
+ m)) - d*(b*c - a*d)*f*h*(4 + m)*x))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-5-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b(2d^2eg-2cd(f\right.}{2bd^2(bc-ad)(4}$$

$$= \frac{(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(}{}$$

$$= \frac{(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(}{}$$

$$= \frac{(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(}{}$$

Mathematica [A]
time = 0.62, size = 279, normalized size = 0.55

$$\frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(adfh(4+m)(c+dx)+b(2d^2eg-c^2fh(2+m)-cd(2fg+2eh+fh(4+m)x))+\frac{(a^2d^2fh(12+7m+m^2)-2abd(4+m)(d(fg+eh)+cfh(1+m))+b^2(6d^2eg+2cd(fg+eh)(1+m)+c^2fh(2+3m+m^2)))(c+dx)(a^2d^2(2+3m+m^2)-2abd(1+m)(c(3+m)+dx)+b^2(c^2(6+5m+m^2)+2cd(3+m)x+2d^2x^2))}{(bc-ad)^3(1+m)(2+m)(3+m)}\right)}{2bd^2(bc-ad)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-4 - m)*(a*d*f*h*(4 + m)*(c + d*x) + b*(2*d^2
*e*g - c^2*f*h*(2 + m) - c*d*(2*f*g + 2*e*h + f*h*(4 + m)*x)) + ((a^2*d^2*f
*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1 + m)) + b^2
*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(a^2*d^2*(2 + 3*m + m^2) - 2*a*b*d*(1 + m)*(c*(3 + m) + d*x) + b^2*(c^2*
(6 + 5*m + m^2) + 2*c*d*(3 + m)*x + 2*d^2*x^2)))/((b*c - a*d)^3*(1 + m)*(2
+ m)*(3 + m))))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. $2(503) = 1006$.
time = 0.12, size = 2343, normalized size = 4.62

method	result	size
gosper	Expression too large to display	2343

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x+c)^(-4-m)*(b*x+a)^(1+m)*(a^3*d^3*f*h*m^3*x^2-3*a^2*b*c*d^2*f*h*m^3*x^
2-a^2*b*d^3*f*h*m^2*x^3+3*a*b^2*c^2*d*f*h*m^3*x^2+2*a*b^2*c*d^2*f*h*m^2*x^3
-b^3*c^3*f*h*m^3*x^2-b^3*c^2*d*f*h*m^2*x^3+a^3*d^3*e*h*m^3*x+a^3*d^3*f*g*m^
3*x+8*a^3*d^3*f*h*m^2*x^2-3*a^2*b*c*d^2*e*h*m^3*x-3*a^2*b*c*d^2*f*g*m^3*x-2
3*a^2*b*c*d^2*f*h*m^2*x^2-2*a^2*b*d^3*e*h*m^2*x^2-2*a^2*b*d^3*f*g*m^2*x^2-7
*a^2*b*d^3*f*h*m*x^3+3*a*b^2*c^2*d*e*h*m^3*x+3*a*b^2*c^2*d*f*g*m^3*x+22*a*b
^2*c^2*d*f*h*m^2*x^2+4*a*b^2*c*d^2*e*h*m^2*x^2+4*a*b^2*c*d^2*f*g*m^2*x^2+10
*a*b^2*c*d^2*f*h*m*x^3+2*a*b^2*d^3*e*h*m*x^3+2*a*b^2*d^3*f*g*m*x^3-b^3*c^3*
e*h*m^3*x-b^3*c^3*f*g*m^3*x-7*b^3*c^3*f*h*m^2*x^2-2*b^3*c^2*d*e*h*m^2*x^2-2
*b^3*c^2*d*f*g*m^2*x^2-3*b^3*c^2*d*f*h*m*x^3-2*b^3*c*d^2*e*h*m*x^3-2*b^3*c*
d^2*f*g*m*x^3+2*a^3*c*d^2*f*h*m^2*x+a^3*d^3*e*g*m^3+7*a^3*d^3*e*h*m^2*x+7*a
^3*d^3*f*g*m^2*x+19*a^3*d^3*f*h*m*x^2-4*a^2*b*c^2*d*f*h*m^2*x-3*a^2*b*c*d^2
*e*g*m^3-22*a^2*b*c*d^2*e*h*m^2*x-22*a^2*b*c*d^2*f*g*m^2*x-58*a^2*b*c*d^2*f
*h*m*x^2-3*a^2*b*d^3*e*g*m^2*x-10*a^2*b*d^3*e*h*m*x^2-10*a^2*b*d^3*f*g*m*x^
2-12*a^2*b*d^3*f*h*x^3+2*a*b^2*c^3*f*h*m^2*x+3*a*b^2*c^2*d*e*g*m^3+23*a*b^2
*c^2*d*e*h*m^2*x+23*a*b^2*c^2*d*f*g*m^2*x+53*a*b^2*c^2*d*f*h*m*x^2+6*a*b^2*
c*d^2*e*g*m^2*x+20*a*b^2*c*d^2*e*h*m*x^2+20*a*b^2*c*d^2*f*g*m*x^2+8*a*b^2*c
*d^2*f*h*x^3+6*a*b^2*d^3*e*g*m*x^2+8*a*b^2*d^3*e*h*x^3+8*a*b^2*d^3*f*g*x^3-
b^3*c^3*e*g*m^3-8*b^3*c^3*e*h*m^2*x-8*b^3*c^3*f*g*m^2*x-14*b^3*c^3*f*h*m*x^
2-3*b^3*c^2*d*e*g*m^2*x-10*b^3*c^2*d*e*h*m*x^2-10*b^3*c^2*d*f*g*m*x^2-2*b^3
*c^2*d*f*h*x^3-6*b^3*c*d^2*e*g*m*x^2-2*b^3*c*d^2*e*h*x^3-2*b^3*c*d^2*f*g*x^
3-6*b^3*d^3*e*g*x^3+a^3*c*d^2*e*h*m^2+a^3*c*d^2*f*g*m^2+10*a^3*c*d^2*f*h*m*
x+6*a^3*d^3*e*g*m^2+14*a^3*d^3*e*h*m*x+14*a^3*d^3*f*g*m*x+12*a^3*d^3*f*h*x^
2-2*a^2*b*c^2*d*e*h*m^2-2*a^2*b*c^2*d*f*g*m^2-20*a^2*b*c^2*d*f*h*m*x-21*a^2
*b*c*d^2*e*g*m^2-53*a^2*b*c*d^2*e*h*m*x-53*a^2*b*c*d^2*f*g*m*x-56*a^2*b*c*d
^2*f*h*x^2-9*a^2*b*d^3*e*g*m*x-8*a^2*b*d^3*e*h*x^2-8*a^2*b*d^3*f*g*x^2+a*b^
```

```
2*c^3*e*h*m^2+a*b^2*c^3*f*g*m^2+10*a*b^2*c^3*f*h*m*x+24*a*b^2*c^2*d*e*g*m^2
+58*a*b^2*c^2*d*e*h*m*x+58*a*b^2*c^2*d*f*g*m*x+34*a*b^2*c^2*d*f*h*x^2+30*a*
b^2*c*d^2*e*g*m*x+34*a*b^2*c*d^2*e*h*x^2+34*a*b^2*c*d^2*f*g*x^2+6*a*b^2*d^3
*e*g*x^2-9*b^3*c^3*e*g*m^2-19*b^3*c^3*e*h*m*x-19*b^3*c^3*f*g*m*x-8*b^3*c^3*
f*h*x^2-21*b^3*c^2*d*e*g*m*x-8*b^3*c^2*d*e*h*x^2-8*b^3*c^2*d*f*g*x^2-24*b^3
*c*d^2*e*g*x^2+2*a^3*c^2*d*f*h*m+3*a^3*c*d^2*e*h*m+3*a^3*c*d^2*f*g*m+8*a^3*
c*d^2*f*h*x+11*a^3*d^3*e*g*m+8*a^3*d^3*e*h*x+8*a^3*d^3*f*g*x-2*a^2*b*c^3*f*
h*m-10*a^2*b*c^2*d*e*h*m-10*a^2*b*c^2*d*f*g*m-34*a^2*b*c^2*d*f*h*x-42*a^2*b
*c*d^2*e*g*m-34*a^2*b*c*d^2*e*h*x-34*a^2*b*c*d^2*f*g*x-6*a^2*b*d^3*e*g*x+7*
a*b^2*c^3*e*h*m+7*a*b^2*c^3*f*g*m+8*a*b^2*c^3*f*h*x+57*a*b^2*c^2*d*e*g*m+56
*a*b^2*c^2*d*e*h*x+56*a*b^2*c^2*d*f*g*x+24*a*b^2*c*d^2*e*g*x-26*b^3*c^3*e*g
*m-12*b^3*c^3*e*h*x-12*b^3*c^3*f*g*x-36*b^3*c^2*d*e*g*x+2*a^3*c^2*d*f*h+2*a
^3*c*d^2*e*h+2*a^3*c*d^2*f*g+6*a^3*d^3*e*g-8*a^2*b*c^3*f*h-8*a^2*b*c^2*d*e*
h-8*a^2*b*c^2*d*f*g-24*a^2*b*c*d^2*e*g+12*a*b^2*c^3*e*h+12*a*b^2*c^3*f*g+36
*a*b^2*c^2*d*e*g-24*b^3*c^3*e*g)/(a^4*d^4*m^4-4*a^3*b*c*d^3*m^4+6*a^2*b^2*c
^2*d^2*m^4-4*a*b^3*c^3*d*m^4+b^4*c^4*m^4+10*a^4*d^4*m^3-40*a^3*b*c*d^3*m^3+
60*a^2*b^2*c^2*d^2*m^3-40*a*b^3*c^3*d*m^3+10*b^4*c^4*m^3+35*a^4*d^4*m^2-140
*a^3*b*c*d^3*m^2+210*a^2*b^2*c^2*d^2*m^2-140*a*b^3*c^3*d*m^2+35*b^4*c^4*m^2
+50*a^4*d^4*m-200*a^3*b*c*d^3*m+300*a^2*b^2*c^2*d^2*m-200*a*b^3*c^3*d*m+50*
b^4*c^4*m+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d^2-96*a*b^3*c^3*d+24*b
^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3440 vs. 2(514) = 1028.
time = 1.60, size = 3440, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] (((b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*h*m^2 + 2*(b^4*c*d^3 - 4*a*
b^3*d^4)*f*g + 2*(b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*f*h + (2*(b^
4*c*d^3 - a*b^3*d^4)*f*g + (3*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + 7*a^2*b^2*d^4)
*f*h)*m)*x^5 - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f*g*m^2 + ((b^4*
c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*f*h*m^3 + 10*(b^4*c^
```

2*d^2 - 4*a*b^3*c*d^3)*f*g + 10*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*
d^3)*f*h + (2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*g + (8*b^4*c^3*
d - 23*a*b^3*c^2*d^2 + 22*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*f*h)*m^2 + (4*(3*b^4
*c^2*d^2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*f*g + (17*b^4*c^3*d - 60*a*b^3*c^
2*d^2 + 55*a^2*b^2*c*d^3 - 12*a^3*b*d^4)*f*h)*m)*x^4 + (((b^4*c^3*d - 3*a*b
^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*f*g + (b^4*c^4 - 2*a*b^3*c^3*d +
2*a^3*b*c*d^3 - a^4*d^4)*f*h)*m^3 + 20*(b^4*c^3*d - 4*a*b^3*c^2*d^2)*f*g +
4*(2*b^4*c^4 - 8*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 - 3*a^4*
d^4)*f*h + (5*(2*b^4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)
*f*g + (7*b^4*c^4 - 16*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 8
*a^4*d^4)*f*h)*m^2 + ((29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 -
 4*a^3*b*d^4)*f*g + (14*b^4*c^4 - 46*a*b^3*c^3*d + 15*a^2*b^2*c^2*d^2 + 36*
a^3*b*c*d^3 - 19*a^4*d^4)*f*h)*m)*x^3 - 2*(6*a^2*b^2*c^4 - 4*a^3*b*c^3*d +
a^4*c^2*d^2)*f*g + 2*(4*a^3*b*c^4 - a^4*c^3*d)*f*h + (((b^4*c^4 - 2*a*b^3*c
^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*f*g + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*
b*c^2*d^2 - a^4*c*d^3)*f*h)*m^3 + 4*(3*b^4*c^4 - 12*a*b^3*c^3*d - 12*a^2*b^
2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*f*g + 20*(4*a^3*b*c^2*d^2 - a^4*c*d^
3)*f*h + ((8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3
- 7*a^4*d^4)*f*g + 5*(a*b^3*c^4 - 4*a^2*b^2*c^3*d + 5*a^3*b*c^2*d^2 - 2*a^4
*c*d^3)*f*h)*m^2 + ((19*b^4*c^4 - 36*a*b^3*c^3*d - 15*a^2*b^2*c^2*d^2 + 46*
a^3*b*c*d^3 - 14*a^4*d^4)*f*g + (4*a*b^3*c^4 - 41*a^2*b^2*c^3*d + 66*a^3*b*
c^2*d^2 - 29*a^4*c*d^3)*f*h)*m)*x^2 - ((7*a^2*b^2*c^4 - 10*a^3*b*c^3*d + 3*
a^4*c^2*d^2)*f*g - 2*(a^3*b*c^4 - a^4*c^3*d)*f*h)*m + ((a*b^3*c^4 - 3*a^2*b
^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*f*g*m^3 - 10*(6*a^2*b^2*c^3*d - 4*a
^3*b*c^2*d^2 + a^4*c*d^3)*f*g + 10*(4*a^3*b*c^3*d - a^4*c^2*d^2)*f*h + ((7*
a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 - 8*a^4*c*d^3)*f*g - 2*(a^2
*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f*h)*m^2 + ((12*a*b^3*c^4 - 55*a^2*
b^2*c^3*d + 60*a^3*b*c^2*d^2 - 17*a^4*c*d^3)*f*g - 4*(2*a^2*b^2*c^4 - 5*a^3
*b*c^3*d + 3*a^4*c^2*d^2)*f*h)*m)*x + (2*(3*b^4*d^4*g + (b^4*c*d^3 - a*b^3*
d^4)*h*m + (b^4*c*d^3 - 4*a*b^3*d^4)*h)*x^5 + (a*b^3*c^4 - 3*a^2*b^2*c^3*d
+ 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g*m^3 + 2*(15*b^4*c*d^3*g + (b^4*c^2*d^2 - 2
*a*b^3*c*d^3 + a^2*b^2*d^4)*h*m^2 + 5*(b^4*c^2*d^2 - 4*a*b^3*c*d^3)*h + (3*
(b^4*c*d^3 - a*b^3*d^4)*g + 2*(3*b^4*c^2*d^2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^
4)*h)*m)*x^4 + (60*b^4*c^2*d^2*g + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2
*c*d^3 - a^3*b*d^4)*h*m^3 + (3*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*
g + 5*(2*b^4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*h)*m^2
+ 20*(b^4*c^3*d - 4*a*b^3*c^2*d^2)*h + (3*(9*b^4*c^2*d^2 - 10*a*b^3*c*d^3 +
 a^2*b^2*d^4)*g + (29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a
^3*b*d^4)*h)*m)*x^3 + (3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^2 -
 2*a^4*c*d^3)*g - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*h)*m^2 + (60*
b^4*c^3*d*g + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*
g + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*h)*m^3 + (3*(4*b^4*
c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - a^3*b*d^4)*g + (8*b^4*c^4 - 14*
a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 - 7*a^4*d^4)*h)*m^2 + 4*(3
*b^4*c^4 - 12*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)

```
*h + ((47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*g
+ (19*b^4*c^4 - 36*a*b^3*c^3*d - 15*a^2*b^2*c^2*d^2 + 46*a^3*b*c*d^3 - 14*a
^4*d^4)*h)*m)*x^2 + 6*(4*a*b^3*c^4 - 6*a^2*b^2*c^3*d + 4*a^3*b*c^2*d^2 - a^
4*c*d^3)*g - 2*(6*a^2*b^2*c^4 - 4*a^3*b*c^3*d + a^4*c^2*d^2)*h + ((26*a*b^3
*c^4 - 57*a^2*b^2*c^3*d + 42*a^3*b*c^2*d^2 - 11*a^4*c*d^3)*g - (7*a^2*b^2*c
^4 - 10*a^3*b*c^3*d + 3*a^4*c^2*d^2)*h)*m + (((b^4*c^4 - 2*a*b^3*c^3*d + 2*
a^3*b*c*d^3 - a^4*d^4)*g + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
 a^4*c*d^3)*h)*m^3 + (3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*
a^3*b*c*d^3 - 2*a^4*d^4)*g + (7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2
*d^2 - 8*a^4*c*d^3)*h)*m^2 + 6*(4*b^4*c^4 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d
^2 + 4*a^3*b*c*d^3 - a^4*d^4)*g - 10*(6*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2 + a
^4*c*d^3)*h + ((26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40*a^3*b
*c*d^3 - 11*a^4*d^4)*g + (12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^
2 - 17*a^4*c*d^3)*h)*m)*x)*e)*(b*x + a)^m*(d*x + c)^(-m - 5)/(24*b^4*c^4 -
96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^4 + 10*
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-5-m)*(f*x+e)*(h*x+g),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

Mupad [**B**]
time = 6.75, size = 2500, normalized size = 4.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 5),x)
```

```
[Out] (x^5*(a + b*x)^m*(6*b^4*d^4*e*g - 8*a*b^3*d^4*e*h - 8*a*b^3*d^4*f*g + 2*b^4
*c*d^3*e*h + 2*b^4*c*d^3*f*g + 12*a^2*b^2*d^4*f*h + 2*b^4*c^2*d^2*f*h + a^2
*b^2*d^4*f*h*m^2 + b^4*c^2*d^2*f*h*m^2 - 8*a*b^3*c*d^3*f*h - 2*a*b^3*d^4*e*
h*m - 2*a*b^3*d^4*f*g*m + 2*b^4*c*d^3*e*h*m + 2*b^4*c*d^3*f*g*m + 7*a^2*b^2
*d^4*f*h*m + 3*b^4*c^2*d^2*f*h*m - 2*a*b^3*c*d^3*f*h*m^2 - 10*a*b^3*c*d^3*f
*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
) - (x*(a + b*x)^m*(6*a^4*d^4*e*g - 24*b^4*c^4*e*g + 10*a^4*c*d^3*e*h + 10*
a^4*c*d^3*f*g + 11*a^4*d^4*e*g*m - 26*b^4*c^4*e*g*m + 10*a^4*c^2*d^2*f*h +
6*a^4*d^4*e*g*m^2 - 9*b^4*c^4*e*g*m^2 + a^4*d^4*e*g*m^3 - b^4*c^4*e*g*m^3 +
 36*a^2*b^2*c^2*d^2*e*g + 2*a^2*b^2*c^4*f*h*m^2 + 2*a^4*c^2*d^2*f*h*m^2 - 2
4*a*b^3*c^3*d*e*g - 24*a^3*b*c*d^3*e*g - 40*a^3*b*c^3*d*f*h - 12*a*b^3*c^4*
e*h*m - 12*a*b^3*c^4*f*g*m + 17*a^4*c*d^3*e*h*m + 17*a^4*c*d^3*f*g*m + 60*a
^2*b^2*c^3*d*e*h + 60*a^2*b^2*c^3*d*f*g - 40*a^3*b*c^2*d^2*e*h - 40*a^3*b*c
^2*d^2*f*g - 7*a*b^3*c^4*e*h*m^2 - 7*a*b^3*c^4*f*g*m^2 - a*b^3*c^4*e*h*m^3
- a*b^3*c^4*f*g*m^3 + 8*a^2*b^2*c^4*f*h*m + 8*a^4*c*d^3*e*h*m^2 + 8*a^4*c*d
^3*f*g*m^2 + a^4*c*d^3*e*h*m^3 + a^4*c*d^3*f*g*m^3 + 12*a^4*c^2*d^2*f*h*m +
 12*a*b^3*c^3*d*e*g*m^2 - 18*a^3*b*c*d^3*e*g*m^2 + 2*a*b^3*c^3*d*e*g*m^3 -
2*a^3*b*c*d^3*e*g*m^3 + 55*a^2*b^2*c^3*d*e*h*m + 55*a^2*b^2*c^3*d*f*g*m - 6
0*a^3*b*c^2*d^2*e*h*m - 60*a^3*b*c^2*d^2*f*g*m - 4*a^3*b*c^3*d*f*h*m^2 + 45
*a^2*b^2*c^2*d^2*e*g*m + 22*a^2*b^2*c^3*d*e*h*m^2 + 22*a^2*b^2*c^3*d*f*g*m^
2 - 23*a^3*b*c^2*d^2*e*h*m^2 - 23*a^3*b*c^2*d^2*f*g*m^2 + 3*a^2*b^2*c^3*d*e
*h*m^3 + 3*a^2*b^2*c^3*d*f*g*m^3 - 3*a^3*b*c^2*d^2*e*h*m^3 - 3*a^3*b*c^2*d^
2*f*g*m^3 + 10*a*b^3*c^3*d*e*g*m - 40*a^3*b*c*d^3*e*g*m - 20*a^3*b*c^3*d*f*
h*m + 9*a^2*b^2*c^2*d^2*e*g*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m +
35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x)^m*(6*a^4*c*d^3*e*g - 8*a^3*b*c^4*
f*h - 24*a*b^3*c^4*e*g + 2*a^4*c^3*d*f*h + 12*a^2*b^2*c^4*e*h + 12*a^2*b^2*
c^4*f*g + 2*a^4*c^2*d^2*e*h + 2*a^4*c^2*d^2*f*g + a^2*b^2*c^4*e*h*m^2 + a^2
*b^2*c^4*f*g*m^2 + a^4*c^2*d^2*e*h*m^2 + a^4*c^2*d^2*f*g*m^2 - 8*a^3*b*c^3*
d*e*h - 8*a^3*b*c^3*d*f*g - 26*a*b^3*c^4*e*g*m - 2*a^3*b*c^4*f*h*m + 11*a^4
*c*d^3*e*g*m + 2*a^4*c^3*d*f*h*m + 36*a^2*b^2*c^3*d*e*g - 24*a^3*b*c^2*d^2*
e*g - 9*a*b^3*c^4*e*g*m^2 - a*b^3*c^4*e*g*m^3 + 7*a^2*b^2*c^4*e*h*m + 7*a^2
*b^2*c^4*f*g*m + 6*a^4*c*d^3*e*g*m^2 + a^4*c*d^3*e*g*m^3 + 3*a^4*c^2*d^2*e*
h*m + 3*a^4*c^2*d^2*f*g*m + 57*a^2*b^2*c^3*d*e*g*m - 42*a^3*b*c^2*d^2*e*g*m
 - 2*a^3*b*c^3*d*e*h*m^2 - 2*a^3*b*c^3*d*f*g*m^2 + 24*a^2*b^2*c^3*d*e*g*m^2
 - 21*a^3*b*c^2*d^2*e*g*m^2 + 3*a^2*b^2*c^3*d*e*g*m^3 - 3*a^3*b*c^2*d^2*e*g
*m^3 - 10*a^3*b*c^3*d*e*h*m - 10*a^3*b*c^3*d*f*g*m))/((a*d - b*c)^4*(c + d*
x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(8*b^4*c
^4*f*h - 12*a^4*d^4*f*h + 20*b^4*c^3*d*e*h + 20*b^4*c^3*d*f*g - 19*a^4*d^4*
f*h*m + 14*b^4*c^4*f*h*m + 60*b^4*c^2*d^2*e*g - 8*a^4*d^4*f*h*m^2 + 7*b^4*c
^4*f*h*m^2 - a^4*d^4*f*h*m^3 + b^4*c^4*f*h*m^3 + 48*a^2*b^2*c^2*d^2*f*h + 3
*a^2*b^2*d^4*e*g*m^2 + 3*b^4*c^2*d^2*e*g*m^2 - 32*a*b^3*c^3*d*f*h + 48*a^3*
b*c*d^3*f*h - 4*a^3*b*d^4*e*h*m - 4*a^3*b*d^4*f*g*m + 29*b^4*c^3*d*e*h*m +
29*b^4*c^3*d*f*g*m - 80*a*b^3*c^2*d^2*e*h - 80*a*b^3*c^2*d^2*f*g + 3*a^2*b^
2*d^4*e*g*m - 5*a^3*b*d^4*e*h*m^2 - 5*a^3*b*d^4*f*g*m^2 - a^3*b*d^4*e*h*m^3
 - a^3*b*d^4*f*g*m^3 + 27*b^4*c^2*d^2*e*g*m + 10*b^4*c^3*d*e*h*m^2 + 10*b^4
```

```
*c^3*d*f*g*m^2 + b^4*c^3*d*e*h*m^3 + b^4*c^3*d*f*g*m^3 + 3*a^2*b^2*c^2*d^2*
f*h*m^2 - 6*a*b^3*c*d^3*e*g*m^2 - 66*a*b^3*c^2*d^2*e*h*m - 66*a*b^3*c^2*d^2
*f*g*m + 41*a^2*b^2*c*d^3*e*h*m + 41*a^2*b^2*c*d^3*f*g*m - 16*a*b^3*c^3*d*f
*h*m^2 + 14*a^3*b*c*d^3*f*h*m^2 - 2*a*b^3*c^3*d*f*h*m^3 + 2*a^3*b*c*d^3*f*h
*m^3 - 25*a*b^3*c^2*d^2*e*h*m^2 - 25*a*b^3*c^2*d^2*f*g*m^2 + 20*a^2*b^2*c*d
^3*e*h*m^2 + 20*a^2*b^2*c*d^3*f*g*m^2 - 3*a*b^3*c^2*d^2*e*h*m^3 - 3*a*b^3*c
^2*d^2*f*g*m^3 + 3*a^2*b^2*c*d^3*e*h*m^3 + 3*a^2*b^2*c*d^3*f*g*m^3 + 15*a^2
*b^2*c^2*d^2*f*h*m - 30*a*b^3*c*d^3*e*g*m - 46*a*b^3*c^3*d*f*h*m + 36*a^3*b
*c*d^3*f*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)) - (x^2*(a + b*x)^m*(8*a^4*d^4*e*h + 8*a^4*d^4*f*g - 12*b^4*c^4*e*
h - 12*b^4*c^4*f*g - 60*b^4*c^3*d*e*g + 20*a^4*c*d^3*f*h + 14*a^4*d^4*e*h*m
 + 14*a^4*d^4*f*g*m - 19*b^4*c^4*e*h*m - 19*b^4*c^4*f*g*m + 7*a^4*d^4*e*h*m
^2 + 7*a^4*d^4*f*g*m^2 - 8*b^4*c^4*e*h*m^2 - 8*b^4*c^4*f*g*m^2 + a^4*d^4*e*
h*m^3 + a^4*d^4*f*g*m^3 - b^4*c^4*e*h*m^3 - b^4*c^4*f*g*m^3 + 48*a^2*b^2*c^
2*d^2*e*h + 48*a^2*b^2*c^2*d^2*f*g + 48*a*b^3*c^3*d*e*h + 48*a*b^3*c^3*d*f*
g - 32*a^3*b*c*d^3*e*h - 32*a^3*b*c*d^3*f*g + 2*a^3*b*d^4*e*g*m - 4*a*b^3*c
^4*f*h*m - 47*b^4*c^3*d*e*g*m + 29*a^4*c*d^3*f*h*m - 80*a^3*b*c^2*d^2*f*h +
 3*a^3*b*d^4*e*g*m^2 + a^3*b*d^4*e*g*m^3 - 5*a*b^3*c^4*f*h*m^2 - a*b^3*c^4*
f*h*m^3 - 12*b^4*c^3*d*e*g*m^2 - b^4*c^3*d*e*g*m^3 + 10*a^4*c*d^3*f*h*m^2 +
 a^4*c*d^3*f*h*m^3 + 3*a^2*b^2*c^2*d^2*e*h*m^2 + 3*a^2*b^2*c^2*d^2*f*g*m^2
+ 60*a*b^3*c^2*d^2*e*g*m - 15*a^2*b^2*c*d^3*e*g*m + 14*a*b^3*c^3*d*e*h*m^2
+ 14*a*b^3*c^3*d*f*g*m^2 - 16*a^3*b*c*d^3*e*h*m...
```

3.132 $\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=815

$$\frac{(bc-ad)^2(adf+b(cf(2+m)-de(3+m)))(cfh(4+m)-d(fg+eh(3+m)))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^4f^2(de-cf)(3+m)}$$

```
[Out] (-a*d+b*c)^2*(a*d*f+b*(c*f*(2+m)-d*e*(3+m)))*(c*f*h*(4+m)-d*(f*g+e*h*(3+m))
)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^4/f^2/(-c*f+d*e)/(3+m)-b*(-a*d+b*c)*(c*f*h
*(4+m)-d*(f*g+e*h*(3+m)))*(b*x+a)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^3/f^2+h*(b
*x+a)^3*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/f-(-a*d+b*c)^2*(3*a*d*f*h-b*(c*f*h*(
4+m)-d*(e*h*m+f*g)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)/(2+m)+(-
a*d+b*c)*(c*f*h*(4+m)-d*(f*g+e*h*(3+m)))*(2*a^2*d^2*f^2+2*a*b*d*f*(c*f*(1+m
)-d*e*(3+m))+b^2*(c^2*f^2*(m^2+3*m+2)-2*c*d*e*f*(m^2+4*m+3)+d^2*e^2*(m^2+5*
m+6)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^4/f^2/(-c*f+d*e)^2/(2+m)/(3+m)-(-a*d+
b*c)*(a*d*f-b*(2*d*e*(2+m)-c*f*(3+2*m)))*(3*a*d*f*h-b*(c*f*h*(4+m)-d*(e*h*m
+f*g)))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)^2/(1+m)/(2+m)-(-a*d+b
*c)*(c*f*h*(4+m)-d*(f*g+e*h*(3+m)))*(2*a^2*d^2*f^2+2*a*b*d*f*(c*f*(1+m)-d*e
*(3+m))+b^2*(c^2*f^2*(m^2+3*m+2)-2*c*d*e*f*(m^2+4*m+3)+d^2*e^2*(m^2+5*m+6))
)*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)-b^2*(3*
a*d*f*h-b*(c*f*h*(4+m)-d*(e*h*m+f*g)))*(f*x+e)^m*hypergeom([-m, -m],[1-m],-
f*(d*x+c)/(-c*f+d*e))/d^5/f/m/((d*x+c)^m)/((d*(f*x+e)/(-c*f+d*e))^m)
```

Rubi [A]
time = 0.90, antiderivative size = 803, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {158, 165, 91, 80, 72, 71, 92, 47, 37}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)^2*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c*f*h*(4 + m) - d*(
f*g + e*h*(3 + m)))*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c
*f)*(3 + m)) - (b*(b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*h*(3 + m)))*(a +
b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^3*f^2) + (h*(a + b*x)^3*(c +
d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*f) - ((b*c - a*d)^2*(b*d*f*g + 3*a*d*f*
h + b*d*e*h*m - b*c*f*h*(4 + m))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4
*f*(d*e - c*f)*(2 + m)) - ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) - c*f*h*(4 +
m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^2*(c^2*f^2*(
2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m + m^2)))*(c +
 d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c*f)^2*(2 + m)*(3 + m)) -
 ((b*c - a*d)*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))*(a*d*f -
```

```
2*b*d*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d
^4*f*(d*e - c*f)^2*(1 + m)*(2 + m)) + ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) -
 c*f*h*(4 + m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^
2*(c^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m +
 m^2)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^4*f*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))
*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f*(c + d*x))/(d*e - c*f))]
)/(d^5*f*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} + \frac{\int (a+bx)^2(c+dx)}{}$$

$$= \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} - \frac{((bc-ad)(dfg+d}{}$$

$$= \frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+}{d^3f^2}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3}{d^4f^2(de-cf)(3}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3}{d^4f^2(de-cf)(3}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3}{d^4f^2(de-cf)(3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 36.87, size = 3579, normalized size = 4.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] (3*a*b^2*c^3*g*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^(3 + m)*(
-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*x - 6*c*d^2*e^
3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m*x^2 - c*d^2*e
^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*d^2*e^2*f*x^3
+ 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8*c^2*d*e*f^2*m
*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*m^2*x^3 - c^3*
f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c^2*d*e^3*x*((c
*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)/(c*(e + f*x))
)^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d*e - c*f)^3*(1
 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f*x)/e)^m*(1 +
(f*x)/e)^3) + (3*a^2*b*c^3*h*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e +
f*x)^(3 + m)*(-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*
x - 6*c*d^2*e^3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m
*x^2 - c*d^2*e^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*
d^2*e^2*f*x^3 + 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8
*c^2*d*e*f^2*m*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*
```

```
m^2*x^3 - c^3*f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c
^2*d*e^3*x*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)
/(c*(e + f*x)))^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d
*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f
*x)/e)^m*(1 + (f*x)/e)^3) + (b^3*g*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4
+ m)*(e + f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e
+ f*x)/e)^m) + (3*a*b^2*h*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(e +
 f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e + f*x)/e)
^m) + (b^3*h*x^5*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^m*Appel
lF1[5, 4 + m, -m, 6, -((d*x)/c), -((f*x)/e)])/(5*((e + f*x)/e)^m) + (3*a^2*
b*g*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(1 + (d*x)/c)^(-4 - m)*(e + f*
x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 - m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m
)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*e^2*f*m*x*((c*(e + f*x))/(e*(c + d*
x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(2 + 3*m
 + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e^3*(-1 + ((c*(e + f*x))/(e
*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3 + m)*x*((c*(e + f*x))/(e*(c + d*x)
))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m - e^2*(-3 +
3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c
*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*(-6 +
 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m*((c*(e + f*x))/(e*(c + d*x)))^m +
m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*Gamma[4 + m] - (2*d^4*e^4*(1 + m)*x^
4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((
c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*m*(1 + m)*x^3*((c*(e + f*x))/(e*(c
+ d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c*(e + f*x))/(e*(c + d*x)))^m + 6*e^
4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) - 2*e^3*f*x*(4 + m - 4*((c*(e + f*
x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^3*d*e*x*
(2*e*f^2*m*(4 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(4 + 5*m + m^2
)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + e^2*f*(4 + m)*x*(3 - 3*((c*(e + f*x
))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m) - e^3*(-8 + m + 8*
((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c
^2*d^2*e^2*x^2*(f^2*(12 + 7*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m +
2*e*f*(4 + m)*x*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/
(e*(c + d*x)))^m) + e^2*(m^2*((c*(e + f*x))/(e*(c + d*x)))^m + 12*(-1 + ((c
*(e + f*x))/(e*(c + d*x)))^m) + m*(6 + 7*((c*(e + f*x))/(e*(c + d*x)))^m)))
)*Gamma[5 + m]))/(2*c*e*(-(d*e) + c*f)^3*(1 + m)*(2 + m)*(3 + m)*(4 + m)*x*
((e + f*x)/e)^m*Gamma[4 + m]) + (a^3*h*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4
+ m)*(1 + (d*x)/c)^(-4 - m)*(e + f*x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 -
 m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*
e^2*f*m*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*
x))/(e*(c + d*x)))^m + f^3*(2 + 3*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x))
)^m + 2*e^3*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3
 + m)*x*((c*(e + f*x))/(e*(c + d*x)))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e +
f*x))/(e*(c + d*x)))^m - e^2*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((
c*(e + f*x))/(e*(c + d*x)))^m)) + c*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e
 + f*x))/(e*(c + d*x)))^m + e*(-6 + 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m
```

```
*((c*(e + f*x))/(e*(c + d*x)))^m + m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*G
amma[4 + m] - (2*d^4*e^4*(1 + m)*x^4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*
x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*
m*(1 + m)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c
*(e + f*x))/(e*(c + d*x)))^m + 6*e^4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)
 - 2*e^3*f*x*(4 + m - 4*((c*(e + f*x))/(e*(c + ...
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (bx+a)^3 (dx+c)^{-4-m} (fx+e)^m (hx+g)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] int((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^3*h*x^4 + a^3*g + (b^3*g + 3*a*b^2*h)*x^3 + 3*(a*b^2*g + a^2*b*
h)*x^2 + (3*a^2*b*g + a^3*h)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^m\,(g+h\,x)\,(a+b\,x)^3}{(c+d\,x)^{m+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x)^3)/(c + d*x)^(m + 4),x)
[Out] int(((e + f*x)^m*(g + h*x)*(a + b*x)^3)/(c + d*x)^(m + 4), x)
```

3.133 $\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=572

$$\frac{(bc-ad)(dg-ch)(adf+b(cf(2+m)-de(3+m)))(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3f(de-cf)(3+m)}-\frac{b(dg-ch)(a+bx)(c+dx)}{d^2f}$$

```
[Out] (-a*d+b*c)*(-c*h+d*g)*(a*d*f+b*(c*f*(2+m)-d*e*(3+m)))*(d*x+c)^(-3-m)*(f*x+e
)^(1+m)/d^3/f/(-c*f+d*e)/(3+m)-b*(-c*h+d*g)*(b*x+a)*(d*x+c)^(-3-m)*(f*x+e)^
(1+m)/d^2/f-(-a*d+b*c)^2*h*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)/(2+m
)-(-c*h+d*g)*(b^2*(-c*f+d*e)*(2+m)*(c*f*(1+m)-d*e*(3+m))-2*d*f*(b^2*c*e+a^2
*d*f+a*b*(c*f*(1+m)-d*e*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^3/f/(-c*f+d
*e)^2/(2+m)/(3+m)-(-a*d+b*c)*h*(a*d*f-b*(2*d*e*(2+m)-c*f*(3+2*m)))*(d*x+c)^
(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)^2/(1+m)/(2+m)+(-c*h+d*g)*(b^2*(-c*f+d*e
)*(2+m)*(c*f*(1+m)-d*e*(3+m))-2*d*f*(b^2*c*e+a^2*d*f+a*b*(c*f*(1+m)-d*e*(3+
m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)-b^2*h
*(f*x+e)^m*hypergeom([-m, -m],[1-m],-f*(d*x+c)/(-c*f+d*e))/d^4/m/((d*x+c)^m
)/((d*(f*x+e)/(-c*f+d*e))^m)
```

Rubi [A]

time = 0.42, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {165, 91, 80, 72, 71, 92, 47, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-2}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df\left(a^2df+b(acf(m+1)-ade(m+3)+bce)\right)\right)}{d^3f(m+2)(m+3)(de-cf)^2}+\frac{(dg-ch)(c+dx)^{-m-1}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df\left(a^2df+b(acf(m+1)-ade(m+3)+bce)\right)\right)}{d^3(m+1)(m+2)(m+3)(de-cf)^3}+\frac{(bc-ad)(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}(adf+bcf(m+2)-bde(m+3))}{d^3f(m+3)(de-cf)}-\frac{h(bc-ad)^2(c+dx)^{-m-2}(e+fx)^{m+1}}{d^3(m+2)(de-cf)}-\frac{h(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{d^3(m+1)(m+2)(de-cf)^2}-\frac{b(a+bx)(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d^2f}-\frac{b^2h(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{f(c+dx)}{de-cf}\right)}{d^4m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(d*g - c*h)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c + d*x)^
(-3 - m)*(e + f*x)^(1 + m))/(d^3*f*(d*e - c*f)*(3 + m)) - (b*(d*g - c*h)*(a
 + b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^2*f) - ((b*c - a*d)^2*h*(c
 + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)*(2 + m)) - ((d*g - c*h
)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b
*(b*c*e + a*c*f*(1 + m) - a*d*e*(3 + m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1
+ m))/(d^3*f*(d*e - c*f)^2*(2 + m)*(3 + m)) - ((b*c - a*d)*h*(a*d*f - 2*b*d
*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d
*e - c*f)^2*(1 + m)*(2 + m)) + ((d*g - c*h)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(
1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b*(b*c*e + a*c*f*(1 + m) - a*d*e*(
3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*h*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f
*(c + d*x))/(d*e - c*f))])/(d^4*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
```

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h\int (a+bx)^2(c+dx)^{-3-m}(e+fx)^m\,dx}{d} + \frac{(dg-ch)\int (a+$$

$$= -\frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f} - \frac{(bc-ad$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+a}{d^3f(de-cf)(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+d}{d^3f(de-cf)(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+a}{d^3f(de-cf)(3+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 7.02, size = 3413, normalized size = 5.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
[Out] (b^2*c^3*g*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^(3 + m)*(-2*c
^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*x - 6*c*d^2*e^3*x^
2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m*x^2 - c*d^2*e^3*m
^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*d^2*e^2*f*x^3 + 6*
c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8*c^2*d*e*f^2*m*x^3
 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*m^2*x^3 - c^3*f^3*
m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c^2*d*e^3*x*((c*e +
 d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)/(c*(e + f*x)))^m
 + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d*e - c*f)^3*(1 + m
)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f*x)/e)^m*(1 + (f*x
)/e)^3) + (2*a*b*c^3*h*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^(
3 + m)*(-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*x - 6*
c*d^2*e^3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m*x^2 -
 c*d^2*e^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*d^2*e^
2*f*x^3 + 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8*c^2*d
*e*f^2*m*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*m^2*x^
3 - c^3*f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c^2*d*e
^3*x*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)/(c*(e
 + f*x)))^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d*e - c
*f)^3*(1 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f*x)/e)
^m*(1 + (f*x)/e)^3) + (b^2*h*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(
e + f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e + f*x)
/e)^m) + (a*b*g*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(1 + (d*x)/c)^(-4
- m)*(e + f*x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 - m)*(1 + (f*x)/e)^(1 +
m)*(c*(4 + m)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*e^2*f*m*x*((c*(e + f*x)
)/(e*(c + d*x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m +
f^3*(2 + 3*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e^3*(-1 + ((c*(
e + f*x))/(e*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3 + m)*x*((c*(e + f*x))/
(e*(c + d*x)))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m
- e^2*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*
x)))^m)) + c*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e + f*x))/(e*(c + d*x)))
^m + e*(-6 + 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m*((c*(e + f*x))/(e*(c +
 d*x)))^m + m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*Gamma[4 + m] - (2*d^4*e^
4*(1 + m)*x^4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*x) + c^4*(e^2*f^2*(-5 +
 m)*m*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*m*(1 + m)*x^3*((c*(e +
f*x))/(e*(c + d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c*(e + f*x))/(e*(c + d*x
)))^m + 6*e^4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) - 2*e^3*f*x*(4 + m - 4
*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) -
2*c^3*d*e*x*(2*e*f^2*m*(4 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(4
 + 5*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + e^2*f*(4 + m)*x*(3 - 3*
((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m) - e^3*
```

```
(-8 + m + 8*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*
x)))^m)) + c^2*d^2*e^2*x^2*(f^2*(12 + 7*m + m^2)*x^2*((c*(e + f*x))/(e*(c +
 d*x)))^m + 2*e*f*(4 + m)*x*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c
*(e + f*x))/(e*(c + d*x)))^m) + e^2*(m^2*((c*(e + f*x))/(e*(c + d*x)))^m +
12*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) + m*(6 + 7*((c*(e + f*x))/(e*(c +
 d*x)))^m))))*Gamma[5 + m]))/(c*e*(-(d*e) + c*f)^3*(1 + m)*(2 + m)*(3 + m)*
(4 + m)*x*((e + f*x)/e)^m*Gamma[4 + m]) + (a^2*h*(c + d*x)^(-4 - m)*((c + d
*x)/c)^(4 + m)*(1 + (d*x)/c)^(-4 - m)*(e + f*x)^m*((c*(e + f*x))/(e*(c + d*
x)))^(-1 - m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m)*(3*e + f*x)*(-2*d^3*e^3*x^3
+ c^3*(-2*e^2*f*m*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*f^2*m*(1 + m)*x^2*(
(c*(e + f*x))/(e*(c + d*x)))^m + f^3*(2 + 3*m + m^2)*x^3*((c*(e + f*x))/(e*
(c + d*x)))^m + 2*e^3*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^2*d*e*x
*(e*f*m*(3 + m)*x*((c*(e + f*x))/(e*(c + d*x)))^m + f^2*(3 + 4*m + m^2)*x^2
*((c*(e + f*x))/(e*(c + d*x)))^m - e^2*(-3 + 3*((c*(e + f*x))/(e*(c + d*x))
)^m + m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c*d^2*e^2*x^2*(f*(6 + 5*m + m^2
)*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*(-6 + 6*((c*(e + f*x))/(e*(c + d*x)
))^m + 5*m*((c*(e + f*x))/(e*(c + d*x)))^m + m^2*((c*(e + f*x))/(e*(c + d*x
)))^m)))*Gamma[4 + m] - (2*d^4*e^4*(1 + m)*x^4 - 2*c*d^3*e^3*x^3*(-3*e*m +
f*(4 + m)*x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((c*(e + f*x))/(e*(c + d*x)))^m
+ 2*e*f^3*m*(1 + m)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + f^4*(2 + 3*m + m^
2)*x^4*((c*(e + f*x))/(e*(c + d*x)))^m + 6*e^4*(-1 + ((c*(e + f*x))/(e*(c +
 d*x)))^m) - 2*e^3*f*x*(4 + m - 4*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c
*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^3*d*e*x*(2*e*f^2*m*(4 + m)*x^2*((c*(e
+ f*x))/(e*(c + d*x)))^m + f^3*(4 + 5*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d
*x)))^m + e^2*f*(4 + m)*x*(3 - 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e
 + f*x))/(e*(c + d*x)))^m) - e^3*(-8 + m + 8*((...
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(b x +a \right)^{2} \left(d x +c \right)^{-4-m} \left(f x +e \right)^{m} \left(h x +g \right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
[Out] integral((b^2*h*x^3 + a^2*g + (b^2*g + 2*a*b*h)*x^2 + (2*a*b*g + a^2*h)*x)*
(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^m\,(g+h\,x)\,(a+b\,x)^2}{(c+d\,x)^{m+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4),x)
[Out] int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4), x)
```

3.134 $\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=363

$$\frac{(b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m))+cdf(1+m)(fg-2eh(3+m)))+adf(cfh(1+m)}{d^2f(de-cf)^2(2+m)(3+m)}$$

```
[Out] (b*(c^2*f^2*h*(m^2+3*m+2)-d^2*e*(3+m)*(f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*
h*(3+m)))+a*d*f*(c*f*h*(1+m)+d*(2*f*g-e*h*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(
1+m)/d^2/f/(-c*f+d*e)^2/(2+m)/(3+m)-(b*(c^2*f^2*h*(m^2+3*m+2)-d^2*e*(3+m)*(
f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*h*(3+m)))+a*d*f*(c*f*h*(1+m)+d*(2*f*g-e
*h*(3+m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^2/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)
-(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(a*d*f*(-c*h+d*g)-b*c*(c*f*h*(2+m)+d*(f*g-e*h
*(3+m)))+b*d*(-c*f+d*e)*h*(3+m)*x)/d^2/f/(-c*f+d*e)/(3+m)
```

Rubi [A]
time = 0.24, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {151, 47, 37}

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b(c^2f^2h(m^2+3m+2)+cdf(m+1)(fg-2eh(m+3))+d^2(-e)(m+3)(fg-eh(m+2)))\right)}{d^2f(m+2)(m+3)(de-cf)^2}-\frac{(c+dx)^{-m-1}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b(c^2f^2h(m^2+3m+2)+cdf(m+1)(fg-2eh(m+3))+d^2(-e)(m+3)(fg-eh(m+2)))\right)}{d^2(m+1)(m+2)(m+3)(de-cf)^3}-\frac{(c+dx)^{-m-3}(e+fx)^{m+1}(adf(dg-ch)-bc(cfh(m+2)-deh(m+3)+dfg)+bdh(m+3)x(de-cf))}{d^2f(m+3)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m +
 m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 +
 m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^2*f*(d*e - c*f)^2*(2 + m)*(
3 + m)) - ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*
(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g -
2*e*h*(3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^2*(d*e - c*f)^3*(
1 + m)*(2 + m)*(3 + m)) - ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g
 - c*h) - b*c*(d*f*g + c*f*h*(2 + m) - d*e*h*(3 + m)) + b*d*(d*e - c*f)*h*(
3 + m)*x))/(d^2*f*(d*e - c*f)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
```

```
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh}{d^2f(de-cf)(}$$

$$= \frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m}{}$$

$$= \frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m}{}$$

Mathematica [A]

time = 0.52, size = 227, normalized size = 0.63

$$\frac{(c+dx)^{-3-m}(e+fx)^{1+m}\left(adf(dg-ch)+\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)+d^2e(3+m)(-fg+eh(2+m))+cdf(1+m)(fg-2eh(3+m))))(c+dx)(cf(2+m)-d(e+em-fx))}{(de-cf)^2(1+m)(2+m)}-b(c^2fh(2+m)-d^2eh(3+m)x+cd(-eh(3+m)+f(g+h(3+m)x)))\right)}{d^2f(-de+cf)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) + ((a*d*f*(2*d*f*g
 + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m + m^2) + d^2*e*(3
 + m)*(-(f*g) + e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))))*(c + d
*x)*(c*f*(2 + m) - d*(e + e*m - f*x)))/((d*e - c*f)^2*(1 + m)*(2 + m)) - b*
(c^2*f*h*(2 + m) - d^2*e*h*(3 + m)*x + c*d*(-(e*h*(3 + m)) + f*(g + h*(3 +
m)*x)))))/(d^2*f*(-(d*e) + c*f)*(3 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(363) = 726$.

time = 0.12, size = 906, normalized size = 2.50

method	result
gosper	$-\frac{(dx+c)^{-3-m}(fx+e)^{1+m}(-b\,c^2f^2h\,m^2x^2+2bcdefh\,m^2x^2-b\,d^2e^2h\,m^2x^2-a\,c^2f^2h\,m^2x+2acdefh\,m^2x-acd\,f^2hm\,x^2-a\,d^2e^2h\,m^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(-b*c^2*f^2*h*m^2*x^2+2*b*c*d*e*f*h*m^2*x^2-b
*d^2*e^2*h*m^2*x^2-a*c^2*f^2*h*m^2*x+2*a*c*d*e*f*h*m^2*x-a*c*d*f^2*h*m*x^2-
a*d^2*e^2*h*m^2*x+a*d^2*e*f*h*m*x^2-b*c^2*f^2*g*m^2*x-3*b*c^2*f^2*h*m*x^2+2
*b*c*d*e*f*g*m^2*x+8*b*c*d*e*f*h*m*x^2-b*c*d*f^2*g*m*x^2-b*d^2*e^2*g*m^2*x-
5*b*d^2*e^2*h*m*x^2+b*d^2*e*f*g*m*x^2-a*c^2*f^2*g*m^2-4*a*c^2*f^2*h*m*x+2*a
*c*d*e*f*g*m^2+8*a*c*d*e*f*h*m*x-2*a*c*d*f^2*g*m*x-a*c*d*f^2*h*x^2-a*d^2*e^
2*g*m^2-4*a*d^2*e^2*h*m*x+2*a*d^2*e*f*g*m*x+3*a*d^2*e*f*h*x^2-2*a*d^2*f^2*g
*x^2+2*b*c^2*e*f*h*m*x-4*b*c^2*f^2*g*m*x-2*b*c^2*f^2*h*x^2-2*b*c*d*e^2*h*m*
x+8*b*c*d*e*f*g*m*x+6*b*c*d*e*f*h*x^2-b*c*d*f^2*g*x^2-4*b*d^2*e^2*g*m*x-6*b
*d^2*e^2*h*x^2+3*b*d^2*e*f*g*x^2+a*c^2*e*f*h*m-5*a*c^2*f^2*g*m-3*a*c^2*f^2*
h*x-a*c*d*e^2*h*m+8*a*c*d*e*f*g*m+10*a*c*d*e*f*h*x-6*a*c*d*f^2*g*x-3*a*d^2*
e^2*g*m-3*a*d^2*e^2*h*x+2*a*d^2*e*f*g*x+b*c^2*e*f*g*m+2*b*c^2*e*f*h*x-3*b*c
^2*f^2*g*x-b*c*d*e^2*g*m-6*b*c*d*e^2*h*x+10*b*c*d*e*f*g*x-3*b*d^2*e^2*g*x+3
*a*c^2*e*f*h-6*a*c^2*f^2*g-a*c*d*e^2*h+6*a*c*d*e*f*g-2*a*d^2*e^2*g-2*b*c^2*
e^2*h+3*b*c^2*e*f*g-b*c*d*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2
*f*m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^
3*e^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3
*f^3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(381) = 762$.

time = 1.55, size = 1586, normalized size = 4.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((b*c^2*d*f^3*h*m^2 + (b*c*d^2 + 2*a*d^3)*f^3*g + (2*b*c^2*d + a*c*d^2)*f^3
*h + (b*c*d^2*f^3*g + (3*b*c^2*d + a*c*d^2)*f^3*h)*m)*x^4 + (4*(b*c^2*d + 2
*a*c*d^2)*f^3*g + 2*(b*c^3 + 2*a*c^2*d)*f^3*h + (b*c^2*d*f^3*g + (b*c^3 + a
*c^2*d)*f^3*h)*m^2 + ((5*b*c^2*d + 2*a*c*d^2)*f^3*g + (3*b*c^3 + 5*a*c^2*d)
*f^3*h)*m)*x^3 + (3*a*c^3*f^3*h + 3*(b*c^3 + 4*a*c^2*d)*f^3*g + (a*c^3*f^3*
h + (b*c^3 + a*c^2*d)*f^3*g)*m^2 + (4*a*c^3*f^3*h + (4*b*c^3 + 7*a*c^2*d)*f
^3*g)*m)*x^2 + (a*c^3*f^3*g*m^2 + 5*a*c^3*f^3*g*m + 6*a*c^3*f^3*g)*x + (a*c
*d^2*g*m^2 + (b*d^3*h*m^2 + 5*b*d^3*h*m + 6*b*d^3*h)*x^3 + (3*b*d^3*g + (b*
d^3*g + (b*c*d^2 + a*d^3)*h)*m^2 + 3*(4*b*c*d^2 + a*d^3)*h + (4*b*d^3*g + (
7*b*c*d^2 + 4*a*d^3)*h)*m)*x^2 + (b*c^2*d + 2*a*c*d^2)*g + (2*b*c^3 + a*c^2
*d)*h + (a*c^2*d*h + (b*c^2*d + 3*a*c*d^2)*g)*m + ((a*c*d^2*h + (b*c*d^2 +
a*d^3)*g)*m^2 + 2*(2*b*c*d^2 + a*d^3)*g + 4*(2*b*c^2*d + a*c*d^2)*h + ((5*b
*c*d^2 + 3*a*d^3)*g + (2*b*c^2*d + 5*a*c*d^2)*h)*m)*x)*e^3 - (2*a*c^2*d*f*g
*m^2 + 3*a*c^3*f*h - (b*d^3*f*h*m^2 + 5*b*d^3*f*h*m + 6*b*d^3*f*h)*x^4 - (6
*b*c*d^2*f*h + (b*d^3*f*g - (b*c*d^2 - a*d^3)*f*h)*m^2 + (3*b*d^3*f*g - (b*
c*d^2 - 3*a*d^3)*f*h)*m)*x^3 + 3*(b*c^3 + 2*a*c^2*d)*f*g + (9*b*c*d^2*f*g +
 9*a*c*d^2*f*h + ((b*c*d^2 - a*d^3)*f*g + (2*b*c^2*d + a*c*d^2)*f*h)*m^2 +
((4*b*c*d^2 - a*d^3)*f*g + 4*(2*b*c^2*d + a*c*d^2)*f*h)*m)*x^2 + (a*c^3*f*h
 + (b*c^3 + 8*a*c^2*d)*f*g)*m + (12*a*c^2*d*f*h + 6*(2*b*c^2*d + a*c*d^2)*f
*g + (2*a*c^2*d*f*h + (2*b*c^2*d + a*c*d^2)*f*g)*m^2 + ((8*b*c^2*d + 7*a*c*
d^2)*f*g + 2*(b*c^3 + 4*a*c^2*d)*f*h)*m)*x)*e^2 + (a*c^3*f^2*g*m^2 + 5*a*c^
3*f^2*g*m + 6*a*c^3*f^2*g - (2*b*c*d^2*f^2*h*m^2 + 3*b*d^3*f^2*g + 3*(2*b*c
*d^2 + a*d^3)*f^2*h + (b*d^3*f^2*g + (8*b*c*d^2 + a*d^3)*f^2*h)*m)*x^4 - (1
2*b*c*d^2*f^2*g + 6*(b*c^2*d + 2*a*c*d^2)*f^2*h + (2*b*c*d^2*f^2*g + (b*c^2
*d + 2*a*c*d^2)*f^2*h)*m^2 + (2*(4*b*c*d^2 + a*d^3)*f^2*g + (7*b*c^2*d + 8*
a*c*d^2)*f^2*h)*m)*x^3 - (9*b*c^2*d*f^2*g + 9*a*c^2*d*f^2*h + ((b*c^2*d + 2
*a*c*d^2)*f^2*g - (b*c^3 - a*c^2*d)*f^2*h)*m^2 + (4*(b*c^2*d + 2*a*c*d^2)*f
^2*g - (b*c^3 - 4*a*c^2*d)*f^2*h)*m)*x^2 + (6*a*c^2*d*f^2*g + (a*c^3*f^2*h
+ (b*c^3 - a*c^2*d)*f^2*g)*m^2 + (3*a*c^3*f^2*h + (3*b*c^3 - a*c^2*d)*f^2*g
)*m)*x)*e)*(d*x + c)^(-m - 4)*(f*x + e)^m/(c^3*f^3*m^3 + 6*c^3*f^3*m^2 + 11
*c^3*f^3*m + 6*c^3*f^3 - (d^3*m^3 + 6*d^3*m^2 + 11*d^3*m + 6*d^3)*e^3 + 3*(
c*d^2*f*m^3 + 6*c*d^2*f*m^2 + 11*c*d^2*f*m + 6*c*d^2*f)*e^2 - 3*(c^2*d*f^2*
m^3 + 6*c^2*d*f^2*m^2 + 11*c^2*d*f^2*m + 6*c^2*d*f^2)*e)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [B]
time = 4.28, size = 1890, normalized size = 5.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x))/(c + d*x)^(m + 4),x)
[Out] ((e + f*x)^m*(2*b*c^3*e^3*h + 2*a*c*d^2*e^3*g + a*c^2*d*e^3*h + b*c^2*d*e^3
*g + 6*a*c^3*e*f^2*g - 3*a*c^3*e^2*f*h - 3*b*c^3*e^2*f*g - 6*a*c^2*d*e^2*f*
g + 3*a*c*d^2*e^3*g*m + a*c^2*d*e^3*h*m + b*c^2*d*e^3*g*m + 5*a*c^3*e*f^2*g
*m - a*c^3*e^2*f*h*m - b*c^3*e^2*f*g*m + a*c*d^2*e^3*g*m^2 + a*c^3*e*f^2*g*
m^2 - 2*a*c^2*d*e^2*f*g*m^2 - 8*a*c^2*d*e^2*f*g*m))/((c*f - d*e)^3*(c + d*x
)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*a*c^3*f^3*g + 2*a*d
^3*e^3*g + 4*a*c*d^2*e^3*h + 4*b*c*d^2*e^3*g + 8*b*c^2*d*e^3*h + 5*a*c^3*f^
3*g*m + 3*a*d^3*e^3*g*m + a*c^3*f^3*g*m^2 + a*d^3*e^3*g*m^2 - 6*a*c*d^2*e^2
*f*g + 6*a*c^2*d*e*f^2*g - 12*a*c^2*d*e^2*f*h - 12*b*c^2*d*e^2*f*g + 5*a*c*
d^2*e^3*h*m + 5*b*c*d^2*e^3*g*m + 2*b*c^2*d*e^3*h*m + 3*a*c^3*e*f^2*h*m + 3
*b*c^3*e*f^2*g*m - 2*b*c^3*e^2*f*h*m + a*c*d^2*e^3*h*m^2 + b*c*d^2*e^3*g*m^
2 + a*c^3*e*f^2*h*m^2 + b*c^3*e*f^2*g*m^2 - a*c*d^2*e^2*f*g*m^2 - a*c^2*d*e
*f^2*g*m^2 - 2*a*c^2*d*e^2*f*h*m^2 - 2*b*c^2*d*e^2*f*g*m^2 - 7*a*c*d^2*e^2*
f*g*m - a*c^2*d*e*f^2*g*m - 8*a*c^2*d*e^2*f*h*m - 8*b*c^2*d*e^2*f*g*m))/((c
*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x^4*(e + f*x)^m*
(2*a*d^3*f^3*g + a*c*d^2*f^3*h + b*c*d^2*f^3*g + 2*b*c^2*d*f^3*h - 3*a*d^3*
e*f^2*h - 3*b*d^3*e*f^2*g + 6*b*d^3*e^2*f*h - 6*b*c*d^2*e*f^2*h + a*c*d^2*f
^3*h*m + b*c*d^2*f^3*g*m + 3*b*c^2*d*f^3*h*m - a*d^3*e*f^2*h*m - b*d^3*e*f^
2*g*m + 5*b*d^3*e^2*f*h*m + b*c^2*d*f^3*h*m^2 + b*d^3*e^2*f*h*m^2 - 2*b*c*d
^2*e*f^2*h*m^2 - 8*b*c*d^2*e*f^2*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11
*m + 6*m^2 + m^3 + 6)) + (x^2*(e + f*x)^m*(3*a*c^3*f^3*h + 3*a*d^3*e^3*h +
3*b*c^3*f^3*g + 3*b*d^3*e^3*g + 12*a*c^2*d*f^3*g + 12*b*c*d^2*e^3*h + 4*a*c
^3*f^3*h*m + 4*a*d^3*e^3*h*m + 4*b*c^3*f^3*g*m + 4*b*d^3*e^3*g*m + a*c^3*f^
3*h*m^2 + a*d^3*e^3*h*m^2 + b*c^3*f^3*g*m^2 + b*d^3*e^3*g*m^2 - 9*a*c*d^2*e
^2*f*h - 9*a*c^2*d*e*f^2*h - 9*b*c*d^2*e^2*f*g - 9*b*c^2*d*e*f^2*g + 7*a*c^
2*d*f^3*g*m + 7*b*c*d^2*e^3*h*m + a*d^3*e^2*f*g*m + b*c^3*e*f^2*h*m + a*c^2
```

```
*d*f^3*g*m^2 + b*c*d^2*e^3*h*m^2 + a*d^3*e^2*f*g*m^2 + b*c^3*e*f^2*h*m^2 -
2*a*c*d^2*e*f^2*g*m^2 - a*c*d^2*e^2*f*h*m^2 - a*c^2*d*e*f^2*h*m^2 - b*c*d^2
*e^2*f*g*m^2 - b*c^2*d*e*f^2*g*m^2 - 2*b*c^2*d*e^2*f*h*m^2 - 8*a*c*d^2*e*f^
2*g*m - 4*a*c*d^2*e^2*f*h*m - 4*a*c^2*d*e*f^2*h*m - 4*b*c*d^2*e^2*f*g*m - 4
*b*c^2*d*e*f^2*g*m - 8*b*c^2*d*e^2*f*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)
*(11*m + 6*m^2 + m^3 + 6)) + (x^3*(e + f*x)^m*(2*b*c^3*f^3*h + 6*b*d^3*e^3*
h + 8*a*c*d^2*f^3*g + 4*a*c^2*d*f^3*h + 4*b*c^2*d*f^3*g + 3*b*c^3*f^3*h*m +
 5*b*d^3*e^3*h*m + b*c^3*f^3*h*m^2 + b*d^3*e^3*h*m^2 - 12*a*c*d^2*e*f^2*h -
 12*b*c*d^2*e*f^2*g + 6*b*c*d^2*e^2*f*h - 6*b*c^2*d*e*f^2*h + 2*a*c*d^2*f^3
*g*m + 5*a*c^2*d*f^3*h*m + 5*b*c^2*d*f^3*g*m - 2*a*d^3*e*f^2*g*m + 3*a*d^3*
e^2*f*h*m + 3*b*d^3*e^2*f*g*m + a*c^2*d*f^3*h*m^2 + b*c^2*d*f^3*g*m^2 + a*d
^3*e^2*f*h*m^2 + b*d^3*e^2*f*g*m^2 - 2*a*c*d^2*e*f^2*h*m^2 - 2*b*c*d^2*e*f^
2*g*m^2 - b*c*d^2*e^2*f*h*m^2 - b*c^2*d*e*f^2*h*m^2 - 8*a*c*d^2*e*f^2*h*m -
 8*b*c*d^2*e*f^2*g*m - b*c*d^2*e^2*f*h*m - 7*b*c^2*d*e*f^2*h*m))/((c*f - d*
e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

3.135 $\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=188

$$-\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)}+\frac{(cfh(1+m)+d(2fg-eh(3+m)))(c+dx)^{-2-m}(e+fx)^{1+m}}{d(de-cf)^2(2+m)(3+m)}-\frac{f(c}{}$$

```
[Out] -(-c*h+d*g)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)/(3+m)+(c*f*h*(1+m)+d*
(2*f*g-e*h*(3+m)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)^2/(2+m)/(3+m)-
f*(c*f*h*(1+m)+d*(2*f*g-e*h*(3+m)))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d/(-c*f+d*
e)^3/(1+m)/(2+m)/(3+m)
```

Rubi [A]
time = 0.06, antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {80, 47, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+1)(m+2)(m+3)(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] -(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)
)) + ((2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m))*(c + d*x)^(-2 - m)*(e + f*x
)^(1 + m))/(d*(d*e - c*f)^2*(2 + m)*(3 + m)) - (f*(2*d*f*g + c*f*h*(1 + m)
- d*e*h*(3 + m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)^3*(1
+ m)*(2 + m)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rubi steps

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} - \frac{(2dfg+cfh(1+m)-de}{d(}$$

$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-de}{d(de-c}$$

$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-de}{d(de-c}$$

Mathematica [A]
time = 0.16, size = 181, normalized size = 0.96

$$\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(-3-m)} - \frac{(-2dfg-h(de(-3-m)+cf(1+m)))\left(\frac{(c+dx)^{-2-m}(e+fx)^{1+m}}{(de-cf)(-2-m)} + \frac{f(c+dx)^{-1-m}(e+fx)^{1+m}}{(de-cf)^2(-2-m)(-1-m)}\right)}{d(de-cf)(-3-m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(-3 - m))
 - ((-2*d*f*g - h*(d*e*(-3 - m) + c*f*(1 + m)))*(((c + d*x)^(-2 - m)*(e + f
*x)^(1 + m))/((d*e - c*f)*(-2 - m)) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1 +
m))/((d*e - c*f)^2*(-2 - m)*(-1 - m))))/(d*(d*e - c*f)*(-3 - m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(188) = 376$.
time = 0.11, size = 509, normalized size = 2.71

method	result
gosper	$-\frac{(dx+c)^{-3-m}(fx+e)^{1+m}\left(-c^2f^2h\,m^2x+2cdefh\,m^2x-cd\,f^2hm\,x^2-d^2e^2h\,m^2x+d^2efhm\,x^2-c^2f^2g\,m^2-4c^2f^2hmx+2cdefg\,m^2\right.}{c^3f^3m^3-3c^2de\,f^2m^3+3c\,d^2e^2f\,m}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(-c^2*f^2*h*m^2*x+2*c*d*e*f*h*m^2*x-c*d*f^2*h
*m*x^2-d^2*e^2*h*m^2*x+d^2*e*f*h*m*x^2-c^2*f^2*g*m^2-4*c^2*f^2*h*m*x+2*c*d*
```

```
e*f*g*m^2+8*c*d*e*f*h*m*x-2*c*d*f^2*g*m*x-c*d*f^2*h*x^2-d^2*e^2*g*m^2-4*d^2
*e^2*h*m*x+2*d^2*e*f*g*m*x+3*d^2*e*f*h*x^2-2*d^2*f^2*g*x^2+c^2*e*f*h*m-5*c^
2*f^2*g*m-3*c^2*f^2*h*x-c*d*e^2*h*m+8*c*d*e*f*g*m+10*c*d*e*f*h*x-6*c*d*f^2*
g*x-3*d^2*e^2*g*m-3*d^2*e^2*h*x+2*d^2*e*f*g*x+3*c^2*e*f*h-6*c^2*f^2*g-c*d*e
^2*h+6*c*d*e*f*g-2*d^2*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f*
m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3*e
^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3*f^
3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(194) = 388.
time = 1.42, size = 898, normalized size = 4.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((c*d^2*f^3*h*m + 2*d^3*f^3*g + c*d^2*f^3*h)*x^4 + (c^2*d*f^3*h*m^2 + 8*c*d
^2*f^3*g + 4*c^2*d*f^3*h + (2*c*d^2*f^3*g + 5*c^2*d*f^3*h)*m)*x^3 + (12*c^2
*d*f^3*g + 3*c^3*f^3*h + (c^2*d*f^3*g + c^3*f^3*h)*m^2 + (7*c^2*d*f^3*g + 4
*c^3*f^3*h)*m)*x^2 + (c^3*f^3*g*m^2 + 5*c^3*f^3*g*m + 6*c^3*f^3*g)*x + (c*d
^2*g*m^2 + 2*c*d^2*g + c^2*d*h + (d^3*h*m^2 + 4*d^3*h*m + 3*d^3*h)*x^2 + (3
*c*d^2*g + c^2*d*h)*m + (2*d^3*g + 4*c*d^2*h + (d^3*g + c*d^2*h)*m^2 + (3*d
^3*g + 5*c*d^2*h)*m)*x)*e^3 - (2*c^2*d*f*g*m^2 + 6*c^2*d*f*g + 3*c^3*f*h -
(d^3*f*h*m^2 + 3*d^3*f*h*m)*x^3 + (9*c*d^2*f*h - (d^3*f*g - c*d^2*f*h)*m^2
- (d^3*f*g - 4*c*d^2*f*h)*m)*x^2 + (8*c^2*d*f*g + c^3*f*h)*m + (6*c*d^2*f*g
 + 12*c^2*d*f*h + (c*d^2*f*g + 2*c^2*d*f*h)*m^2 + (7*c*d^2*f*g + 8*c^2*d*f*
h)*m)*x)*e^2 + (c^3*f^2*g*m^2 + 5*c^3*f^2*g*m + 6*c^3*f^2*g - (d^3*f^2*h*m
+ 3*d^3*f^2*h)*x^4 - 2*(c*d^2*f^2*h*m^2 + 6*c*d^2*f^2*h + (d^3*f^2*g + 4*c*
d^2*f^2*h)*m)*x^3 - (9*c^2*d*f^2*h + (2*c*d^2*f^2*g + c^2*d*f^2*h)*m^2 + 4*
(2*c*d^2*f^2*g + c^2*d*f^2*h)*m)*x^2 + (6*c^2*d*f^2*g - (c^2*d*f^2*g - c^3*
f^2*h)*m^2 - (c^2*d*f^2*g - 3*c^3*f^2*h)*m)*x)*e)*(d*x + c)^(-m - 4)*(f*x +
 e)^m/(c^3*f^3*m^3 + 6*c^3*f^3*m^2 + 11*c^3*f^3*m + 6*c^3*f^3 - (d^3*m^3 +
6*d^3*m^2 + 11*d^3*m + 6*d^3)*e^3 + 3*(c*d^2*f*m^3 + 6*c*d^2*f*m^2 + 11*c*d
```

```
^2*f*m + 6*c*d^2*f)*e^2 - 3*(c^2*d*f^2*m^3 + 6*c^2*d*f^2*m^2 + 11*c^2*d*f^2
*m + 6*c^2*d*f^2)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [B]
time = 3.41, size = 869, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x))/(c + d*x)^(m + 4),x)
[Out] (x^2*(e + f*x)^m*(3*c^3*f^3*h + 3*d^3*e^3*h + c^3*f^3*h*m^2 + d^3*e^3*h*m^2
 + 12*c^2*d*f^3*g + 4*c^3*f^3*h*m + 4*d^3*e^3*h*m - 9*c*d^2*e^2*f*h - 9*c^2
*d*e*f^2*h + 7*c^2*d*f^3*g*m + d^3*e^2*f*g*m + c^2*d*f^3*g*m^2 + d^3*e^2*f*
g*m^2 - 8*c*d^2*e*f^2*g*m - 4*c*d^2*e^2*f*h*m - 4*c^2*d*e*f^2*h*m - 2*c*d^2
*e*f^2*g*m^2 - c*d^2*e^2*f*h*m^2 - c^2*d*e*f^2*h*m^2))/((c*f - d*e)^3*(c +
d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*c^3*f^3*g + 2*d^
3*e^3*g + c^3*f^3*g*m^2 + d^3*e^3*g*m^2 + 4*c*d^2*e^3*h + 5*c^3*f^3*g*m + 3
*d^3*e^3*g*m - 6*c*d^2*e^2*f*g + 6*c^2*d*e*f^2*g - 12*c^2*d*e^2*f*h + 5*c*d
^2*e^3*h*m + 3*c^3*e*f^2*h*m + c*d^2*e^3*h*m^2 + c^3*e*f^2*h*m^2 - 7*c*d^2*
e^2*f*g*m - c^2*d*e*f^2*g*m - 8*c^2*d*e^2*f*h*m - c*d^2*e^2*f*g*m^2 - c^2*d
*e*f^2*g*m^2 - 2*c^2*d*e^2*f*h*m^2))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m
 + 6*m^2 + m^3 + 6)) + (c*e*(e + f*x)^m*(6*c^2*f^2*g + 2*d^2*e^2*g + c^2*f^
2*g*m^2 + d^2*e^2*g*m^2 + c*d*e^2*h - 3*c^2*e*f*h + 5*c^2*f^2*g*m + 3*d^2*e
^2*g*m - 6*c*d*e*f*g + c*d*e^2*h*m - c^2*e*f*h*m - 2*c*d*e*f*g*m^2 - 8*c*d*
e*f*g*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d^2
*f^2*x^4*(e + f*x)^m*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/((c*f
 - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d*f*x^3*(e + f*x)^
m*(4*c*f + c*f*m - d*e*m)*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/
((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

3.136 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx$

Optimal. Leaf size=177

$$-\frac{(Ab-aB)(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(1+n;1,-p;2+n;\frac{b(c+dx)}{bc-ad},-\frac{f(c+dx)}{de-cf}\right)}{b(bc-ad)(1+n)}-\frac{B(c+dx)^{1+n}(e+\dots}{}$$

```
[Out] -(A*b-B*a)*(d*x+c)^(1+n)*(f*x+e)^p*AppellF1(1+n,1,-p,2+n,b*(d*x+c)/(-a*d+b*
c),-f*(d*x+c)/(-c*f+d*e))/b/(-a*d+b*c)/(1+n)/((d*(f*x+e)/(-c*f+d*e))^p)-B*(
d*x+c)^(1+n)*(f*x+e)^(1+p)*hypergeom([1, 2+n+p],[2+p],d*(f*x+e)/(-c*f+d*e))
/b/(-c*f+d*e)/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {163, 72, 71, 142, 141}

$$\frac{B(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}{}_2F_1\left(n+1,-p;n+2;-\frac{f(c+dx)}{de-cf}\right)}{bd(n+1)}-\frac{(Ab-aB)(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{f(c+dx)}{de-cf},\frac{b(c+dx)}{bc-ad}\right)}{b(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] -(((A*b - a*B)*(c + d*x)^(1 + n)*(e + f*x)^p*AppellF1[1 + n, -p, 1, 2 + n,
-((f*(c + d*x))/(d*e - c*f)), (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*(1
 + n)*((d*(e + f*x))/(d*e - c*f))^p)) + (B*(c + d*x)^(1 + n)*(e + f*x)^p*Hy
pergeometric2F1[1 + n, -p, 2 + n, -((f*(c + d*x))/(d*e - c*f))])/(b*d*(1 +
n)*((d*(e + f*x))/(d*e - c*f))^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rubi steps

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx = \frac{B\int (c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{a+bx}\,dx}{b}$$

$$= \frac{\left(B(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\right)\int (c+dx)^n\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^p dx}{b} + \frac{\Big((Ab}{}$$

$$= -\frac{(Ab-aB)(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p} F_1\Big(1+n;-p,1;2+n}{b(bc-ad)(1+n)}$$

Mathematica [A]

time = 0.38, size = 199, normalized size = 1.12

$$\frac{(c+dx)^n(e+fx)^p\left(\frac{(Ab-aB)\left(\frac{b(c+dx)}{d(a+bx)}\right)^{-n}\left(\frac{b(e+fx)}{f(a+bx)}\right)^{-p}F_1\left(-n-p;-n,-p;1-n-p;\frac{-bc+ad}{d(a+bx)},\frac{-be+af}{f(a+bx)}\right)}{n+p}+\frac{bB\left(\frac{f(c+dx)}{-de+cf}\right)^{-n}(e+fx)\,{}_2F_1\left(-n,1+p;2+p;\frac{d(e+fx)}{de-cf}\right)}{f(1+p)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] ((c + d*x)^n*(e + f*x)^p*(((A*b - a*B)*AppellF1[-n - p, -n, -p, 1 - n - p,
(-(b*c) + a*d)/(d*(a + b*x)), (-(b*e) + a*f)/(f*(a + b*x))])/((n + p)*((b*(
c + d*x))/(d*(a + b*x)))^n*((b*(e + f*x))/(f*(a + b*x)))^p) + (b*B*(e + f*x
)*Hypergeometric2F1[-n, 1 + p, 2 + p, (d*(e + f*x))/(d*e - c*f)])/(f*(1 + p
)*((f*(c + d*x))/(-(d*e) + c*f))^n)))/b^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)\,(dx+c)^{n}\,(fx+e)^{p}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

```
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="giac")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e+f\,x)^p\,(A+B\,x)\,(c+d\,x)^n}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x),x)
[Out] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x), x)
```

3.137 $\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$

Optimal. Leaf size=233

$$-\frac{d(Be-Af)(a+bx)^{1+m}(c+dx)^{-m}}{(bc-ad)f^2m}-\frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,-m;1-m;\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}-\frac{(\ldots}{\ldots}$$

```
[Out] -d*(-A*f+B*e)*(b*x+a)^(1+m)/(-a*d+b*c)/f^2/m/((d*x+c)^m)-(-A*f+B*e)*(b*x+a)
^m*hypergeom([1, -m],[1-m],(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f^2/m/((d
*x+c)^m)-(a*B*d*f*m-b*(B*c*f*m-A*d*f+B*d*e))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d
+b*c))^m*hypergeom([m, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)/f^2/m
/(1+m)/((d*x+c)^m)
```

Rubi [A]
time = 0.15, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {161, 133, 80, 72, 71}

$$-\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\,{}_2F_1\left(1,-m;1-m;\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}-\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\,{}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)(aBdfm-b(-Adf+Bcfm+Bde))}{bf^2m(m+1)(bc-ad)}-\frac{d(a+bx)^{m+1}(Be-Af)(c+dx)^{-m}}{f^2m(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] -((d*(B*e - A*f)*(a + b*x)^(1 + m))/((b*c - a*d)*f^2*m*(c + d*x)^m)) - ((B*
e - A*f)*(a + b*x)^m*Hypergeometric2F1[1, -m, 1 - m, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))])/(f^2*m*(c + d*x)^m) - ((a*B*d*f*m - b*(B*d*e -
A*d*f + B*c*f*m))*(a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeom
etric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*f^2
*m*(1 + m)*(c + d*x)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 161

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((g_.) + (h_.)*(x_
)))/((e_.) + (f_.)*(x_)), x_Symbol] :> Dist[(f*g - e*h)*((c*f - d*e)^(m + n
 + 1)/f^(m + n + 2)), Int[(a + b*x)^m/((c + d*x)^(m + 1)*(e + f*x)), x], x]
 + Dist[1/f^(m + n + 2), Int[((a + b*x)^m/(c + d*x)^(m + 1))*ExpandToSum[(f
^(m + n + 2)*(c + d*x)^(m + n + 1)*(g + h*x) - (f*g - e*h)*(c*f - d*e)^(m +
 n + 1))/(e + f*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGt
Q[m + n + 1, 0] && (LtQ[m, 0] || SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \frac{B\int (a+bx)^m(c+dx)^{-m}\,dx}{f} + \frac{(-Be+Af)\int \frac{(a+bx)^m(c+dx)^{-m}}{e+fx}\,dx}{f}$$

$$= -\frac{(b(Be-Af))\int (a+bx)^{-1+m}(c+dx)^{-m}\,dx}{f^2} + \frac{((be-af)(Be-}{}$$

$$= \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m} +$$

$$= \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m} -$$

Mathematica [A]

time = 0.20, size = 174, normalized size = 0.75

$$\frac{(a+bx)^m(c+dx)^{-m}\left(b(Be-Af)(1+m)\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)+\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(-b(Be-Af)(1+m)\,{}_2F_1\left(m,m;1+m;\frac{d(a+bx)}{-bc+ad}\right)+Bfm(a+bx)\,{}_2F_1\left(m,1+m;2+m;\frac{d(a+bx)}{-bc+ad}\right)\right)\right)}{bf^2m(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] ((a + b*x)^m*(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[1, m, 1 + m, ((d*e -
c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))] + ((b*(c + d*x))/(b*c - a*d))^m*(-
(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[m, m, 1 + m, (d*(a + b*x))/(-(b*c)
 + a*d)]) + B*f*m*(a + b*x)*Hypergeometric2F1[m, 1 + m, 2 + m, (d*(a + b*x)
)/(-(b*c) + a*d)])))/(b*f^2*m*(1 + m)*(c + d*x)^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m (Bx+A) (dx+c)^{-m}}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)/((d*x+c)**m)/(f*x+e),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,{(a+b\,x)}^m}{(e+f\,x)\,{(c+d\,x)}^m} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m), x)
```

3.138 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=250

$$\frac{2(Ab-aB)\sqrt{a+bx}\,(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2}+\frac{2B(}{}$$

```
[Out] 2/3*B*(b*x+a)^(3/2)*(d*x+c)^n*(f*x+e)^p*AppellF1(3/2,-n,-p,5/2,-d*(b*x+a)/(
-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)
/(-a*f+b*e))^p)+2*(A*b-B*a)*(d*x+c)^n*(f*x+e)^p*AppellF1(1/2,-n,-p,3/2,-d*(
b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))*(b*x+a)^(1/2)/b^2/((b*(d*x+c)/(-a*
d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A]
time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {165, 145, 144, 143}

$$\frac{2\sqrt{a+bx}\,(Ab-aB)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2}+\frac{2B(a+bx)^{3/2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{3}{2};-n,-p;\frac{5}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*AppellF1[1/2, -n, -p,
3/2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (2*B*(a + b*x)^(
3/2)*(c + d*x)^n*(e + f*x)^p*AppellF1[3/2, -n, -p, 5/2, -((d*(a + b*x))/(b*
c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(3*b^2*((b*(c + d*x))/(b*c - a*d)
)^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \frac{B\int \sqrt{a+bx}\,(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\right.}{b}$$

$$= \frac{2(Ab-aB)\sqrt{a+bx}\,(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\right.}{b^2}$$

Mathematica [A]
time = 0.35, size = 184, normalized size = 0.74

$$\frac{2\sqrt{a+bx}\,(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\left(3(Ab-aB)F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)+B(a+bx)F_1\left(\frac{3}{2};-n,-p;\frac{5}{2};\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*(3*(A*b - a*B)*AppellF1[1/2, -n, -
p, 3/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + B*(a
+ b*x)*AppellF1[3/2, -n, -p, 5/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x
))/(-(b*e) + a*f)]))/(3*b^2*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b
*e - a*f))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(Bx + A)(dx + c)^n (fx + e)^p}{\sqrt{bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
```

```
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a)**(1/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e+f\,x\right)}^p\,\left(A+B\,x\right)\,{\left(c+d\,x\right)}^n}{\sqrt{a+b\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x)^(1/2),x)
```

```
[Out] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x)^(1/2), x)
```

3.139 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$

Optimal. Leaf size=530

$$\frac{(bg-ah)^3(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+}{be-}\right.}{b^4(1+m)}$$

```
[Out] (-a*h+b*g)^3*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b
*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+3*h*(-a*h+b*g)^2*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+
e)^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^
4/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+3*h^2*(-a*h+b
*g)*(b*x+a)^(3+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a
*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*
x+e)/(-a*f+b*e))^p)+h^3*(b*x+a)^(4+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(4+m,-n,-
p,5+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(4+m)/((b*(d*x+c)/(-
a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A]
time = 0.79, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {187, 165, 145, 144, 143}

$$\frac{3h^2(bg-ah)(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+3;-n,-p;m+4;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+3)}+\frac{(bg-ah)^3(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+1)}+\frac{3h(bg-ah)^2(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+2;-n,-p;m+3;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+2)}+\frac{h^3(a+bx)^{m+4}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+4;-n,-p;m+5;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
```

```
[Out] ((b*g - a*h)^3*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(
b^4*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(3*h*(b*g - a*h)^2*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m
, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))
])/(b^4*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p
) + (3*h^2*(b*g - a*h)*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3
 + m, -n, -p, 4 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a
*f))])/(b^4*(3 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f
))^p) + (h^3*(a + b*x)^(4 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[4 + m, -n,
-p, 5 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
4*(4 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
```

```
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rule 187

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)^2\,dx}{b} + \frac{(bg-ah)\int}{}$$

$$= \frac{h^2\int (a+bx)^{2+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b^2} + 2\frac{(h(bg-a}{}$$

$$= \frac{h^3\int (a+bx)^{3+m}(c+dx)^n(e+fx)^p\,dx}{b^3} + \frac{(h^2(bg-ah))\int (a+}{}$$

$$= \frac{\left(h^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{3+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e}{b^3}$$

$$= \frac{\left(h^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{3+}}{b^3}$$

$$= \frac{(bg-ah)^3(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx}{be-af}\right.}{b^4(1+m)}$$

Mathematica [F]
time = 2.81, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,(hx+g)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="maxima")
[Out] integrate((h*x + g)^3*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="fricas")
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*(b*x + a)^m*(d*x + c)^n*
(f*x + e)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**3,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^p\,(g+h\,x)^3\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(g + h*x)^3*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)^3*(a + b*x)^m*(c + d*x)^n, x)
```

3.140 $\int (a+bx)^m (c+dx)^n (e+fx)^p (g+hx)^2 \, dx$

Optimal. Leaf size=393

$$\frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+}{be-}\right.}{b^3(1+m)}$$

```
[Out] (-a*h+b*g)^2*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b
*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+2*h*(-a*h+b*g)*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)
^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/
(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+h^2*(b*x+a)^(3+
m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x
+a)/(-a*f+b*e))/b^3/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e)
)^p)
```

Rubi [A]
time = 0.31, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {187, 165, 145, 144, 143}

$$\frac{(bg-ah)^2(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+1)}+\frac{2h(bg-ah)(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+2;-n,-p;m+3;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+2)}+\frac{h^2(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+3;-n,-p;m+4;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

```
[Out] ((b*g - a*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(2*h*(b*g - a*h)*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m,
-n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])
/(b^3*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
+ (h^2*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3 + m, -n, -p, 4
+ m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^3*(3 +
 m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rule 187

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b} + \frac{(bg-ah)\int (\ldots}{\ldots}$$

$$= \frac{h^2\int (a+bx)^{2+m}(c+dx)^n(e+fx)^p\,dx}{b^2} + 2\frac{(h(bg-ah))\int (a+\ldots}{\ldots}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e\ldots}{b^2}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{2+\ldots}}{b^2}$$

$$= \frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right.\ldots}{b^3(1+m)\ldots}$$

Mathematica [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,(hx+g)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="maxima")
[Out] integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="fricas")
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="giac")
[Out] integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^p\,(g+h\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n, x)
```

3.141 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$

Optimal. Leaf size=256

$$\frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+b}{be-a}\right.}{b^2(1+m)}$$

```
[Out] (-a*h+b*g)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x
+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/
((b*(f*x+e)/(-a*f+b*e))^p)+h*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(2+m
,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+
c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A]

time = 0.13, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {165, 145, 144, 143}

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{h(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+2;-n,-p;m+3;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```

```
[Out] ((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n,
-p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(b^
2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (h
*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m, -n, -p, 3 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(b^2*(2 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
```

```
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(bg-ah)\int (a+bx)^{\cdots}}{}$$

$$= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e\cdots}{b}$$

$$= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{1+\cdots}}{b}$$

$$= \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\cdots\right.}{b^2(1+m)\cdots}$$

Mathematica [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (bx+a)^{m} (dx+c)^{n} (fx+e)^{p} (hx+g)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="maxima")
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="fricas")
[Out] integral((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^p\,(g+h\,x)\,\left(a+b\,x\right)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

3.142 $\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx$

Optimal. Leaf size=123

$$\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(1+m)}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x+a)/(-a*d+b
*c),-f*(b*x+a)/(-a*f+b*e))/b/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(
-a*f+b*e))^p)
```

Rubi [A]

time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {145, 144, 143}

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+$$

$$= \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^m\Big($$

$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\Big(1+m;-}{b(1+m)}$$

Mathematica [A]
time = 0.09, size = 121, normalized size = 0.98

$$\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m;\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, (
d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="fricas")
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="giac")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e+f\,x)^p\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n, x)
```

3.143 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx},x\right)$$

```
[Out] CannotIntegrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
[Out] Defer[Int][((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
[Out] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)^p}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="fricas")
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p/(h*x+g),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(e+f\,x)^p\,(a+b\,x)^m\,(c+d\,x)^n}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x),x)
[Out] int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x), x)
```

3.144 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$

Optimal. Leaf size=268

$$\frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,m+n;2+m;-\frac{d(a+bx)}{bc-ad},\ldots\right)}{b^2(1+m)}$$

```
[Out] (A*b-B*a)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+
n)*AppellF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2
/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)+B*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^(-m-n)*
(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(2+m,-n,m+n,3+m,-d*(b*x+a)/(-a*d+b*c),
-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A]

time = 0.14, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {165, 145, 144, 143}

$$\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{B(a+bx)^{m+2}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+2;-n,m+n;m+3;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

```
[Out] ((A*b - a*B)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x)
)/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b
*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(b*c
 - a*d))^n) + (B*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[2 + m, -n, m + n, 3 + m, -((d*(a + b*x)
)/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((b*(c + d*x))/
(b*c - a*d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \frac{B\int (a+bx)^{1+m}(c+dx)^n(e+fx)^{-m-n}\,dx}{b} + \frac{(Ab-aB)}{}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int}{b}$$

$$= \frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-}}{b}$$

Mathematica [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
[Out] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n), x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (bx+a)^{m} (Bx+A) (dx+c)^{n} (fx+e)^{-m-n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="maxima")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="fricas")
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-m-n),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n}{\left(e+f\,x\right)^{m+n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n), x)
```

3.145 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$

Optimal. Leaf size=283

$$\frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,m+n;2+m;-\frac{d(a+bx)}{bc-ad},-\frac{f}{b}\right.}{bf(1+m)}$$

```
[Out] B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appel
lF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b/f/(1+m)/(
(b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)
*(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*
c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A]
time = 0.14, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {165, 145, 144, 143}

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{bf(m+1)}-\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n+1;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b*c - a*d))
, -((f*(a + b*x))/(b*e - a*f))])/(b*f*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n
) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a +
b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*(
(b*(c + d*x))/(b*c - a*d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}\,dx}{f} + \frac{(-Be+A\ldots}{}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)\ldots}{f}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\ldots}{f\ldots}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e-\ldots}{be-\ldots}\right.\ldots}{bf(\ldots}$$

Mathematica [A]
time = 0.35, size = 208, normalized size = 0.73

$$\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{1-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{-1+m+n}\left(B(be-af)F_1\left(1+m;-n,m+n;2+m;\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)+b(-Be+Af)F_1\left(1+m;-n,1+m+n;2+m;\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)\right)}{f(be-af)^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(1 - m - n)*((b*(e + f*x))/(b*e -
a*f))^(-1 + m + n)*(B*(b*e - a*f)*AppellF1[1 + m, -n, m + n, 2 + m, (d*(a +
 b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + b*(-(B*e) + A*f)*App
ellF1[1 + m, -n, 1 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*
x))/(-(b*e) + a*f)]))/(f*(b*e - a*f)^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^
n)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{m} \left(Bx+A\right) \left(dx+c\right)^{n} \left(fx+e\right)^{-1-m-n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-1-m-n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,{(a+b\,x)}^m\,{(c+d\,x)}^n}{{(e+f\,x)}^{m+n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1), x)
```

3.146 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$

Optimal. Leaf size=277

$$\frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(1+m;-n,1+m+n;2+m;-\frac{d(a+bx)}{bc-ad}\right.}{f(be-af)(1+m)}$$

```
[Out] B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appel
lF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b
*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*
x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x
+e))/f/(-a*f+b*e)/(1+m)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A]
time = 0.10, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {165, 145, 144, 143, 134}

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n+1;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)}-\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}\right)}{f(m+1)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a + b*x))/(b*c - a
*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1
- m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*
c - a*d)*(e + f*x)))])/(f*(b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*
c - a*d)*(e + f*x)))^n)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
```

```
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-1-m-n}\,dx}{f} + \frac{(-Be+\cdots}{\cdots}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(\cdots}{f(be-\cdots}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(\cdots}{f(be-\cdots}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(\cdots}{\cdots}\right.}{f(be\cdots}$$

Mathematica [A]
time = 0.42, size = 215, normalized size = 0.78

$$-\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-1-m-n}\left(\frac{b(e+fx)}{be-af}\right)^n\left(B(e+fx)\left(\frac{b(e+fx)}{be-af}\right)^m F_1\left(1+m;-n,1+m+n;2+m;\frac{d(a+bx)}{-bc+ad},\frac{f(a+bx)}{-be+af}\right)+(-Be+Af)\,{}_2F_1\left(1+m,-n;2+m;\frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right)\right)}{f(-be+af)(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*((b*(e + f*x))/(b*e
 - a*f))^n*(B*(e + f*x)*((b*(e + f*x))/(b*e - a*f))^m*AppellF1[1 + m, -n, 1
 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)
] + (-(B*e) + A*f)*Hypergeometric2F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a +
 b*x))/((b*c - a*d)*(e + f*x))]))/(f*(-(b*e) + a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(Bx+A)\,(dx+c)^n\,(fx+e)^{-2-m-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-2-m-n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n}{\left(e+f\,x\right)^{m+n+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2), x)
```

3.147 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$

Optimal. Leaf size=263

$$\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{(b(Bce(1+m)+A(cf(1+n)-de(2+m+n)))+a}{}$$

```
[Out] (-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m-n)/(-a*f+b*e)/(-c*f+d*
e)/(2+m+n)-(b*(B*c*e*(1+m)+A*(c*f*(1+n)-d*e*(2+m+n)))+a*(A*d*f*(1+m)+B*(d*e
*(1+n)-c*f*(2+m+n))))*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([-
n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^2/(-c*f+d*
e)/(1+m)/(2+m+n)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A]
time = 0.15, antiderivative size = 261, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {160, 12, 134}

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1)-Bcf(m+n+2)+Bde(n+1))+b(Acf(n+1)-Ade(m+n+2)+Bce(m+1)))\,{}_2F_1\left(m+1,-n;m+2;-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}\right)}{(m+1)(m+n+2)(be-af)^2(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/((
b*e - a*f)*(d*e - c*f)*(2 + m + n)) - ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) -
A*d*e*(2 + m + n)) + a*(A*d*f*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))
*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 +
 m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e -
 a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*
d)*(e + f*x)))^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \ldots$$
$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \ldots$$
$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \ldots$$

Mathematica [A]
time = 0.32, size = 223, normalized size = 0.85

$$-\frac{(a+bx)^{1+m}(c+dx)^n(e+fx)^{-2-m-n}\left((-Be+Af)(c+dx)+\frac{(b(Bce(1+m)+Acf(1+n)-Ade(2+m+n))+a(Adf(1+m)+Bde(1+n)-Bcf(2+m+n)))\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)\,{}_2F_1\left(1+m,-n;2+m;\frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right)}{(be-af)(1+m)}\right)}{(be-af)(de-cf)(2+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-2 - m - n)*((-(B*e) + A*f)*(c
+ d*x) + ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) - A*d*e*(2 + m + n)) + a*(A*d*f
*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))*(e + f*x)*Hypergeometric2F1[
1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*
e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n)))/((b
*e - a*f)*(d*e - c*f)*(2 + m + n)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(Bx+A)\,(dx+c)^n\,(fx+e)^{-3-m-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-3-m-n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n}{\left(e+f\,x\right)^{m+n+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3), x)
```

3.148 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx$

Optimal. Leaf size=558

$$\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)}+\frac{(af(Adf(2+m)+B(de(1+n)-cf(3+m+n)))+}{}$$

```
[Out] (-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-3-m-n)/(-a*f+b*e)/(-c*f+d*
e)/(3+m+n)+(a*f*(A*d*f*(2+m)+B*(d*e*(1+n)-c*f*(3+m+n)))+b*(B*e*(d*e+c*f*(1+
m))+A*f*(c*f*(2+n)-d*e*(4+m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m
-n)/(-a*f+b*e)^2/(-c*f+d*e)^2/(2+m+n)/(3+m+n)+((2+m+n)*(a*b*c*d*f*(-A*f+B*e
)+b*d*e*((a*B*c*f+A*(-a*d*f-b*c*f+b*d*e))*(3+m+n)-(-A*f+B*e)*(b*c*(1+m)+a*d
*(1+n)))-(a*d+b*c)*f*((a*B*c*f+A*(-a*d*f-b*c*f+b*d*e))*(3+m+n)-(-A*f+B*e)*(
b*c*(1+m)+a*d*(1+n))))-(b*c*(1+m)+a*d*(1+n))*(a*f*(A*d*f*(2+m)+B*(d*e*(1+n)
-c*f*(3+m+n)))+b*(B*e*(d*e+c*f*(1+m))+A*f*(c*f*(2+n)-d*e*(4+m+n)))))*(b*x+a
)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b
*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^3/(-c*f+d*e)^2/(1+m)/(2+m+n)/(3+m+n)/(
((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A]

time = 0.64, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {160, 12, 134}

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}\left((m+n+2)\left(-bde\left(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Acf(n+2)-Ade(m+n+3)+Bce(m+1))\right)+f(ad+bc)\left(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Acf(n+2)-Ade(m+n+3)+Bce(m+1))\right)+abcdf(Be-Af)\right)-(ad(n+1)+bc(m+1))\left(af(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Af(cf(n+2)-de(m+n+4))+Be(cf(m+1)+de))\right)\right){}_2F_1\left(m+1,-n;m+2;-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}\right)}{(m+1)(m+n+2)(m+n+3)(be-af)^3(de-cf)^2}+\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-3}}{(m+n+3)(be-af)(de-cf)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(e+fx)^{-m-n-2}(af(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Af(cf(n+2)-de(m+n+4))+Be(cf(m+1)+de)))}{(m+n+2)(m+n+3)(be-af)^2(de-cf)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-3 - m - n))/((
b*e - a*f)*(d*e - c*f)*(3 + m + n)) + ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n)
- B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*
(4 + m + n))))*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/
((b*e - a*f)^2*(d*e - c*f)^2*(2 + m + n)*(3 + m + n)) + (((2 + m + n)*(a*b*
c*d*f*(B*e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m
+ n)) + a*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d
)*f*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 +
m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*
f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(
1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(a + b*x)^(1 + m)*(c + d*x
)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*
f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)^3*(d*e - c*f)^2*(1 +
m)*(2 + m + n)*(3 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x))
)^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} - \ldots$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \ldots$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \ldots$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \ldots$$

Mathematica [A]
time = 1.48, size = 508, normalized size = 0.91

$$-\frac{(a+bx)^{1+m}(c+dx)^n(e+fx)^{-3-m-n}\left(-((Be-Af)(c+dx))-\frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(de+cf(1+m))+Af(cf(2+n)-de(4+m+n))))(c+dx)(e+fx)}{(be-af)(de-cf)(2+m+n)}-\frac{((2+m+n)(abcdf(Be-Af)-bde(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(Adf(2+m)+Bde(1+n)-Bcf(3+m+n)))+(bc+ad)f(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))))-(bc(1+m)+ad(1+n))(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(de+cf(1+m))+Af(cf(2+n)-de(4+m+n)))))\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^2\,{}_2F_1\left(1+m,-n;2+m;\frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right)}{(be-af)^2(de-cf)(1+m)(2+m+n)}\right)}{(be-af)(de-cf)(3+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-3 - m - n)*(-((B*e - A*f)*(c +
 d*x)) - ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e
*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n))))*(c + d*x)*(e +
 f*x))/((b*e - a*f)*(d*e - c*f)*(2 + m + n)) - (((2 + m + n)*(a*b*c*d*f*(B*
e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a
*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d)*f*(b*(B
*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 + m) + B*d*
e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f*(A*d*f*
(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) +
 A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(e + f*x)^2*Hypergeometric2F1[1 + m
, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*e - a
*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)
*(e + f*x)))^n)))/((b*e - a*f)*(d*e - c*f)*(3 + m + n)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{m} \left(Bx+A\right) \left(dx+c\right)^{n} \left(fx+e\right)^{-4-m-n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-4-m-n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,{(a+b\,x)}^m\,{(c+d\,x)}^n}{{(e+f\,x)}^{m+n+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4), x)
```

3.149 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*b*arcsin(d*x)/d^3-1/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/6*(3*b*d^2*x+6*a*d
^2+4*c)*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1623, 1823, 794, 222}

$$-\frac{\sqrt{1-d^2x^2}\,(2(3ad^2+2c)+3bd^2x)}{6d^4}+\frac{b\text{ArcSin}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*Sqrt[1 - d^2*x^2])/d^2 - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x(-2c-3ad^2-3bd^2x)}{\sqrt{1-d^2x^2}}\,dx}{3d^2}\\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 89, normalized size = 1.13

$$\frac{-\sqrt{1-d^2x^2}\,(3d^2(2a+bx)+2c(2+d^2x^2))+3b\sqrt{-d^2}\,\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*Sqrt[-d
^2]*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(6*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 139, normalized size = 1.76

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\,\mathrm{csgn}(d)c\,d^2x^2\sqrt{-d^2x^2+1}+3\,\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,b\,d^2x+6\,\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,a\right.}{6d^4\sqrt{-d^2x^2+1}}$

risch	$\frac{(2c\,x^2d^2+3b\,d^2x+6a\,d^2+4c)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{6d^4\sqrt{-(dx+1)(dx-1)}\sqrt{-dx+1}}+\frac{b\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1}}{2d^2\sqrt{d^2}\sqrt{-dx+1}\sqrt{dx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(-d^2*x^2+1)^(1/2)+3
*csgn(d)*(-d^2*x^2+1)^(1/2)*b*d^2*x+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*cs
gn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Maxima [**A**]
time = 0.50, size = 87, normalized size = 1.10

$$-\frac{\sqrt{-d^2x^2+1}\,cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}\,bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,a}{d^2}+\frac{b\arcsin\left(dx\right)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

Fricas [**A**]
time = 0.89, size = 78, normalized size = 0.99

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.15, size = 76, normalized size = 0.96

$$\frac{6\,bd\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)-\left(6\,ad^2+\left(2\,(dx+1)c+3\,bd-4\,c\right)(dx+1)-3\,bd+6\,c\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*d*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^2 + (2*(d*x + 1)*c +
3*b*d - 4*c)*(d*x + 1) - 3*b*d + 6*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Mupad [B]
time = 7.44, size = 244, normalized size = 3.09

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

3.150 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 1829, 655, 222}

$$\frac{(2ad^2+c)\,\text{ArcSin}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-c-2ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(c+2ad^2\right)\sin^{-1}(dx)}{2d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 82, normalized size = 1.30

$$
\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2} + \frac{\sqrt{-d^2}\left(c+2ad^2\right)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{2d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + (Sqrt[-d^2]*(c + 2*a*d^2)*Log[-(
Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(2*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 117, normalized size = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)dcx-2\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a\,d^2+2\,\mathrm{csgn}(d)d\sqrt{-d^2x^2+1}\,b-\right.}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(cx+2b)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}} + \frac{\left(\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)a}{\sqrt{d^2}}+\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)c}{2d^2\sqrt{d^2}}\right)\sqrt{}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A]
time = 0.51, size = 57, normalized size = 0.90

$$\frac{a\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,b}{d^2}+\frac{c\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

Fricas [A]
time = 1.05, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.97, size = 60, normalized size = 0.95

$$-\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\,\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2 +
 c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B]
time = 6.99, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.151 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))-c*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1623, 1823, 858, 222, 272, 65, 214}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\text{ArcSin}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}-\frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx+b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right) \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-\frac{a\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A]

time = 0.26, size = 93, normalized size = 1.94

$$-\frac{c\sqrt{1-d^2x^2}}{d^2}+2a\tanh^{-1}\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)-\frac{b\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + 2*a*ArcTanh[Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]
] - (b*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/Sqrt[-d^2]
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 96, normalized size = 2.00

method	result
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-\mathrm{csgn}(d)\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2-\mathrm{csgn}(d)\sqrt{-d^2x^2+1}\,c+\arctan\left(\frac{\mathrm{csgn}(}{\sqrt{-(dx+}}\right.\right.}{d^2\sqrt{-d^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^2*(-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*
d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2
))*b*d)*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Maxima [**A**]
time = 0.53, size = 57, normalized size = 1.19

$$-a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

Fricas [**A**]
time = 0.82, size = 81, normalized size = 1.69

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\,\sqrt{-dx+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] Result contains complex when optimal does not.
time = 43.37, size = 245, normalized size = 5.10

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13]1/sageVA
```

Mupad [B]
time = 3.92, size = 122, normalized size = 2.54

$$a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\sqrt{1-d\,x}\,\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

3.152 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\sin^{-1}(dx)}{d}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))-a*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1623, 1821, 858, 222, 272, 65, 214}

$$-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\text{ArcSin}(dx)}{d}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right) \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A]

time = 0.32, size = 93, normalized size = 1.94

$$-\frac{a\sqrt{1-d^2x^2}}{x}+2b\tanh^{-1}\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)-\frac{c\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{\sqrt{-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + 2*b*ArcTanh[Sqrt[-d^2]*x - Sqrt[1 - d^2*x^2]]
- (c*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/Sqrt[-d^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 97, normalized size = 2.02

method	result
default	$\frac{\left(-\mathrm{csgn}(d)d\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)bx-\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)da+\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)cx\right)\sqrt{-dx+1}\,\sqrt{dx}}{\sqrt{-d^2x^2+1}\,xd}$
risch	$\frac{a\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{x\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{c\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}-b\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{(-}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*d*arctanh(1/(-d^2*x^2+1)^(1/2))*b*x-(-d^2*x^2+1)^(1/2)*csgn(d)*d*
a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c*x)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*
csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

Maxima [A]
time = 0.52, size = 57, normalized size = 1.19

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^2x^2+1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

Fricas [A]
time = 1.09, size = 84, normalized size = 1.75

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}\,ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 41.36, size = 221, normalized size = 4.60

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13](-2*sage
```

Mupad [B]
time = 3.74, size = 114, normalized size = 2.38

$$b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

3.153 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] -1/2*(a*d^2+2*c)*arctanh((-d^2*x^2+1)^(1/2))-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-b
*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1623, 1821, 821, 272, 65, 214}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[1 - d^2*x^2])/x^2 - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{1}{2}\int \frac{-2b-(2c+ad^2)\,x}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}(-2c-ad^2)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{4}(-2c-ad^2)\,\text{Subst}\Bigg(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}\left(a+\frac{2c}{d^2}\right)\text{Subst}\Bigg(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-} \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}(2c+ad^2)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 69, normalized size = 0.97

$$\frac{(-a-2bx)\sqrt{1-d^2x^2}}{2x^2} + (2c+ad^2)\tanh^{-1}\left(\sqrt{-d^2}\,x-\sqrt{1-d^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-a - 2*b*x)*Sqrt[1 - d^2*x^2])/(2*x^2) + (2*c + a*d^2)*ArcTanh[Sqrt[-d^2]
*x - Sqrt[1 - d^2*x^2]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 108, normalized size = 1.52

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^2\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2x^2+2\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)c\,x^2+2\sqrt{-d^2x^2}\right.}{2\sqrt{-d^2x^2+1}\,x^2}$
risch	$\frac{\sqrt{dx+1}\,(dx-1)(2bx+a)\sqrt{(-dx+1)(dx+1)}}{2x^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}-\frac{\left(c+\frac{a\,d^2}{2}\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
a*d^2*x^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*c*x^2+2*(-d^2*x^2+1)^(1/2)*b*x+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Maxima [A]
time = 0.49, size = 98, normalized size = 1.38

$$-\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^2x^2+1}\,b}{x}-\frac{\sqrt{-d^2x^2+1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

Fricas [A]
time = 0.95, size = 65, normalized size = 0.92

$$\frac{(ad^2+2\,c)x^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\,\sqrt{-dx+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13](-1/2*(s
```

Mupad [**B**]
time = 5.85, size = 312, normalized size = 4.39

$$c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\frac{a\,d^2\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\left(\sqrt{1-d\,x}-1\right)^4}{2\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}+\frac{a\,d^2\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2}-\frac{a\,d^2\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2}-\frac{b\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\left(\sqrt{1-d\,x}-1\right)^2}{32\left(\sqrt{d\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.154 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{cx^2\sqrt{-1+dx}\,\sqrt{1+dx}}{3d^2}+\frac{\sqrt{-1+dx}\,\sqrt{1+dx}\,(2(2c+3ad^2)+3bd^2x)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*b*arccosh(d*x)/d^3+1/3*c*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/6*(3*b*d
^2*x+6*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4
```

Rubi [A]
time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1624, 1823, 794, 223, 212}

$$-\frac{(1-d^2x^2)\,(2(3ad^2+2c)+3bd^2x)}{6d^4\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c +
3*a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) +
 (b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 +
d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{x(a+bx+cx^2)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{x(2c+3ad^2+3bd^2x)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}{2d^2\sqrt{}}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}{}$$

$$= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2}}{2d^3}$$

Mathematica [A]
time = 0.19, size = 74, normalized size = 0.85

$$\frac{\sqrt{-1+dx}\sqrt{1+dx}\,(3d^2(2a+bx)+2c(2+d^2x^2))+6bd\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{6d^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2)) + 6*b
*d*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 137, normalized size = 1.57

method	result
risch	$\frac{(2c x^2 d^2+3b d^2 x+6a d^2+4c)\sqrt{dx+1}\,\sqrt{dx-1}}{6d^4}+\frac{b\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\,\mathrm{csgn}(d)c\,d^2x^2\sqrt{d^2x^2-1}+3\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)b\,d^2x+6\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)a\,d^2+4\sqrt{d^2 \ldots}\right.}{6d^4\sqrt{d^2x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(d^2*x^2-1)^(1/2)+3*(d
^2*x^2-1)^(1/2)*csgn(d)*b*d^2*x+6*(d^2*x^2-1)^(1/2)*csgn(d)*a*d^2+4*(d^2*x^
2-1)^(1/2)*csgn(d)*c+3*ln(((d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*csg
n(d)/d^4/(d^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.30, size = 100, normalized size = 1.15

$$\frac{\sqrt{d^2x^2-1}\,cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}\,bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}+\frac{2\,\sqrt{d^2x^2-1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

Fricas [A]
time = 1.41, size = 73, normalized size = 0.84

$$-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.22, size = 105, normalized size = 1.21

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\,(2\,ad^{11}-bd^{10}+2\,cd^9)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [**B**]
time = 12.35, size = 318, normalized size = 3.66

$$\frac{\sqrt{dx-1}\,\left(\frac{2c}{3d^4}+\frac{cx^3}{3d}+\frac{cx^2}{3d^2}+\frac{2cx}{3d^3}\right)}{\sqrt{dx+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{dx-1}-\mathrm{i}}{\sqrt{dx+1}-1}\right)}{d^3}-\frac{\frac{14\,b\left(\sqrt{dx-1}-\mathrm{i}\right)^3}{\left(\sqrt{dx+1}-1\right)^3}+\frac{14\,b\left(\sqrt{dx-1}-\mathrm{i}\right)^5}{\left(\sqrt{dx+1}-1\right)^5}+\frac{2\,b\left(\sqrt{dx-1}-\mathrm{i}\right)^7}{\left(\sqrt{dx+1}-1\right)^7}+\frac{2\,b\left(\sqrt{dx-1}-\mathrm{i}\right)}{\sqrt{dx+1}-1}}{d^3-\frac{4\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+\frac{6\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^4}{\left(\sqrt{dx+1}-1\right)^4}-\frac{4\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^6}{\left(\sqrt{dx+1}-1\right)^6}+\frac{d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^8}{\left(\sqrt{dx+1}-1\right)^8}}+\frac{a\,\sqrt{dx-1}\,\sqrt{dx+1}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.155 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{(2b+cx)\sqrt{-1+dx}\,\sqrt{1+dx}}{2d^2}+\frac{(c+2ad^2)\cosh^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arccosh(d*x)/d^3+1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/
d^2
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(52) = 104.
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {915, 1829, 655, 223, 212}

$$\frac{\sqrt{d^2x^2-1}\,(2ad^2+c)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
```

```
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((c+2ad^2)\sqrt{-1+d\right.}{2d^2\sqrt{-1+}}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left((c+2ad^2)\sqrt{-1+d\right.}{2}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{(c+2ad^2)\sqrt{-1+d^2}}{2d^3\sqrt{-1}}$$

Mathematica [A]
time = 0.18, size = 63, normalized size = 1.21

$$\frac{d(2b+cx)\sqrt{-1+dx}\,\sqrt{1+dx}+2(c+2ad^2)\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{2d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x] + 2*(c + 2*a*d^2)*ArcTanh[Sqrt[
(-1 + d*x)/(1 + d*x)]])/(2*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 120, normalized size = 2.31

method	result
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)dcx+2\ln\left(\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)a\,d^2+2\,\mathrm{csgn}(d)d\sqrt{d^2x^2-}\right.}{2d^3\sqrt{d^2x^2-1}}$
risch	$\frac{(cx+2b)\sqrt{dx-1}\,\sqrt{dx+1}}{2d^2}+\frac{\left(\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)a}{\sqrt{d^2}}+\frac{\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)c}{2d^2\sqrt{d^2}}\right)\sqrt{(dx+1)\,(dx-}}{\sqrt{dx-1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^3*((d^2*x^2-1)^(1/2)*csgn(d)*d*c*x+2*ln((
(d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*a*d^2+2*csgn(d)*d*(d^2*x^2-1)^(1/2)
*b+ln(((d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*c)/(d^2*x^2-1)^(1/2)*csgn(d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(44) = 88.
time = 0.33, size = 90, normalized size = 1.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,cx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

Fricas [A]
time = 1.21, size = 61, normalized size = 1.17

$$\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{dx-1}\,-(2\,ad^2+c)\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.69, size = 80, normalized size = 1.54

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\,(2\,ad^2+c)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B]
time = 12.40, size = 312, normalized size = 6.00

$$\frac{b\sqrt{dx-1}\,\sqrt{dx+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{dx-1}-\mathrm{i}}{\sqrt{dx+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{dx-1}-\mathrm{i}\right)}{\left(\sqrt{dx+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-\frac{\frac{14\,c\left(\sqrt{dx-1}-\mathrm{i}\right)^3}{\left(\sqrt{dx+1}-1\right)^3}+\frac{14\,c\left(\sqrt{dx-1}-\mathrm{i}\right)^5}{\left(\sqrt{dx+1}-1\right)^5}+\frac{2\,c\left(\sqrt{dx-1}-\mathrm{i}\right)^7}{\left(\sqrt{dx+1}-1\right)^7}+\frac{2\,c\left(\sqrt{dx-1}-\mathrm{i}\right)}{\sqrt{dx+1}-1}}{d^3-\frac{4\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+\frac{6\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^4}{\left(\sqrt{dx+1}-1\right)^4}-\frac{4\,d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^6}{\left(\sqrt{dx+1}-1\right)^6}+\frac{d^3\left(\sqrt{dx-1}-\mathrm{i}\right)^8}{\left(\sqrt{dx+1}-1\right)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.156 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}} dx$

Optimal. Leaf size=55

$$\frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+\frac{b\cosh^{-1}(dx)}{d}+a\tan^{-1}\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+c*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/d^2
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {1624, 1823, 858, 223, 212, 272, 65, 211}

$$\frac{a\sqrt{d^2x^2-1}\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}-\frac{c(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$
$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}}$$
$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-\right.}{}$$
$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x\right.}{2\sqrt{-1+dx}\,\sqrt{1+dx}}$$
$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\right.}{}$$
$$= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\,\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1}}{}$$

Mathematica [A]
time = 0.16, size = 69, normalized size = 1.25

$$\frac{c\sqrt{-1+dx}\,\sqrt{1+dx}}{d^2} + 2a\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2b\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + 2*a*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)
]] + (2*b*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 95, normalized size = 1.73

method	result
default	$\frac{\left(-\mathrm{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)c+\ln\left(\left(\sqrt{(dx+1)(dx-1)}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)bd\right)}{d^2\sqrt{d^2x^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*csgn(d)*c+ln(
(((d*x+1)*(d*x-1))^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Maxima [A]
time = 0.51, size = 56, normalized size = 1.02

$$-a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

Fricas [A]
time = 1.03, size = 73, normalized size = 1.33

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] Result contains complex when optimal does not.
time = 41.28, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
```

```
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [A]
time = 0.69, size = 71, normalized size = 1.29

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

Mupad [B]
time = 3.97, size = 118, normalized size = 2.15

$$\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-a\,\left(\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

3.157 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{x}+\frac{c\cosh^{-1}(dx)}{d}+b\tan^{-1}\left(\sqrt{-1+dx}\,\sqrt{1+dx}\right)$$

```
[Out] c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+a*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/x
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {1624, 1821, 858, 223, 212, 272, 65, 211}

$$-\frac{a(1-d^2x^2)}{x\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(c\sqrt{-1\right.}}{}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, \right.}{2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\,\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{\right.}}{}$$

$$= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{c\sqrt{-1}}{}$$

Mathematica [A]
time = 0.15, size = 69, normalized size = 1.25

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{x} + 2b\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2c\tanh^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + 2*b*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
 + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 96, normalized size = 1.75

method	result
risch	$\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{x} + \frac{\left(\frac{c\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)}{\sqrt{d^2}} - b\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$

default	$\frac{\left(-\mathrm{csgn}(d) d \arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right) bx+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d) da+\ln\left(\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right) cx\right)\sqrt{dx-1}}{\sqrt{d^2x^2-1}\,xd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*d*arctan(1/(d^2*x^2-1)^(1/2))*b*x+(d^2*x^2-1)^(1/2)*csgn(d)*d*a+l
n(((d^2*x^2-1)^(1/2)*csgn(d)+d*x)*csgn(d))*c*x)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
*csgn(d)/(d^2*x^2-1)^(1/2)/x/d
```

Maxima [A]
time = 0.53, size = 56, normalized size = 1.02

$$-b\arcsin\left(\frac{1}{d|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

Fricas [A]
time = 1.03, size = 82, normalized size = 1.49

$$\frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,ad-cx\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 40.18, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|{\frac{1}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|{\frac{e^{2i\pi}}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|{\frac{1}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|{\frac{e^{2i\pi}}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|{\frac{1}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|{\frac{e^{2i\pi}}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

Giac [A]
time = 1.13, size = 83, normalized size = 1.51

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)-\frac{8\,ad^{2}}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

Mupad [B]
time = 3.86, size = 118, normalized size = 2.15

$$\frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

3.158 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{2x^2}+\frac{b\sqrt{-1+dx}\,\sqrt{1+dx}}{x}+\frac{1}{2}(2c+ad^2)\tan^{-1}\left(\sqrt{-1+dx}\,\sqrt{1+dx}\right)$$

```
[Out] 1/2*(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/2*a*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1624, 1821, 821, 272, 65, 211}

$$\frac{\sqrt{d^2x^2-1}\,(ad^2+2c)\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{2x^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{x\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*(1 - d^2*x^2))/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*Arc
Tan[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+(2c+ad^2)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+\cdots}\right.}{2\sqrt{-1\cdots}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+\cdots}\right.}{\cdots}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+\cdots}\right.}{\cdots}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(2c+ad^2\right)\sqrt{-1+\cdots}}{2\sqrt{-1\cdots}}$$

Mathematica [A]
time = 0.14, size = 60, normalized size = 0.72

$$\frac{(a+2bx)\sqrt{-1+dx}\,\sqrt{1+dx}}{2x^2}+\left(2c+ad^2\right)\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (2*c + a*d^2)*ArcTan[S
qrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 103, normalized size = 1.24

method	result
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(2bx+a)}{2x^2}+\frac{\left(-\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)c-\frac{\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2}{2}\right)\sqrt{(dx+1)\,(dx-1}}{\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^2\left(\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2x^2+2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)c\,x^2-2\sqrt{d^2x^2-1}\,bx-\right.}{2\sqrt{d^2x^2-1}\,x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctan(1/(d^2*x^2-1)^(1/2))*a*d
^2*x^2+2*arctan(1/(d^2*x^2-1)^(1/2))*c*x^2-2*(d^2*x^2-1)^(1/2)*b*x-(d^2*x^2
-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^2
```

Maxima [A]
time = 0.50, size = 61, normalized size = 0.73

$$-\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right)-c\arcsin\left(\frac{1}{d|x|}\right)+\frac{\sqrt{d^2x^2-1}\,b}{x}+\frac{\sqrt{d^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Fricas [**A**]
time = 1.06, size = 69, normalized size = 0.83

$$\frac{2\,bdx^2+2\,(ad^2+2\,c)x^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\,\sqrt{dx-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(67) = 134.
time = 1.35, size = 145, normalized size = 1.75

$$-\frac{(ad^3+2\,cd)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-16\,bd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

Mupad [**B**]
time = 9.89, size = 316, normalized size = 3.81

$$\frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}}-c\,\left(\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}-\frac{a\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.159 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{a\sqrt{-1+dx}\,\sqrt{1+dx}}{3x^3}+\frac{b\sqrt{-1+dx}\,\sqrt{1+dx}}{2x^2}+\frac{(3c+2ad^2)\,\sqrt{-1+dx}\,\sqrt{1+dx}}{3x}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{-1+dx}\right.$$

```
[Out] 1/2*b*d^2*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/3*a*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+1/3*(2*a*d^2+3*c)*(d*x-1)^(1
/2)*(d*x+1)^(1/2)/x
```

Rubi [A]
time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1624, 1821, 849, 821, 272, 65, 211}

$$-\frac{(1-d^2x^2)\,(2ad^2+3c)}{3x\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{3x^3\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\,\text{ArcTan}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(a*(1 - d^2*x^2))/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(
3*x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-
1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+(3c+2ad^2)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2(}{x^2}}{6\sqrt{-1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{1}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{1}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{1}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-}{3x\sqrt{-1+dx}\,\sqrt{1}}$$

Mathematica [A]
time = 0.21, size = 71, normalized size = 0.61

$$\frac{\sqrt{-1+dx}\,\sqrt{1+dx}\,(3x(b+2cx)+a(2+4d^2x^2))}{6x^3} + bd^2\tan^{-1}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/(6*x^3
) + b*d^2*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 123, normalized size = 1.06

method	result
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(4a\,d^2x^2+6c\,x^2+3bx+2a)}{6x^3} - \frac{b\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\sqrt{(dx+1)(dx-1)}}{2\sqrt{dx-1}\,\sqrt{dx+1}}$

default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^{2}\left(3\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)b\,d^{2}x^{3}-4\sqrt{d^{2}x^{2}-1}\,a\,d^{2}x^{2}-6\sqrt{d^{2}x^{2}-1}\,c\,x^{2}-3\sqrt{d^{2}}\right.}{6\sqrt{d^{2}x^{2}-1}\,x^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(3*arctan(1/(d^2*x^2-1)^(1/2))*b
*d^2*x^3-4*(d^2*x^2-1)^(1/2)*a*d^2*x^2-6*(d^2*x^2-1)^(1/2)*c*x^2-3*(d^2*x^2
-1)^(1/2)*b*x-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3
```

Maxima [A]
time = 0.51, size = 86, normalized size = 0.74

$$-\frac{1}{2}\,bd^{2}\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\sqrt{d^{2}x^{2}-1}\,ad^{2}}{3\,x}+\frac{\sqrt{d^{2}x^{2}-1}\,c}{x}+\frac{\sqrt{d^{2}x^{2}-1}\,b}{2\,x^{2}}+\frac{\sqrt{d^{2}x^{2}-1}\,a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

Fricas [A]
time = 1.26, size = 90, normalized size = 0.78

$$\frac{6\,bd^{2}x^{3}\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+2\,(2\,ad^{3}+3\,cd)x^{3}+(2\,(2\,ad^{2}+3\,c)x^{2}+3\,bx+2\,a)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(92) = 184.
time = 2.04, size = 197, normalized size = 1.70

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^8-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-96\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-48\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-128\,ad^4-192\,cd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

Mupad [B]
time = 9.44, size = 304, normalized size = 2.62

$$\frac{\frac{bd^2\,1\mathrm{i}}{32}+\frac{bd^2\left(\sqrt{dx-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\left(\sqrt{dx+1}-1\right)^2}-\frac{bd^2\left(\sqrt{dx-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\left(\sqrt{dx+1}-1\right)^4}}{\frac{\left(\sqrt{dx-1}-\mathrm{i}\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+\frac{2\left(\sqrt{dx-1}-\mathrm{i}\right)^4}{\left(\sqrt{dx+1}-1\right)^4}+\frac{\left(\sqrt{dx-1}-\mathrm{i}\right)^6}{\left(\sqrt{dx+1}-1\right)^6}}-\frac{bd^2\ln\left(\frac{\left(\sqrt{dx-1}-\mathrm{i}\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)1\mathrm{i}}{2}+\frac{bd^2\ln\left(\frac{\sqrt{dx-1}-\mathrm{i}}{\sqrt{dx+1}-1}\right)1\mathrm{i}}{2}+\frac{c\sqrt{dx-1}\,\sqrt{dx+1}}{x}+\frac{\sqrt{dx-1}\,\left(\frac{2ad^3x^3}{3}+\frac{2ad^2x^2}{3}+\frac{adx}{3}+\frac{a}{3}\right)}{x^3\sqrt{dx+1}}+\frac{bd^2\left(\sqrt{dx-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\left(\sqrt{dx+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```